Multi-Objective Optimization for Online Train Trajectory Planning with Moving Window Method

Zhiyu He*, Yinan Li, Hui Li, Ning Xu

***Abstract*—Optimization on trajectory planning is significant offering the driver or automatic train operation system a guidance to drive the train efficiently. An optimal trajectory is subject to operational, geographic, physical and dynamic constraints. In previous studies, researchers have put much effort into offline optimization. However, operational errors in real situation are rarely taken into consideration. Thus, this paper proposes a dynamic trajectory planning to address the deviation. Specifically, first, we design an online optimization framework to display model predictive control-based train trajectory planning theory. Various constraints are set for optimization problem. Second, taking energy consumption and punctuality into consideration, we propose a moving window method to search the optimal variables, which are applied to evaluate the fitness function. Plus, to improve global search ability and convergence rate, we present a multi-swarm particle swarm optimization by dividing the population into several parts. On the basis of operational time error at reallocation positions, we design a novel weight allocation mechanism for the revaluation of fitness function. At last, taking real data from Beijing-Shanghai high-speed railway as an example, the robustness and effectiveness of the proposed algorithm is proved by comparing multiple parameters with some numerical results of simulations. The results show that the proposed method can adjust operating strategy dynamically so that the train runs in an energy-efficient way. Additionally, in delay scene, the proposed method can also guide the train to catch up with the scheduled time and save energy consumption.**

***Index Terms*—Model predictive control, dynamic trajectory planning, high-speed train, evolutionary algorithm, energy consumption**

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I. INTRODUCTION

High-speed train of China has become a country representative in recent years because of its safety, speed, comfort, punctuality and environmental friendly way[1]. Generally speaking, high-speed train is operated by centralized controlling from Train Control Center (TCC) or Radio Block Center (RBC), and automatically formed trajectory planning by knowing the train position, speed and other state data. However, operational errors are inevitable, and may accumulate to cause irretrievable consequences. Thus, automatic train operation (ATO) has become a trend for high-speed railway development in the future [2-3]. It is an intelligent automatic control system which requests traction/braking force in order to track the reference profile precisely. The reference profile is calculated by energy consumption and punctuality. Therefore, energy-efficient driving is critical for high-speed trains to operate optimally.

The study on energy-saving optimization can be originated to 1960s when Ichikawa started to deal with the optimization theory in the operation of train [4]. Since then, a number of researchers and scholars have paid attention to save traction energy for electric trains [5-9]. Some auxiliary systems, such as FreightMiser, MetroMiser and driving style manager, have been studied to assist drivers to operate more efficiently [10]. In 1980, Milroy reformulated the dynamic model, which facilitates the train optimal control theory [11]. Based on this model, Howlett proved that an optimal strategy exists and must satisfy a Pontryagin type criterion [12]. So in early relevant studies, the control variables are considered as continuous inputs at the beginning for simplicity [13-14]. However, the control input of most locomotives in the world is discrete and traction/braking force cannot be continuous. Rodrigo introduced a semi analytical discretized solution and applied the Lagrange multipliers approach to realize the optimization of n-tuples of speed [15].

In recent years, because of the increasing computational calculating speed and accuracy, more and more numerical algorithms have been brought up, such as genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), differential evolution (DE) and so on. Han et al. applied GA to calculate the optimal train trajectory considering non-constant gradients, curves and speed limits [16]. Dynamic programming (DP) was able to deal with complicated actual train running preconditions, e.g. local constraints and variable grade profile, and to search the optimal speed code sequence to save operational energy

[17-18]. Dominguez designed a GA to regulate an train operational profile to minimize the traction consumption by taking the storage technique of regenerative power into consideration[19-20]. Yang introduced a cooperative scheduling theory to optimize the timetable generation, and fully considered recovery energy by coinciding the accelerating and braking time of neighboring trains [21]. Shangguan proposed a multi-objective optimization method with hybrid evolutionary algorithm[22]. Li established a hybrid system model to present the new characteristics of high-speed electric trains, and a minimal-energy driving strategy is obtained[23].

Due to high-density operating vehicles, short headway time, coupled with frequently changing speed restrictions and geographical conditions， the railway operating network has been turned to a time-varying non-linear system. Consequently, train trajectory regulation is a dynamic procedure essentially. Ke presented a new optimization method based on the MAX-MIN ant system algorithm, which made the computational time reduced to an acceptable level for online calculation[24]. Song utilized a dual speed curve optimization method with the information of line slope to reduce energy consumption[25]. Yan presented a state-space mode for train operation and MPC based online planning strategy to obtain real-time driving trajectory[26]. Due to the long distance between successive stations in China, plus the weather, crosswind and other unknown disturbances, the resistance on the train is dynamic. As a result, driving trajectory will deviate from scheduled profile causing unexpected consequences as time delay or excessive energy consumption. However, existing research rarely takes in account the time-varying parameters and random disturbances of high-speed trains for online trajectory planning.

In this paper, dynamic trajectory planning for high-speed trains is proposed. Different from offline trajectory optimization in previous studies, uncertain operational error in real operating situation is taken into account. Given operating information, real-time train trajectory planning is presented for energy-efficient and punctual driving. We exploit model predictive control (MPC)-based evolutionary theory to search the optimal solution and output reasonable control variables for high-speed trains. The salient features are summarized as follows.

(1) MPC-based theory is introduced to adjust optimal operating strategy and dynamically regulate train operating trajectory.

(2) A novel weight allocation mechanism for objective function is presented to adjust to real operating situation based on time error evaluation.

(3) Passenger comfort parameter is taken into evaluation of the optimal trajectory planning.

(4) In each moving window optimization process, MSPSO is utilized to obtain the optimal strategy addressing energy-saving function and punctuality.

The rest of paper is organized as follows. In Section 2, we formulate the train dynamic and energy consumption model. Then, we introduce discretization method of trip route. In Section 3, we take into account real operating constraints and propose moving window optimization approach for dynamic train trajectory planning. In Section 4, we present optimization approaches including weight allocation of objective function and MSPSO algorithm. In Section 5, with real data of Beijing-Shanghai High-speed Railway, simulation experiments are carried out to verify the effectiveness of the proposed approach. In Section 6, conclusions are presented.

II. PREPARATION

A. Parameter declaration

For clarification of following description, some basic parameters are declared first.

TABLE I
PARAMETERS

Symbol	Description
M	train mass, t
s	train running distance, m
v	train operating speed, m/s
$F_0(v)$	basic resistance, kN
$W(v)$	additional resistance, kN
N	horizon window
X_0	starting point
X_d	destination point
k	kth moving window
L	the length between successive stations, m
T_d	trip time at the timetable, s
L_u	the length of a simulation unit, m

[a]t = ton, m = meter, s = second, kN = kilo newton.

B. Fundamental train model

Multiple elements will influence the movement of high-speed trains, involving speed limits, operating resistance and so on. The train dynamic model can be formulated as

$$\begin{cases} \dot{s} = v \\ M(1+\gamma)\dot{v} = u(t) - F_0(v) - W_0(v) \end{cases} \tag{1}$$

where γ denotes the rotary mass coefficient, which is 0.06. $u(t) \in [-u_{b,\max}, u_{t,\max}]$, $u_{b,\max}$, $u_{t,\max}$ are the maximum traction and braking force, respectively. $u(t)$ represents the output value of power supply system, which determines the train to accelerate or decelerate. In the arithmetic expression, the value of $u(t)$ is positive in traction phase while negative in braking phase.

Thus, we can conclude that train trajectory planning strongly depends on the distance between successive stations and a set of control variables Ψ_k. In an optimization horizon N, the control variables can be formulated as

$$\Psi_k \left\{u_k^*, u_{k+1}^*, ..., u_{k+N-2}^*, u_{k+N-1}^*\right\} \tag{2}$$

where u_k^* denotes as the output control at subspace k. Note that there are many feasible plans for optimal driving trajectory, which could result in different operating time and energy consumption. It should be noticed that for subspace k, only u_k^* is applied for practical use.

$F_0(v)$ is the basic resistance, including rolling resistance, mechanical resistance and air resistance. For modern electric trains, air resistance increases tremendously with growing operating speed. The expression of the basic resistance can be described as

$$F_0(v)=\left(a_0+a_1v+a_2v^2\right)\cdot Mg\cdot 10^{-3} \tag{3}$$

Where a_0, a_1, a_2 are resistance coefficients, which may jitter due to weather, component wear, crosswind and other vehicle-related situation. Regarding to dynamic trajectory planning, the resistance coefficients should be considered as variables rather than constants.

W(v) is denoted as additional resistance consisting of gradient, curve and tunnel. Unlike train automatic control subsystem, train trajectory planning subsystem regards the vehicle as a mass particle for model simplicity and speed calculation. So the additional resistance performs mutability instead of scale. It can be expressed as

$$W(v)=\left(f_g+f_c+f_t\right)\cdot Mg\cdot 10^{-3} \tag{4}$$

where f_g, f_c, f_t are the gradient, curve, tunnel resistance, respectively.

C. Energy Consumption Model Formulation

The energy consumption of high-speed train is mainly ascribed to traction force, which accounts for 80% of the total amount [26]. For each subspace k, the energy consumption of horizon N is related to the output power and operating time. The expression can be formulated as

$$E(\Psi_k)=\int_{t_k}^{t_{k+N}}\eta u(t)v(t)dt \tag{5}$$

where η is described as

$$\eta=\begin{cases}1, & u(t)\geq 0\\ \mu, & u(t)<0\end{cases} \tag{6}$$

where $\mu\in[0,1)$ denotes the regenerative efficiency.

Since the only practical output force is the first optimization value, written as u_k^1, with a total number of n output values, the combined energy consumption can be presented as

$$E_{\text{total}}=\sum_{k=1}^{n}\int_{t_k}^{t_{k+1}}\eta u_k^1(t)v(t)dt \tag{7}$$

D. Discretization Analysis

The basis of this paper is route discretization depending on the geographical conditions and speed constraints. As shown in Fig.1, the whole line is divided into *P* parts, namely subsections. Each shares the same gradient, curve, tunnel and speed limit. Then, a subsection is separated into smaller units for moving horizon optimization.

$$L=\left\{L_{s,1},L_{s,2},\ldots,L_{s,P-1},L_{s,P}\right\} \tag{8}$$

In order to meet the dynamic planning requirement and adjust to random disturbances, the length of an optimization unit needs to be small and the value is set as 100m. Thus, the parametric adjustment can be more precise and timely in the optimizing procedure. In terms of the operation in each unit, we assume the acceleration rate is seen as a constant approximately.

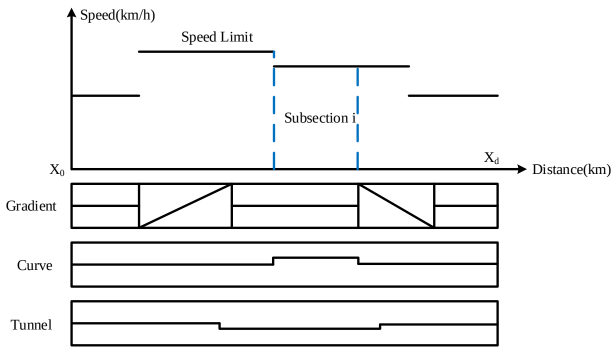

Fig.1. Route discretization on gradient, curve, tunnel and speed limit, X_0 and X_d denote the starting and destination point, respectively.

III. Optimization Model Establishment

A. Optimization Flow Design

As the essential principle of dynamic train trajectory planning in this paper, model predictive control (MPC) relies on the power supply system, positioning system and communication system. The diagram of the optimization process is described in Fig.2.

As shown in Fig.2, the dynamic trajectory optimization process consists of passenger comfort analyzer, fitness function weight allocation and dynamic speed constraints calculation. Passenger comfort analyzer ensures real-time calculation of train operating comfort. Weight allocation is the balance between energy consumption and punctuality. The various constraints set the boundary for train trajectory regulation and operation. The distance-based unit for moving horizon optimization and dynamic speed constraints calculation is set as 100m. Note that the weight allocation refreshing period is set to be 9000m.

B. Constraints

Given the optimal train trajectory planning in this paper,

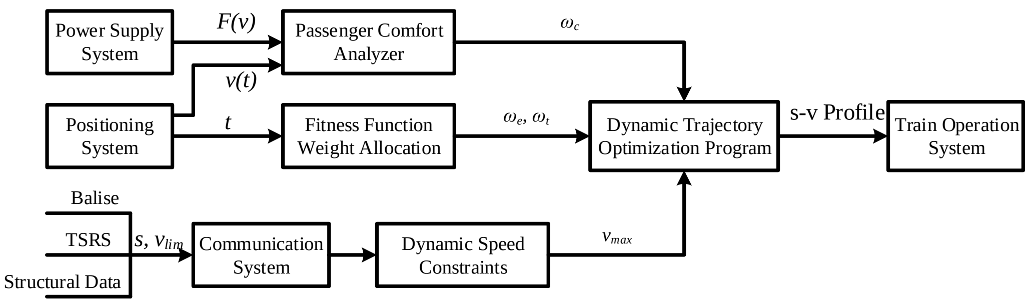

Fig.2. Optimization procedure flow, TSRS is the temporary speed restriction service.

the train operates under various constraints, such as the distance between successive stations, the trip time on the timetable, passenger comfort and multiple dynamic speed restrictions.

The distance-based constraints can be described as

$$X(0)=0,\ \ X(n)=L \tag{9}$$

where $X(0)$ denotes the starting point. $X(n)$ denotes the destination point.

The trip route is separated into n parts for optimization, and each represents a basic unit x_i, which is, as mentioned above, set the length of 100m.

The train should reach the destination at a fixed time according to the timetable. The operating time of any feasible driving strategy is an essential criterion for objective function calculation.

$$T(n)-T(0)\in\left[T_d-\Delta T_0, T_d+\Delta T_0\right] \tag{10}$$

where $T(0)$ denotes the starting time and $T(n)$ denotes the finishing time. ΔT_0 represents the allowed time error.

Dynamic speed constraints are the primary influence factors for online train trajectory planning. With the increasing operating speed, the train should be controlled under a certain restriction at each unit for safety. It is generated mainly by calculating the speed limit from the train ahead, speed range according to the power supply system and temporary speed limit.

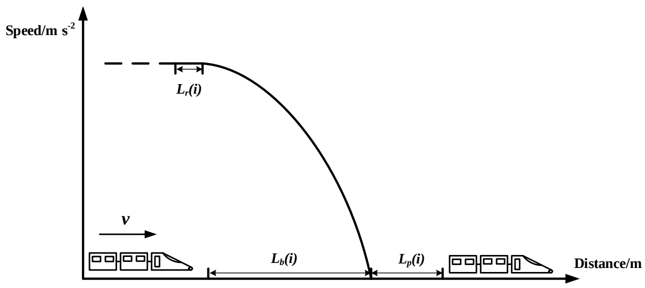

Fig.3. Safety interval between successive trains

(1) Speed constraint from the train ahead

In moving block system, the operating speed is dynamically affected by the forward train. As shown in Fig.3, for safety, the minimum tracking interval can be defined as

$$L(i)=L_r(i)+L_b(i)+L_p(i) \tag{11}$$

where $L_r(i)$ denotes the reflecting distance of ATO or driver. $L_b(i)$ denotes the braking distance when outputting service braking (SB). $L_p(i)$ denotes the reserved distance for measurement error and unknown disturbances.

$L_b(i)$ can be calculated by train current speed and preset deceleration rate, which is generated by braking control unit (BCU) in coordination of the proportion of electric braking and air braking.

$$L_b(i)=v^2(i)/2a_b \tag{12}$$

where $v(i)$ is the current speed. a_b is the preset deceleration rate of SB.

Therefore, the dynamic limited operating speed from forward train at ith unit $v_{ft,\lim}(i)$ can be expressed as

$$v_{ft,\lim}(i)=\left(2a_b\cdot\left(D(i)-L_r(i)-L_p(i)\right)\right)^{1/2} \tag{13}$$

where $D(i)$ denotes the actual distance between two successive trains.

(2) Speed constraint from power supply system

For certain high-speed train category, the output power should follow tractive effort profile. At each optimization unit, the speed is confined within a pre-calculated range, as shown in Fig.4. In view of geographical conditions and operating resistance at unit i, the maximum speed $v_{ps,\max}(i)$ is figured by full traction and the minimum speed $v_{ps,\min}(i)$ is figured by coast phase.

The limited speed range of unit i is presented as

$$v_l(i)\in\left[v_{ps,\min}(i), v_{ps,\max}(i)\right] \tag{14}$$

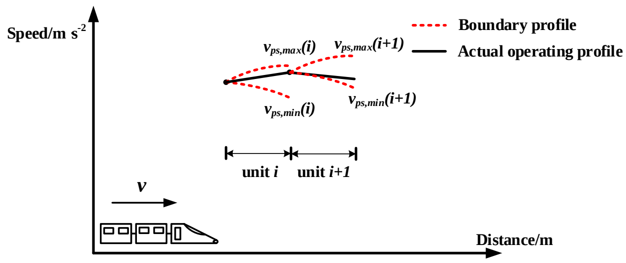

Fig.4. Speed constraints from power supply system

(3) Speed constraint from temporary speed limit

The information of temporary speed limit is transmitted by TSRS, due to track construction, weather change and other track-related conditions. We use $v_{ts,\max}(i)$ to represent the speed limit from TSRS.

Finally, we can draw the maximum speed at unit i by

$$v_{\max}(i)=\min\left\{v_{ft,\max}(i), v_{ps,\max}(i), v_{ts,\max}(i)\right\} \tag{15}$$

C. *Objective Function Establishment*

In this paper, the performance of optimization results are evaluated in terms of punctuality and energy consumption.

For subspace k in optimization horizon N, each planning trajectory may correspond to a different operating time. The planning trip time at section k is denoted as $T_{k+N|k}(\Psi_k)$, and the schedule time according to reference profile is T_k^*. The time error in optimization horizon N is expressed as

$$\Delta T(\Psi_k)=T_{k+N|k}(\Psi_k)-T_k^* \tag{16}$$

Note that ΔT_0 is the allowed time error. Then we can conclude the normalized punctuality objective function $\Phi_t(\Psi_k)$ as

$$\Phi_t(\Psi_k)=\left(\Delta T_{k+N|k}^{\max}-\Delta T(\Psi_k)\right)\Big/\left(\Delta T_{k+N|k}^{\max}-\Delta T_{k+N|k}^{\min}\right) \tag{17}$$

where $\Delta T_{k+N|k}^{\max}$ denotes the difference between maximum trip time and the scheduled time in horizon N at subspace k; $\Delta T_{k+N|k}^{\min}$ denotes the difference between maximum trip time and the scheduled time in horizon N at subspace k.

Regarding the energy consumption of subspace k in

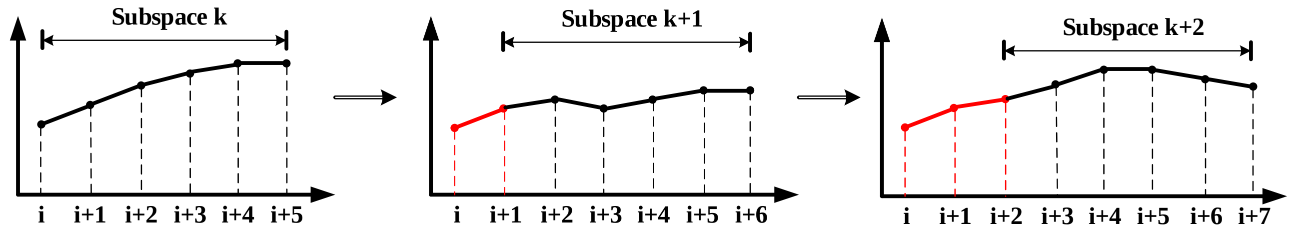

Fig.5. Moving horizon optimization process, when N=5

optimization horizon N, the value at section (k+N) is denoted as $E_{k+N|k}(\Psi_k)$. The normalized energy consumption objective function $\Phi_e(\Psi_k)$ can be expressed as

$$\Phi_e(\Psi_k)=\left(E_{k+N|k}^{\max}-E_{k+N|k}(\Psi_k)\right)\Big/\left(E_{k+N|k}^{\max}-E_{k+N|k}^{\min}\right) \quad (18)$$

where $E_{k+N|k}^{\max}$ and $E_{k+N|k}^{\min}$ are the maximum and minimum energy consumption in horizon N at subspace k, respectively.

Based on the discussion above, we can design the objective function as follows, targeting at achieving the maximum value,

$$\Phi(\Psi_k)=\omega_t(\Psi_k)\Phi_t(\Psi_k)+\omega_e(\Psi_k)\Phi_e(\Psi_k) \quad (19)$$

where $\omega_t(\Psi_k)$ and $\omega_e(\Psi_k)$ denote the weight of trip time, energy consumption and passenger comfort, respectively. $\omega_t(\Psi_k)+\omega_e(\Psi_k)=1$.

D. Moving Horizon Optimization

In this paper, moving horizon theory is applied to search the optimal control variables Ψ_k^* at subspace k step by step based on $\Phi(\Psi_k)$. As shown in Fig.5, the primary process of moving horizon optimization is presented. There are multiple feasible solutions for each subspace, and only the first optimized variable is practical for train operation. At subspace k, the optimal driving profile is calculated according to the objective function $\Phi(\Psi_k)$, and the optimal control variable $u^*(i)$ at unit i is obtained. Then the optimization horizon moves forward to subspace k+1, and the driving profile is recalculated according to $\Phi(\Psi_{k+1})$. In this way, an energy-efficient operation trajectory for high-speed train is generated.

For online train trajectory planning, the performance of objective function and the delay caused by calculation are both critical. Better objective function value ensures more energy-saving driving trajectory. Shorter computational time ensures the less deviation from the planned profile. Thus, the value of optimization horizon N is worth discussing.

In conclusion, based on the various constraints and physical parameters, the optimization function of subspace k can be formulated as follows:

$$\begin{aligned}
&\max \quad \Phi(\Psi_k)\\
&s.t. \quad X(0)=0,\ X(n)=L\\
&\qquad T(n)-T(0)\in\left[T_d-\Delta T_0,\, T_d+\Delta T_0\right]\\
&\qquad v(0)=v(n)=0,\ v(i)\in\left[v_{\min}(i),v_{\max}(i)\right]\\
&\qquad \omega_t(\Psi_k)+\omega_e(\Psi_k)+\omega_c(\Psi_k)=1\\
&\qquad \omega_t(\Psi_k),\omega_e(\Psi_k),\omega_c(\Psi_k)\in(0,1)
\end{aligned}$$

IV. Algorithm Design

In this section, some detailed parametric calculation and the dynamic optimization programming are presented.

A. Weight Allocation Mechanism

To deal with the contradiction between punctuality and energy consumption, an adaptive weight allocation mechanism is proposed in this section. Since the long distance of Chinese high-speed lines, he operating time delay is regarded as the primary criterion for weight calculation.

$$\omega_t=\begin{cases}\dfrac{1}{2}e^{\left(\Delta T^*(i)/\Delta T-1/2\right)}, & \Delta T^*(i)<0\\[2mm] 1-\dfrac{1}{2}e^{\left(-\Delta T^*(i)/\Delta T+1/2\right)}, & \Delta T^*(i)\geq 0\end{cases} \quad (20)$$

where $\Delta T^*(i)=T(i)-T_s(i)$, denotes the difference of current operating time from the scheduled time.

As shown in Fig.6, when the train arrives at the weight reallocated point, the proportion of punctuality and energy consumption will be recalculated. Note that punctuality has a negative correlation with energy consumption. Thus within allowed time error range, the train should extend operating time to save energy. Additionally, in each optimization horizon k, if the weight of punctuality is close to or higher than the one of energy consumption, the train tends to drive in “traction-coast” mode in the performance of objective function. However, since only the first control variable is available for real operating, the train will accelerate to a high speed at the next weight reallocated point. It means the train is more likely to deviate from scheduled timetable and cost more energy consumption. Therefore, we modify the weight allocation profile and set the balanced time error is $\Delta T/2$.

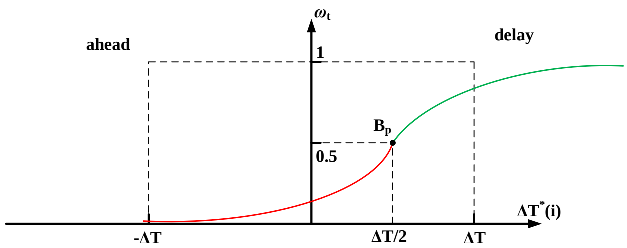

Fig.6. Weight allocation mechanism

B. Multi-swarm Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) is an easy and effective evolutionary algorithm proposed by Eberhart and Kennedy[28]. By generating a population of candidate particles randomly, PSO moves these particles around the iteratively due to the fitness function over the particles’ position and velocity. Each particle's movement is affected by both local best position and global best known position in the search-space in the meantime. MSPSO is a modified PSO algorithm by separating the original population into several parts to increase population diversity and global searching ability. When the distance between particles in sub-populations is less than a fixed value, they combine into one. Step by step, the individual particles finally join into one population as initial. This combination process is applied to avoid the local optimum effectively.

Regarding the dynamic train trajectory planning at subspace k in horizon N, the optimizing procedure can be described as follows:

Step 1: Generate the initial particles randomly based on the target speed at subspace k. In the proposed optimization model, the population can be expressed as

$$\mathrm{P}_{d,c} = \left\{ v_{k,c}, v_{k+1,c}, ..., v_{k+N-1,c} \right\} \tag{21}$$

where $d = \{1, 2, ..., D\}$ denotes the index of individual in current population. $c = \{1, 2, ..., c_m\}$ denotes the iteration time of current population.

The initial population can be expressed as

$$\mathrm{P}_{0,c} = \left\{ \mathrm{P}_{d,c} \mid d = 1, 2, ..., D \right\} \tag{22}$$

Step 2: Separate the population into several parts depending on the affinities. The description of sub-populations can be presented as

$$\mathrm{P}_{d,c} = \left\{ \mathrm{P}^1_{d,c}, \mathrm{P}^2_{d,c}, ..., \mathrm{P}^P_{d,c} \right\} \tag{23}$$

where P denotes the number of sub-population.

The criterion of affinities can be defined as

$$A\left(\mathrm{P}_{1d,c}, \mathrm{P}_{2d,c}\right) = \left\| \mathrm{P}_{1d,c} - \mathrm{P}_{2d,c} \right\| \le \delta \tag{24}$$

Step 3: Calculate the fitness value according to objective function. For each sub-population, we choose the best position of individual $X^m_{pbest,c}$ and population $X^m_{gbest,c}$. m denotes the index of current sub-population.

Step 4: Update the rest of individuals in the sub-population according to

$$\begin{cases} V^m_{d,c+1} = \omega^m_c V^m_{d,c} + \mu^m_1 r^m_1 \left(X^m_{pbest,c} - X^m_{d,c} \right) + \mu^m_2 r^m_2 \left(X^m_{gbest,c} - X^m_{d,c} \right) \\ X^m_{d,c+1} = X^m_{d,c} + V^m_{d,c+1} \end{cases} \tag{25}$$

where ω^m_c denotes the inertia coefficient. μ^m_1, μ^m_2 are preset constants. $r_1, r_2 \sim U(0,1)$. $V^m_{d,c}$, $V^m_{d,c+1}$ denote the moving speed of current particle and updated particle, respectively. $X^m_{d,c}$, $X^m_{d,c+1}$ are the current position and updated position of the particle.

ω^m_c is a linear time-varying coefficient as

$$\omega^m_c = \omega^m_{c,\max} - \left(\omega^m_{c,\max} - \omega^m_{c,\min} \right) \frac{c}{c_m} \tag{26}$$

where $\omega^m_{c,\max}$, $\omega^m_{c,\min}$ denote the maximum and minimum value of ω^m_c.

Step 5: Determine whether the combination of subpopulations can proceed. If the iteration time reaches the maximum value or the affinities of neighbored sub-population is less than δ, we should unite the sub-populations as one. If not, return to step 3.

Step: 6: Initialize the iteration time c and the index of sub-populations. Return to step 3.

Step 7: When the number of sub-population is 1 and the best position of current population is no longer change, or the iteration time reaches c_m, the procedure will stop. If not, execute step 3-4.

V. Sample Simulation

In this part, part of real line data about Beijing-Shanghai High-speed Railway is taken into simulation to verify the effectiveness of proposed algorithm. The total length is 65km and the scheduled time is 18min. We simulate the program on a PC (2.6GHz processor speed and 4GB memory size) with Windows 10 platform. The simulated software is Matlab2016b.

In the simulation process, we apply the CRH-3 high-speed train and the parameters are shown in Table II. Additionally, the parameters of the searching program are shown in Table III.

TABLE II
PARAMETERS FOR CRH-3

Symbol	Quantity
M	536t
γ	0.06
$u_{t,\mathrm{m}}$	300kN
$u_{b,\mathrm{m}}$	300kN
$P_{t,\mathrm{m}}$	8800kW
$P_{b,\mathrm{m}}$	8000kW
$F_0(v)$	$0.79+0.0064v+0.000115v^2$

TABLE III
PARAMETERS FOR MSPSO ALGORITHM

Symbol	Quantity
D	100
u^m_1, u^m_2	1.42
$\omega^m_{c,\max}$	0.9
$\omega^m_{c,\min}$	0.4
δ	0.06
L_u	100m

A. Verification of proposed Algorithm

To expound the performance of proposed algorithm in dynamic train trajectory planning, we take real data as mentioned into simulation. For simplicity, the resistance coefficients we apply are a_1=0.79, a_2=0.0064, a_3=0.000115, the initialized weight allocation $\omega_t = 0.5$, $\omega_e = 0.5$, and $N = 30$, $c_m = 100$. The static speed limit is set as

$$v_l = \begin{cases} 80\text{km/h}, \ 0\text{km} \le s < 2\text{km} \\ 200\text{km/h}, \ 2\text{km} \le s < 5\text{km} \\ 300\text{km/h}, \ 5\text{km} \le s < 60\text{km} \\ 200\text{km/h}, \ 60\text{km} \le s \le 65\text{km} \end{cases} \tag{27}$$

Fig.7 shows the result of dynamic train trajectory planning under certain constraints. We can see from the speed profile that after acceleration at 35km, the dynamic optimization result guides the train to coast or uses low tractive effort to save energy consumption. Considering regenerative mechanism, the energy consumption of the whole trip adds up

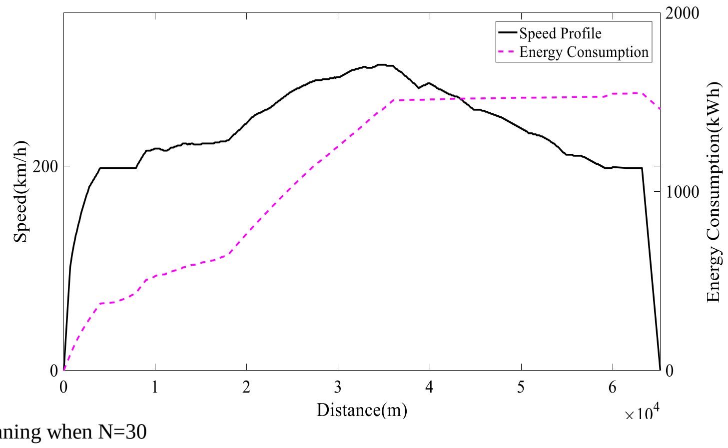

Fig.7. Dynamic train trajectory planning when N=30

to 1458 kWh and the punctuality is 20 seconds. Compared to the regular operating strategy of "traction-cruising-braking", the proposed method saves more than (1710-1458)/1710=14.7% energy consumption.

B. Evaluation of Weight Allocation Mechanism

To demonstrate the rationality of the balanced weight allocation point set, we compare different balanced points with the proposed mechanism. We can tell from Fig.8 that the basic optimized energy-efficient driving of high- speed trains is "traction-coast" mode, so as in each optimization horizon. Note that the balanced weight allocation point set represents the sensitive of train operation mode conversion. The performance mapping to the optimization algorithm is when the proportion of punctuality is larger, the train shall coast or use low tractive effort to save energy consumption, and vice versa.

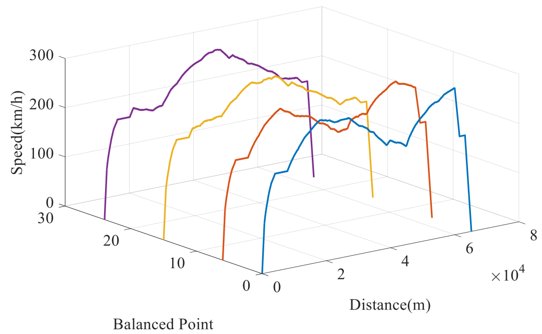

Fig.8. The planned trajectories of different balanced points

TABLE IV
SIMULATION RESULTS OF DIFFERENT BALANCED POINTS

Balanced Point	EC(kWh)	Punctuality(s)
$(0, 0.5)$	1790	-19
$(0.2\cdot\Delta T, 0.5)$	1698	-13
$(0.5\cdot\Delta T, 0.5)$	1458	20
$(0.8\cdot\Delta T, 0.5)$	1453	37

Table IV illustrates the punctuality and energy consumption of different balanced points. The varied balanced points are set as $B_{p1}=(0, 0.5)$, $B_{p2}=(0.2\cdot\Delta T, 0.5)$, $B_{p3}=(0.5\cdot\Delta T, 0.5)$, $B_{p4}=(0.8\cdot\Delta T, 0.5)$. The allowed time error ΔT is 30s. The initial weight allocation is $\omega_t = 0.5$, $\omega_e = 0.5$, and $N = 30$. Note that EC denotes the energy consumption of the whole trip. We can see from Table III that when the value of balanced point is less than $0.5\cdot\Delta T$, the train consumes more electric power and takes up less operating time. However, when the value of balanced point is larger than $0.5\cdot\Delta T$, the train consumes less energy but the punctuality is beyond allowed time error range. The reason is that the train costs too much time in coast mode and the operating time is way behind the scheduled time. Thus the train has to use full traction (See Fig.8) to catch up the time loss and it makes EC increases dramatically before braking point.

C. Analysis of Optimization Parameters

In this section, we will analyze the influence of varied optimization parameters on dynamic train trajectory planning procedure.

Example C.1: The value of weight reallocation period L_{period} affects not only the switch frequency of weight value, but the performance of energy-saving planning. If the period is too short, the weight value updates frequently and it will bring unexpected burden of on-board devices. However, if the period is too long, the train operation will deviate from scheduled time and cost extra electric power. Therefore, we aim to find an optimal L_{period} by conducting some simulation experiments under certain constraints with $\Delta T = 30$. Fig. 9 describes the planned train trajectories of different reallocation period and the numerical results are shown in Table V.

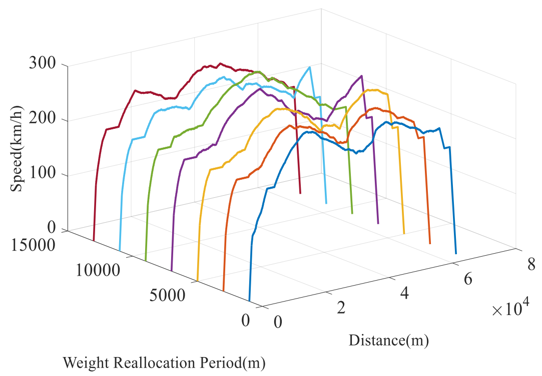

Fig.9. The planned trajectories of varied weight reallocation periods

TABLE V
SIMULATION RESULTS OF DIFFERENT PERIODS

Period(m)	EC(kWh)	Punctuality(s)
1000	1556	11
3000	1512	10
5000	1609	3
7000	1613	13
9000	1460	20
11000	1508	21
13000	1624	15

As shown in Fig.9, we can see that when L_{period} =1000m, the weight value updates frequently and the number of traction stage N_t is three. With the increase of L_{period} , N_t decreases. Note that every period of N_t changing denotes a transition period. We can conclude from Table 4 that in each period, the value of EC rises first, and then falls. It is because in this process, the train has to output full tractive effort to catch up with scheduled time at the last traction stage. If L_{period} is too long, the weight allocation cannot update in time and the train operation will deviate from scheduled time causing extra energy consumption. In consideration of calculating complexity on-board and energy-efficient function, we choose L_{period} =9000m as the optimal weight reallocation period.

Example C.2: Note that N denotes the horizon window, which is crucial for online planning. The optimizing efficiency greatly determines the real-time planning performance. Thus the allowed time error is set as ΔT=30 and weight reallocation period L_{period} =9000m . Fig.10 demonstrates the relationship between optimization horizon N and punctuality, optimization horizon N and EC, respectively. The detailed numerical data is displayed in Table VI.

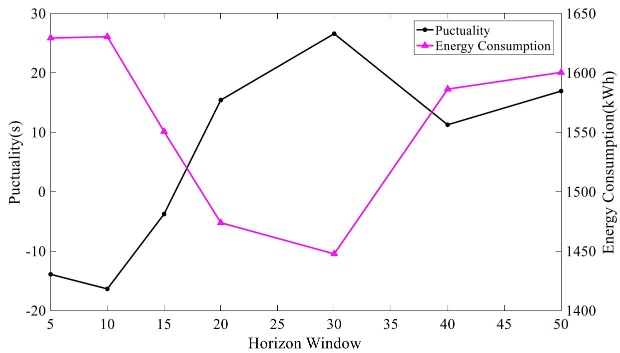

Fig.10. Profiles of EC and punctuality on varied horizon window N

TABLE VI
SIMULATION RESULTS OF DIFFERENT HORIZON WINDOWS

N	ψ_t (s)	EC(kWh)	Punctuality(s)
5	0.092	1635	-14
10	0.126	1629	-16
15	0.159	1538	-2
20	0.214	1474	15
30	0.284	1468	20
40	0.321	1636	11
50	0.398	1589	16

Note that ψ_t denotes the calculation time of each optimization time. We can see from Table VI when horizon window N increases, the calculating time ψ_t rises correspondingly. As shown in Fig.10, we can conclude that when N is from 5 to 30, EC decreases gradually and punctuality is within the allowed time error. The reason is that the risen value of N strengthens the global dynamic planning ability of the proposed method. When N is larger than 30, the simulation results display an over-planned phenomenon and EC increases. EC is reduced by (1635-1468)/1635=10.2% with N from 5 to 30 in acceptable time error. If N rises from 30 to 40 and 50, EC is reduced by (1635-1468)/1635=10.2% and (1589-1468)/1635=7.6%, respectively. Based on the above analysis, we choose N=30 as the optimal horizon window.

D. Evaluation of Delay Scene

To prove the robustness of the proposed method, we introduce a delay departure scene in this example. By comparing to the on time scene, we can see the dynamic allocation of weight coefficients. Note that in minimum time strategy, the operating time of high-speed train is 980s and the total energy consumption is 2087 kWh. Thus, we set the delay departure time as 30s, 60s and 90s, ΔT=30 , L_{period} =9000m and N=30. The simulation results are shown in Fig.11 and the detailed numerical results are displayed in Table VII.

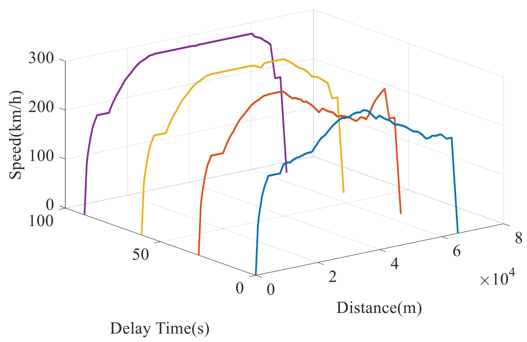

Fig.11. The planned trajectories of different delay time

TABLE VII
SIMULATION RESULTS OF DIFFERENT DELAY TIME

Delay Time(s)	EC(kWh)	Punctuality(s)
0	1458	21
30	1688	23
60	1794	-17
90	2005	-10

We can conclude from Fig.11 that in delay scene, the train can fast adjust to minimum-time strategy to catch up with the scheduled time according to the timetable. Once the operating time error is near the allowed time error, the proposed approach can balance the factors between energy-consumption and punctuality. Comparing to the minimum-time strategy, the proposed method saves (2087-1688)/2087=19.1%, (2087-1794)/2087= 14.0%, (2087-2005)/2087=3.9% more energy consumption when the delay time is 30s, 60s and 90s, respectively.

VI. Conclusion

In this paper, a MPC-based optimization approach and a MSPSO algorithm is proposed to dynamically plan train trajectory for energy-saving function and punctuality. On the basis of the real data from Beijing-Shanghai High-speed Railway, some numerical experiment has been carried out and optimization parameters have been taken into comparison. The result shows that the proposed method saves 14.7% energy consumption comparing to the regular operating strategy of "traction-cruising-braking". In delay scene, the proposed method can also adjust the operating strategy to catch up with scheduled time efficiently.

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