Three-way Decision Based on TODIM Method with Single-valued Neutrosophic Sets

Dongsheng Xu, Xinyang He*, Xiaolan Ni, and Xu Zhen

Abstract—**Three-way decision models have received substantial interest grounded in decision-theoretic rough sets and Bayesian decision theory. Single-valued neutrosophic sets are extremely useful for handling uncertain and inconsistent information, making them a valuable tool that is commonly applied in decision-making. In a three-way decision problem involving a piece of single-value neutrosophic information, the losses of each equivalence class under different actions can usually be identified with some accuracy. A critical aspect of the three-way decision problem centers around appropriate handling of the loss function. This paper proposes a novel approach to rank loss functions in each equivalence class of three-way decisions, based on the TODIM method and operates within a single-valued neutrosophic environment. Furthermore, a numerical experiment on the location of a breakfast restaurant is used to to assess the model compared to some existing related models, with the aim of demonstrating its validity and soundness.**

Index Terms—**Three-way decision, single-valued neutrosophic sets, decision-theoretic rough sets, TODIM method.**

I. Introduction

ROUGH set theory is a theory proposed by the Polish scientist Z. Pawlak[1], which has gained wide attention because it can effectively analyze various kinds of incomplete information such as imprecision, inconsistency and incompleteness. However, the limited applicability in classical rough set theory is attributed to be deficiency in consideration for fault tolerance in dealing with uncertainty. To perfect this classical theory, Yao et al.[2] combined it with the Bayesian decision process and introduced the decision-theoretic rough sets (DTRSs). Afterward, Yao[3] introduced three-way decision making (3WD) using DTRSs, which focuses on decision-making process of individuals. In Yao's view, the 3WD divides a feasible domain into positive, negative, and boundary regions, each of which is pairwise disjoint. Then the decision rules can be generated for each of these regions, that are acceptance, rejection and non-commitment decisions.

Up until now, the 3WD has been developed with the contribution of numerous researchers. They have focused their research on 3WD with respect to conditional probability and loss function. Research on conditional probability is mainly to find new approaches for estimating conditional probability. Among these studies, Yao and Zhou[4] developed the model of Naive Bayesian decision-theoretic rough sets (NBRS), which combines the concepts of Naive Bayesian classification and DTRSs, and can be estimated in accordance with Bayesian theory and the assumption of naive probability independence. In a similar vein, Liu et al.[5] suggested a novel discriminant analysis method that computes the DTRSs' conditional probability through logistic regression, with thresholds calculated using DTRSs and a Bayesian decision procedure. The new method also provides a suitable mechanism for the interpretation of the thresholds. In the study of the 3WD's loss function, Herbert and Yao ([6],[7],[8]) incorporated the game theory of classification metric into the loss function of DTRSs to identify the equilibrium point of the game and optimize the size objective of the decision domain. Deng and Yao[9] suggested an information-theoretic method that employs uncertainty as the objective function to explain and determine the threshold value. This approach introduced information theory into probabilistic rough sets. Liang et al.[10] and Liu ([11],[12],[13]) introduced an uncertainty assessment form into the loss function. They considered different forms of uncertainty for each loss value in the DTRSs and proposed new DTRSs models that broaden the range of possible loss values.

In uncertain problems, there are some biases in using exact numerical sets to describe uncertain phenomena. The American scholar Professor Zadeh[14] established fuzzy sets theory (Fuzzy Sets), considering the degree of membership. After that, some scholars refined the membership degree. Among them, Smarandache([15],[16]) proposed the neutrosophic sets. The elements in the neutrosophic sets can be real numbers, interval values or some mergers and intersections of both, so that the neutrosophic sets can be generalized to single-valued neutrosophic sets (SVNSs)[17]. Fewer studies have been conducted on 3WD in single-valued neutrosophic environments. By utilizing an evaluation function, Abdel Basset et al.[18] conducted research on 3WD that was based on SVNSs. Furthermore, Singh ([19],[20]) succeeded in establishing a correlation between 3WD, SVNSs, and conceptual lattice in his research. The handling of loss is a critical aspect when dealing with the 3WD problem that involves SVNSs information. Using cosine similarity measures and Euclidean distances, Jiao et al. ([21],[22]) developed a 3WD model with SVNSs information. By utilizing these measures, the 3WD model can calculate the metric between the loss functions of different actions and the ideal loss. This enables the determination of the expected behavior for each equivalence class. However, in Jiao et al's model, the risk preferences of decision-makers are somewhat overlooked, while this method also has the

Manuscript received October 20, 2022; revised July 23, 2023. The Natural Science Foundation of Sichuan sponsored this investigation(2022NSFSC1821).

Dongsheng Xu is a professor of College of Science, Southwest Petroleum University, Chengdu, 610500, China, e-mail: (xudongsheng1976@163.com).

Xinyang He is a graduate student in the College of Science of Southwest Petroleum University, Chengdu, 610500, China, e-mail: (2624525547@qq.com).

Xiaolan Ni is a graduate student in the College of Science of Southwest Petroleum University, Chengdu, 610500, China, e-mail: (2169639414@qq.com).

Xu Zhen is a graduate student of Graduate School of Information, Production and Systems from Waseda University, 807-0804, Japan, e-mail:(davidxu@asagi.waseda.jp).

drawback of relying on pre-defined identification of the ideal solution. The TODIM(TOmada de Decisão Iterativa Multicritério) method[23] is a multi-attribute decision-making method informed by prospect theory, in which the relative dominance degree is obtained by the comparison of the relative dominance of two solutions, which can avoid the artificial prior confirmation of the reference point and adapt to the risk appetite of decision-makers to a certain degree. Xu[24] incorporated the cumulative prospect theory into the TODIM method, and took full advantage of the TODIM method to contemplate decision-makers' psychological state. Gong[25] used the TODIM to solve the decision-making problem of interval type II fuzzy sets considering decision-makers' psychological behavioral state. It's clear that the TODIM method has a greater advantage in solving decision-making problem.

Therefore, in this paper, we apply the relative dominance degree in the TODIM method to rank the losses of performing different actions of the 3WD and propose a 3WD model based on the TODIM method with single-valued neutrosophic information.

II. Preliminaries

A. SVNS

Definition 1: Assuming X is a given domain, x is in the domain, the SVNS defined on X is composed of the truth subordinate function $T_Z(x)$, the uncertainty subordinate function $I_Z(x)$ and the distortion subordinate function $F_Z(x)$.

$$Z = \{(x, T_Z(x), I_Z(x), F_Z(x)) | x \in X\},$$

and all three membership functions of A have values between 0 and 1, $0 \le T_Z(x) + I_Z(x) + F_Z(x) \le 3$. To facilitate writing, Z can be simply expressed as (T_Z, I_Z, F_Z).

Definition 2: Suppose Z is a SVNN on X, and Z^c be complement of Z. Z^c satisfies:

$$T_{Z^c} = F_Z, I_{Z^c} = 1 - I_Z, F_{Z^c} = T_Z. \tag{1}$$

Definition 3: On X, Let Z_1 and Z_2 be two SVNNs, the SVNN's calculation rules for addition, number multiplication and the normalized Hamming distance are as follows:

1)

$$Z_1 \oplus Z_2 = (T_{Z_1} + T_{Z_2} - T_{Z_1} \cdot T_{Z_2}, I_{Z_1} \cdot I_{Z_2}, F_{Z_1} \cdot F_{Z_2}).$$

2)

$$\mu Z_1 = \left(1 - (1 - T_{Z_1})^{\mu}, I_{Z_1}^{\mu}, F_{Z_1}^{\mu}\right).$$

3)

$$d(Z_1, Z_2) = \frac{1}{3}(|T_{Z_1} - T_{Z_2}| + |I_{Z_1} - I_{Z_2}| + |F_{Z_1} - F_{Z_2}|). \tag{2}$$

Definition 4: [26] Assuming Z is a SVNN, whose score function S(Z) and accuracy function H(Z) are:

$$S(Z) = \frac{(2 + T_Z - I_Z - F_Z)}{3},$$

$$H(Z) = T_Z - F_Z,$$

where $S(Z) \in (0, 1)$ and $H(Z) \in (-1, 1)$.

Definition 5: Let Z_1 and Z_2 be two SVNNs,

1) If the score function value of Z_1 is less than Z_2, then $Z_1 < Z_2$.

2) When the score function value of Z_1 is equal to that of Z_2, then:

if the accuracy function value of Z_1 is less than Z_2, then $Z_1 < Z_2$;

if the accuracy function value of Z_1 is equal to Z_2, then $Z_1 = Z_2$;

Definition 6: [27] Suppose $Z_1 = (T_{Z_1}, I_{Z_1}, F_{Z_1})$ and $Z_2 = (T_{Z_2}, I_{Z_2}, F_{Z_2})$ are SVNNs, if satisfying the following four axiomatization conditions:

1) $E(Z_1) = 0 \Leftrightarrow T_{Z_1}, I_{Z_1}, F_{Z_1} = 0$ or $T_{Z_1}, I_{Z_1}, F_{Z_1} = 1$;

2) $E(Z_1) = 1 \Leftrightarrow (T_{Z_1}, I_{Z_1}, F_{Z_1}) = (0.5, 0.5, 0.5)$;

3) $E(Z_1) = E(Z_1{}^c)$;

4) If Z_2 is more uncertain than Z_1, then for $T_{Z_2} - T_{Z_2{}^c} \le 0$, $I_{Z_2} - I_{Z_2{}^c} \le 0$,$F_{Z_2} - F_{Z_2{}^c} \le 0$, there is $T_{Z_1} - T_{Z_2} \le 0$, $I_{Z_1} - I_{Z_2} \le 0$, $F_{Z_1} - F_{Z_2} \le 0$; or when $T_{Z_2} - T_{Z_2}^c \ge 0$, $I_{Z_2} - I_{Z_2}^c \ge 0$, $F_{Z_2} - F_{Z_2{}^c} \ge 0$, have $T_{Z_1} - T_{Z_2} \ge 0$, $I_{Z_1} - I_{Z_2}(x) \ge 0$, $F_{Z_1} - F_{Z_2} \ge 0$, $E(Z_1) \le E(Z_2)$,

the E is called SVNN's entropy.

B. The TODIM Method

Here, we consider such a multi-attribute decision problem. In this problem, $A_i (i \in [1, m])$ is option. $G_j (j \in [1, n])$ is the attribute, whose weight is ω_j, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. z_{ij} is the evaluation value of A_i under the G_j, then we can obtain an evaluation matrix $Z = (z_{ij})_{m \times n}$. The TODIM method entails the following steps.

Step 1: Normalize Z into $U = (u_{ij})_{m \times n}$.

Step 2: In accordance with the weights of the attributes, calculate the relative weight of G_j with respect to G_r:

$$\omega_{jr} = \frac{\omega_j}{\omega_r}.$$

The ω_r is the maximum value of ω_j.

Step 3: In accordance with normalisation results and relative weights, compute the dominance of A_i over every alternative A_k under attribute G_j:

$$\phi_j(A_i, A_k) = \begin{cases} \sqrt{\dfrac{\omega_{jr}(u_{ij} - u_{kj})}{\sum_{j=1}^{n} \omega_{jr}}}, u_{ij} - u_{kj} > 0, \\ 0, u_{ij} - u_{kj} = 0, \\ -\dfrac{1}{\theta}\sqrt{\dfrac{\sum_{j=1}^{n} \omega_{jr}(u_{ij} - u_{kj})}{\omega_{jr}}}, u_{ij} - u_{kj} < 0, \end{cases}$$

where $i, k = 1, 2, \cdots, m$.

Step 4: Determine the extent to which alternative A_i is dominant over all other alternatives.

$$\delta(A_i, A_k) = \sum_j \phi_j(A_i, A_k).$$

Step 5: In accordance with the results of the previous step, compute the overall dominance degree of each alternative A_i.

$$\phi(A_i) = \frac{\sum_k \delta(A_i, A_k) - \min_i \left\{\sum_k \delta(A_i, A_k)\right\}}{\max_i \left\{\sum_k \delta(A_i, A_k)\right\} - \min_i \left\{\sum_k \delta(A_i, A_k)\right\}}.$$

TABLE I
LOSS FUNCTIONS IN THE FORM OF SVNSS

	X
$a_P(Accept)$	$\lambda_{PP} = (T_{\lambda_{PP}}, I_{\lambda_{PP}}, F_{\lambda_{PP}})$
$a_B(Accept)$	$\lambda_{PB} = (T_{\lambda_{PB}}, I_{\lambda_{PB}}, F_{\lambda_{PB}})$
$a_N(Accept)$	$\lambda_{PN} = (T_{\lambda_{PN}}, I_{\lambda_{PN}}, F_{\lambda_{PN}})$
	$\neg X$
$a_P(Accept)$	$\lambda_{PN} = (T_{\lambda_{PN}}, I_{\lambda_{PN}}, F_{\lambda_{PN}})$
$a_B(Accept)$	$\lambda_{BN} = (T_{\lambda_{BN}}, I_{\lambda_{BN}}, F_{\lambda_{BN}})$
$a_N(Accept)$	$\lambda_{NN} = (T_{\lambda_{NN}}, I_{\lambda_{NN}}, F_{\lambda_{NN}})$

Step 6: Rank these solutions based on the magnitude of their corresponding $\phi(A_i)$ values. The solution with the largest $\phi(A_i)$ is considered the best.

C. 3WD and DTRS model

The 3WD approach divides a universe into three parts: the ”positive”, ”negative”, and ”boundary” regions, which represent the decision rules of acceptance, noncommitment, and rejection in sequence. These three parts are pairwise disjoint and are represented by Pos, Neg, and Bnd.[21]

The state sets $\Omega = \{X, \neg X\}$ and action sets $A = \{a_P, a_B, a_N\}$ together form the DTRS model. When x belong to X, which can be expressed as $x \in X$, it can be accompanied by three actions in the action sets: $Pos\,(X)\,, Bnd\,(X)\,, Neg\,(X)$, resulting in respective losses denoted as $\lambda_{PP}, \lambda_{BP}, \lambda_{NP}$. Similarly, when x is not in X, denoted as $x \in \neg X$, it can also face three actions, whose loss is denoted as $\lambda_{PN}, \lambda_{BN}, \lambda_{NN}$, respectively. The losses for each action are summarized in Table 1.

The expected loss resulting from performing three different behaviors $(\ \cdot = P,\ B,\ N)$ is determined by:

$$E\left(a.|[x]_R\right) = \lambda_{\cdot P} \Pr\left(X|[x]_R\right) + \lambda_{\cdot N} \Pr\left(\neg X|[x]_R\right),$$

where $Pr\left(*|[x]_R\right) = \frac{\left|* \cap [x]_R\right|}{|*|}$ is conditional probability. Obviously $Pr\left(*|[x]_R\right) \in [0,1], (* \in \Omega)$. And U represent a set of all solutions, $[x]_R$ is a set in U which has the equivalence relation R with x.

The decision rules of the DTRS, in accordance with Bayesian decision theory, are outlined as follows:

For the expected loss of actions taken on equivalence class $[x]_R$, if $a_P < min(a_B, a_N)$, it indicates that $x \in Pos\,(X)$;

For the expected loss of actions taken on equivalence class $[x]_R$, if $a_B < min(a_P, a_N)$, it indicates that $x \in Bnd\,(X)$;

For the expected loss of actions taken on equivalence class $[x]_R$, if $a_N < min(a_P, a_B)$, it indicates that $x \in Neg\,(X)$.

III. THREE-WAY DECISION BASED ON TODIM METHOD WITH SINGLE-VALUED NEUTROSOPHIC SETS

JIAO et al.[21] utilized the Bayesian decision procedure in the development of a single-valued neutrosophic decision-theoretic rough set (SVN-DTRS) method. In the SVN-DTRS model, all loss functions are expressed using SVNN, distinguishing it from the original DTRS model. Then based on the above discussion, we can calculate the average loss of SVN-DTRS under different actions$(\cdot = P, B, N)$:

$$\varepsilon\left(a.|[x]_R\right) = \lambda_{\cdot P} \Pr\left(X|[x]_R\right) \oplus \lambda_{\cdot N} \Pr\left(\neg X|[x]_R\right),$$

TABLE II
LOSSES OF EACH SOLUTION UNDER DIFFERENT ACTIONS

	A_1	$\cdots$	A_n
a_P	$\varepsilon\left(a_P|A_1\right)$	$\cdots$	$\varepsilon\left(a_P|A_n\right)$
a_B	$\varepsilon\left(a_B|A_1\right)$	$\cdots$	$\varepsilon\left(a_B|A_n\right)$
a_P	$\varepsilon\left(a_N|A_1\right)$	$\cdots$	$\varepsilon\left(a_N|A_n\right)$

TABLE III
BENEFITS OF EACH SOLUTION UNDER DIFFERENT ACTIONS

	A_1	$\cdots$	A_n
a_P	$E\left(a_P|A_1\right)$	$\cdots$	$E\left(a_P|A_n\right)$
a_B	$E\left(a_B|A_1\right)$	$\cdots$	$E\left(a_B|A_n\right)$
a_P	$E\left(a_N|A_1\right)$	$\cdots$	$E\left(a_N|A_n\right)$

$\lambda_{\cdot P}$ denotes the evaluation value of performing the corresponding action when $x \in X$, and $\lambda_{\cdot N}$ represents the evaluation value of performing the corresponding action when $x \in \neg X$.

According to Definition 3, the average loss can be calculated as:

$$\varepsilon\left(a.|[x]_R\right) = \begin{pmatrix} 1 - \left(1 - T_{\lambda_{\cdot P}}\right)^{Pr(X|[x]_R)} \left(1 - T_{\lambda_{\cdot N}}\right)^{Pr(\neg X|[x]_R)}, \\ \left(I_{\lambda_{\cdot P}}\right)^{Pr(X|[x]_R)} \cdot \left(I_{\lambda_{\cdot N}}\right)^{Pr(\neg X|[x]_R)}, \\ \left(F_{\lambda_{\cdot P}}\right)^{Pr(X|[x]_R)} \cdot \left(F_{\lambda_{\cdot N}}\right)^{Pr(\neg X|[x]_R)} \end{pmatrix} \tag{3}$$

Through the SVN-DTRS process, we can get the average loss, as shown in Table II.

In the TODIM method, losses are typically converted into gains. In our case, we normalise losses to gains for each equivalence class under different actions. The normalisation of decision data with SVN information is commonly achieved by finding its complement, so we use Equation 1 for each loss function to obtain Table III.

Here we use the relative advantage degree calculation in the TODIM method to measure the relative advantages of three different actions in the case of the same project. This approach avoids the need for artificial determination of the ideal solution in advance. The TODIM method captures decision-makers’ risk attitudes , making it a more practical approach compared to other multi-attribute decision-making methods. Below are specific steps involved in the 3WD based on the TODIM method:

Step 1: In the 3WD problem, the weights of each equivalence class are usually unknown. However, the classical TODIM method requires the weight of A_i. To solve this problem, Majumdar and Samanta[27] proposed a method for calculating the entropy of a SVNS. Furthermore, Biswas et al.[28] introduced an entropy weighting method to ascertain attribute weights, where the entropy of $A_i\,(i = 1, 2, \cdots, n)$ is: (here we consider the solution as the attribute and the individual action as the solution)

$$EN_i = 1 - \frac{1}{n} \sum_{j=P,B,N} \left(T_{ji}) + F_{ji})\right) \left|I_{ji}) + I^c{}_{ji})\right|, \tag{4}$$

where $EN_i \geq 0$, and T_{ji}, I_{ji}, F_{ji} respectively represent the different degrees of membership of the A_i adopting the j-th behavior. Then based on the information entropy of each solution, the weight of each solution A_i can be calculated:

$$w_i = \frac{1 - EN_i}{\sum\limits_i \left(1 - EN_i\right)}. \tag{5}$$

Step 2: Compute the relative weight of A_i:

$$\omega_{ir} = \frac{\omega_i}{\omega_r}, \tag{6}$$

where the ω_r is the maximum value of ω_i.

Step 3: According to the classical TODIM method, calculate A_i in the execution a_P, a_B, a_N relative to the execution $(a_B, a_N), (a_P, a_N), (a_P, a_B)$, whose calculation formula is:

$$\phi_i(a_j, a_k) = \begin{cases} \sqrt{\dfrac{\omega_{jr} d(E(a_j|A_i), E(a_k|A_i))}{\sum\limits_{i=1}^{n} \omega_{ir}}}, E(a_j|A_i) > E(a_k|A_i), \\ 0, E(a_j|A_i) = E(a_k|A_i), \\ -\dfrac{1}{\theta}\sqrt{\dfrac{\left(\sum\limits_{i=1}^{n} \omega_{ir}\right) d(E(a_j|A_i), E(a_k|A_i))}{\omega_{ir}}}, E(a_j|A_i) < E(a_k|A_i), \end{cases} \tag{7}$$

the θ is the attenuation coefficient, the $j, k = P, B, N$. A_i comparison of the expected benefits from the implementation of a_j and a_k is in accordance with definition 5. If $E(a_j|A_i) > E(a_k|A_i)$, then $\phi_i(a_j, a_k)$ means that it is more beneficial to perform action a_j than action a_k. If $E(a_j|A_i) < E(a_k|A_i)$, it means that it is more beneficial to perform action a_k than action a_j.

Step 4: Utilizing the results of the calculations from step 3 and referring to the classic TODIM method, we can calculate the overall advantage of performing different actions under the solution A_i.

$$\phi(a_\cdot|A_i) = \frac{\phi_i(a_\cdot, a_k) - \min\limits_j \left\{\sum\limits_k \phi_i(a_j, a_k)\right\}}{\max\limits_j \left\{\sum\limits_k \phi_i(a_j, a_k)\right\} - \min\limits_j \left\{\sum\limits_k \phi_i(a_j, a_k)\right\}}, \tag{8}$$

where the $\cdot, k = P, B, N$.

Step 5: Repeat steps 3-4 to calculate the overall advantage of performing different actions for all solutions.

Step 6: Make decisions for the different actions of each solution. The decision rule becomes:

If the overall dominance of the equivalence class $[x]_R$ to perform a_P is greater than that of performing a_B and a_N, then $[x]_R$ will perform a_P, namely $x \in Pos(X)$.

If the overall dominance of the equivalence class $[x]_R$ to perform a_B is greater than that of performing a_P and a_N, then $[x]_R$ will perform a_B, namely $x \in Bnd(X)$.

If the overall dominance of the equivalence class $[x]_R$ to perform a_N is greater than that of performing a_P and a_B, then $[x]_R$ will perform a_N, namely $x \in Neg(X)$.

IV. NUMERICAL EXAMPLE AND ANALYSIS

80% of catering industry operators are small-scale entrepreneurs who may not have established social networks or possess significant economic resources. These operators are often faced with the embarrassing fact that they cannot afford to pay rent when choosing locations. Therefore, it is essential for the store to choose the right location. Table IV displays the values of the loss functions corresponding to the four equivalence classes in the 20 recommended locations, where some basic conditions have already been examined.

TABLE IV
VALUES OF LOSS FUNCTION

	$[x_1]_R$	$[x_2]_R$
λ_{PY}	(0.79,0.69,0.11)	(0.65,0.95,0.95)
λ_{BY}	(0.03,0.03,0.34)	(0.93,0.38,0.22)
λ_{NY}	(0.67,0.76,0.25)	(0.93,0.48,0.69)
λ_{PN}	(0.95,0.31,0.49)	(0.65,0.44,0.89)
λ_{BN}	(0.84,0.43,0.58)	(0.03,0.76,0.13)
λ_{NN}	(0.74,0.18,0.50)	(0.04,0.67,0.25)
	$[x_3]_R$	$[x_4]_R$
λ_{PY}	(0.27,027,0.14)	(0.54,0.96,0.97)
λ_{BY}	(0.17,0.64,0.95)	(0.95,0.80,0.42)
λ_{NY}	(0.09,0.65,0.84)	(0.27,0.96,0.97)
λ_{PN}	(0.75,0.79,0.22)	(0.95,0.03,0.93)
λ_{BN}	(0.70,0.70,0.54)	(0.79,0.65,0.84)
λ_{NN}	(0.82,0.16,0.25)	(0.48,0.14,0.91)

TABLE V
THE CONDITIONAL PROBABILITY

	$[x_1]_R$	$[x_2]_R$	$[x_3]_R$	$[x_4]_R$
$\Pr(Y\,\|[x]_R)$	0.75	0.43	0.50	0.60
$\Pr(N\,\|[x]_R)$	0.25	0.57	0.50	0.40

TABLE VI
LOSS FUNCTION UNDER DIFFERENT ACTIONS

	$[x_1]_R$	$[x_2]_R$
a_P	(0.85,0.57,0.16)	(0.65,0.61,0.92)
a_B	(0.38,0.06,0.39)	(0.69,0.56,0.16)
a_N	(0.69,0.53,0.30)	(0.69,0.58,0.39)
	$[x_3]_R$	$[x_4]_R$
a_P	(0.57,0.46,0.18)	(0.81,0.24,0.95)
a_B	(0.50,0.67,0.72)	(0.91,0.74,0.55)
a_N	(0.60,0.32,0.46)	(0.36,0.44,0.95)

TABLE VII
GAIN FUNCTIONS UNDER DIFFERENT ACTIONS

	$[x_1]_R$	$[x_2]_R$
a_P	(0.16,0.44,0.85)	(0.92,0.39,0.65)
a_B	(0.39,0.94,0.38)	(0.16,0.44,0.69)
a_N	(0.30,0.47,0.69)	(0.39,0.42,0.69)
	$[x_3]_R$	$[x_4]_R$
a_P	(0.18,0.54,0.57)	(0.95,0.76,0.81)
a_B	(0.72,0.33,0.50)	(0.55,0.26,0.91)
a_N	(0.46,0.68,0.60)	(0.95,0.56,0.36)

These loss functions are represented in the form of SVNN. Meanwhile, Table V illustrates the probabilities obtained for these four equivalence classes. [21]

Then we can calculate the loss function of each solution under different actions using Equations 3, as presented in Table VI.

According to Equation 1, we can transform the loss function into a gain function, as shown in Table VII.

A. Using the 3WD based on the score function of the SVNSs

For the loss ranking in the 3WD problem, we can compare the losses from different actions in the 3WD problem for ranking purposes by employing SVNSs' score and accuracy function, with a focus on the membership aspect of the

TABLE VIII
SCORE FUNCTIONS FOR EACH EQUIVALENCE CLASS

	$S(a_P)$	$S(a_B)$	$S(a_N)$
$[x_1]_R$	0.2905	0.3550	0.3795
$[x_2]_R$	0.6260	0.3454	0.4262
$[x_3]_R$	0.3548	0.6282	0.3952
$[x_4]_R$	0.4610	0.4697	0.6758

TABLE IX
COSINE SIMILARITY MEASURE FOR EACH EQUIVALENCE CLASSES

	$s(a_P)$	$s(a_B)$	$s(a_N)$
$[x_1]_R$	0.4978	0.5765	0.6451
$[x_2]_R$	0.8449	0.5673	0.6896
$[x_3]_R$	0.5708	0.8899	0.6751
$[x_4]_R$	0.6535	0.0704	0.8887

TABLE X
EUCLIDEAN DISTANCE FOR EACH EQUIVALENCE CLASSES

	$\varepsilon(a_P)$	$\varepsilon(a_B)$	$\varepsilon(a_N)$
$[x_1]_R$	1.2690	1.1860	1.0908
$[x_2]_R$	0.7612	1.1688	1.0242
$[x_3]_R$	1.1639	0.6642	1.0521
$[x_4]_R$	1.1124	1.0481	0.6657

SVNSs. Here we use this method to obtain the score function for each item in Table VII.

Table VIII shows the scores of the benefit function for each equivalence class under different actions. In Table V, $[x_1]_R$ performs each action in the following order of score: $a_N \succ a_B \succ a_P$; $[x_2]_R$ performs each act in the following order of score: $a_P \succ a_N \succ a_B$; $[x_3]_R$ performs each action in the following order of score: $a_B \succ a_N \succ a_P$; $[x_4]_R$ performs each action in the following order of score: $a_N \succ a_B \succ a_P$. Obviously, the higher the scores to the benefit function for each equivalence class under different actions, the greater the degree to which the action is performed. So $[x_2]_R$ is more suitable to choose as the location of the store.

B. Using the 3WD based on Cosine Similarity and Euclidean Distance

Jiao et al.[22] used cosine similarity and Euclidean distance to measure the metric between the loss function and the ideal loss in the 3WD for ranking purposes, and the results using these two methods applied to this data are shown in Tables IX and X, when the ideal loss is chosen as (0,1,1).

From Table IX, $[x_1]_R$ performs each action in the following order of similarity: $a_N \succ a_B \succ a_P$; $[x_2]_R$ performs each action in the following order of similarity: $a_P \succ a_N \succ a_B$; $[x_3]_R$ performs each action in the following order of similarity: $a_B \succ a_N \succ a_P$; $[x_4]_R$ performs each action in the following order of similarity: $a_N \succ a_P \succ a_B$. Obviously, the higher the similarity to the ideal loss, the higher the degree to which the behaviour is performed. So $[x_2]_R$ is more suitable to choose as the location of the store.

From Table X, $[x_1]_R$ performs each action in the following order of distance: $a_N \succ a_B \succ a_P$; $[x_2]_R$ performs each action in the following order of distance: $a_P \succ a_N \succ a_B$; $[x_3]_R$ performs each action in the following order of distance: $a_B \succ a_N \succ a_P$; $[x_4]_R$ performs each action in the following order of distance: $a_N \succ a_P \succ a_B$. Obviously, the smaller the Euclidean distance from the ideal loss, the greater the degree to which the action is performed. So $[x_2]_R$ is more suitable to choose as the location of the store.

TABLE XI
WEIGHTS DERIVED BY ENTROPY WEIGHTING METHOD

	$[x_1]_R$	$[x_2]_R$	$[x_3]_R$	$[x_4]_R$
w	0.2127	0.1531	0.2058	0.4284

TABLE XII
RELATIVE WEIGHTS BETWEEN EQUIVALENCE CLASSES

	$[x_1]_R$	$[x_2]_R$	$[x_3]_R$	$[x_4]_R$
w	0.4965	0.3575	0.4805	1

TABLE XIII
ADVANTAGES OF EACH SOLUTION EQUIVALENCE CLASS UNDER DIFFERENT ACTIONS

	a_P	a_B	a_N
$[x_1]_R$	0	0.4351	1
$[x_2]_R$	1	0	0.4439
$[x_3]_R$	0	1	0.4542
$[x_4]_R$	0.631	0	1

C. Using the 3WD based on the TODIM method

Step 1: According to Equations 4-5, the weights in different actions are computed, as shown in Table XI.

Step 2: In accordance with Equation 6, calculate the relative weights under different actions, as shown in Table XII.

Step 3: As per Equation 7, calculate the relative advantage of each solution under different actions, and the results are shown in $\phi_1 - \phi_4$.

$$\phi_1(a_j, a_k) = \begin{bmatrix} 0 & -0.5501 & -0.2905 \\ 0.2482 & 0 & -0.5506 \\ 0.1519 & 0.2444 & 0 \end{bmatrix}$$

$$\phi_2(a_j, a_k) = \begin{bmatrix} 0 & 0.2443 & 0.2061 \\ -0.5415 & 0 & -0.2929 \\ -0.3941 & 0.1300 & 0 \end{bmatrix}$$

$$\phi_3(a_j, a_k) = \begin{bmatrix} 0 & -0.4535 & -0.3394 \\ 0.2046 & 0 & 0.1889 \\ 0.1746 & -0.4256 & 0 \end{bmatrix}$$

$$\phi_4(a_j, a_k) = \begin{bmatrix} 0 & 0.2658 & -0.4070 \\ -0.5891 & 0 & -0.6550 \\ 0.2129 & 0.2907 & 0 \end{bmatrix}$$

Step 4: In accordance with Equation 8, calculate the combined dominance of each action relative to the others, as shown in Table XIII.

From Table XIII, $[x_1]_R$ performs each action in the following order of advantage: $a_N \succ a_B \succ a_P$; $[x_2]_R$ performs each action in the following order of advantage: $a_P \succ a_N \succ a_B$; $[x_3]_R$ performs each act in the following order of advantage: $a_B \succ a_N \succ a_P$; $[x_4]_R$ performs each action in the following order of advantage: $a_N \succ a_B \succ a_P$. so $[x_2]_R$ is more suitable to choose as the location of the store.

TABLE XIV
THE RESULTS OF BOTH METHODS

	$[x_1]_R$	$[x_2]_R$	$[x_3]_R$	$[x_4]_R$
By the TODIM method	a_N	a_P	a_B	a_N
By the score function	a_N	a_P	a_B	a_N
By the cosine similarity	a_N	a_P	a_B	a_N
By the euclidean distance	a_N	a_P	a_B	a_N

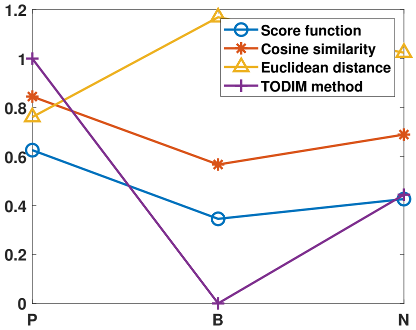

Fig. 1. Comparison of Results from Four Methods Using $[x_2]_R$ as an example

D. Comparative Analysis

From the findings in Table XIV, the model ranking results in this paper are consistent with the other three results. Obviously, $[x_2]_R$ is the most suitable for the decision-makers' interest. From Fig.1, it can be observed that all four methods are acceptable when executing P in $[x_2]_R$. However, the acceptance level is lower for executing B and N compared to P. Among them, the judgment methods based on cosine similarity and score function show relatively small differences for the three behaviors. Although it can be seen that P is the optimal decision, the intuitive perception for decision-makers is not strong. This finding successfully proves the rationality and effectiveness of the 3WD based on the TODIM method. Therefore, our method is effective in helping decision-makers make decisions in complex environments.

Compared with the other three methods, our approach enables a more precise capture of the distinct psychological states experienced by decision-makers, when confronted by gains and losses and make the decision more realistic. At the same time, the TODIM method can avoid the prior selection of reference points compared to Jiao's methods. In summary, our model has wider applicability and implementability.

E. Sensitivity analysis

At the same time, we perform a sensitivity analysis of the parameter θ of this algorithm for $[x_2]_R$, and the result is shown in Fig.2, which shows that as the θ changes, the result does not change. It verifies the stability of the result in terms of the decision-makers' preferences.

V. Conclusions

3WD uses a pair of state and a trio of behavior for characterizing the decision process by finding the expected loss of different behavior sets under the equivalence class and then making decisions in accordance with the Bayesian minimization risk-cost decision principle. In this paper, we extend the 3WD idea and the classical TODIM method to propose a three-way decision model based on the TODIM method. This method factors in the different performances of decision-makers facing losses and gains, and reflects the objective psychological state of decision-makers. Then, we apply our method along with some other validated effective 3WD methods to the 3WD problem with single-value neutrosophic information. Through the results, it is demonstrated that our method is practical and objective. By way of contrast, although the outcomes of our method and other methods are consistent, the three-way decision model based on the TODIM method increases the spacing of the overall degree of dominance of each action, which can help decision-makers make better judgments. Moreover, our method takes into account the reality that decision-makers have different investment intentions when facing different risks. This is reflected in the introduction of the TODIM method relative advantage degree function based on prospect theory. Our method differs from using score functions and accuracy functions in that it considers not only the data, but also the risk preferences of decision-makers. Unlike using the Euclidean distance and similarity function to find measures of expected and ideal losses, our method can take decision-makers' risk preferences into consideration as well as avoid the idea of identifying reference points in advance. At the same time, the sensitivity analysis shows that the parameters have minimal impact on the results, demonstrating the robustness of our proposed method. Overall, our method focuses on decision-makers' risk preferences while considering on the data itself, which for a three-way decision problem can result in a decision that better caters to the preferences of decision-makers. However, despite our approach having some advantages in the site selection problem discussed in this paper, it still has the shortcoming of ignoring the psychological characteristics of decision-makers. In subsequent endeavors, we will be devoted to examining the psychological factors of decision-makers in fuzzy three-way decision problems.

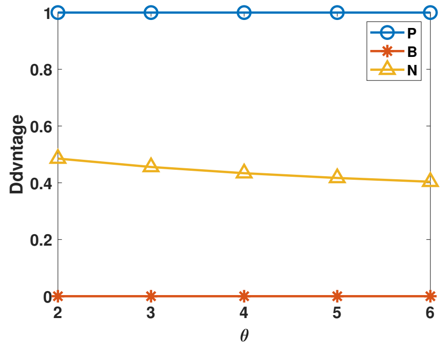

Fig. 2. Sensitivity analysis with $[x_2]_R$ as an example

References

[1] Z. Pawlak, "Rough sets," *International journal of computer & information sciences*, vol. 11, no. 5, pp. 341–356, 1982.

[2] Y. Yao and S. K. M. Wong, "A decision theoretic framework for approximating concepts," *International journal of man-machine studies*, vol. 37, no. 6, pp. 793–809, 1992.

[3] Y. Yao, "Three-way decisions with probabilistic rough sets," *Information sciences*, vol. 180, no. 3, pp. 341–353, 2010.

[4] Y. Yao and B. Zhou, "Naive bayesian rough sets," in *Rough Set and Knowledge Technology: 5th International Conference, RSKT 2010, Beijing, China, October 15-17, 2010. Proceedings 5*. Springer, 2010, pp. 719–726.

[5] D. Liu, T. Li, and D. Liang, "Incorporating logistic regression to decision-theoretic rough sets for classifications," *International Journal of Approximate Reasoning*, vol. 55, no. 1, pp. 197–210, 2014.

[6] J. P. Herbert and J. Yao, "Game-theoretic risk analysis in decision-theoretic rough sets," in *Rough Sets and Knowledge Technology: Third International Conference, RSKT 2008, Chengdu, China, May 17-19, 2008. Proceedings 3*. Springer, 2008, pp. 132–139.

[7] ——, "Game-theoretic rough sets," *Fundamenta Informaticae*, vol. 108, no. 3-4, pp. 267–286, 2011.

[8] ——, "Analysis of data-driven parameters in game-theoretic rough sets," in *Rough Sets and Knowledge Technology: 6th International Conference, RSKT 2011, Banff, Canada, October 9-12, 2011. Proceedings 6*. Springer, 2011, pp. 447–456.

[9] X. Deng and Y. Yao, "An information-theoretic interpretation of thresholds in probabilistic rough sets," in *Rough Sets and Knowledge Technology: 7th International Conference, RSKT 2012, Chengdu, China, August 17-20, 2012. Proceedings 7*. Springer, 2012, pp. 369–378.

[10] D. Liang, D. Liu, W. Pedrycz, and P. Hu, "Triangular fuzzy decision-theoretic rough sets," *International Journal of Approximate Reasoning*, vol. 54, no. 8, pp. 1087–1106, 2013.

[11] D. Liu, T. Li, P. Hu, and H. Li, "Multiple-category classification with decision-theoretic rough sets," in *Rough Set and Knowledge Technology: 5th International Conference, RSKT 2010, Beijing, China, October 15-17, 2010. Proceedings 5*. Springer, 2010, pp. 703–710.

[12] D. Liu, T. Li, and D. Liang, "Three-way government decision analysis with decision-theoretic rough sets," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 20, no. supp01, pp. 119–132, 2012.

[13] ——, "Decision-theoretic rough sets with probabilistic distribution," in *Rough Sets and Knowledge Technology: 7th International Conference, RSKT 2012, Chengdu, China, August 17-20, 2012. Proceedings 7*. Springer, 2012, pp. 389–398.

[14] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[15] F. Smarandache, "A unifying field in logics: Neutrosophic logic." in *Philosophy*. American Research Press, 1999, pp. 1–141.

[16] ——, *A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability: neutrsophic logic. Neutrosophy, neutrosophic set, neutrosophic probability*. Infinite Study, 2005.

[17] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single valued neutrosophic sets*. Infinite study, 2010.

[18] M. Abdel-Basset, G. Manogaran, M. Mohamed, and N. Chilamkurti, "Three-way decisions based on neutrosophic sets and ahp-qfd framework for supplier selection problem," *Future Generation Computer Systems*, vol. 89, no. 6, pp. 19–30, 2018.

[19] P. K. Singh, "Three-way n-valued neutrosophic concept lattice at different granulation," *International Journal of Machine Learning and Cybernetics*, vol. 9, no. 11, pp. 1839–1855, 2018.

[20] ——, "Three-way fuzzy concept lattice representation using neutrosophic set," *International Journal of Machine Learning and Cybernetics*, vol. 8, no. 1, pp. 69–79, 2017.

[21] L. Jiao, H.-L. Yang, and S.-G. Li, "Three-way decision based on decision-theoretic rough sets with single-valued neutrosophic information," *International Journal of Machine Learning and Cybernetics*, vol. 11, no. 3, pp. 657–665, 2020.

[22] F. Jia and P. Liu, "A novel three-way decision model under multiple-criteria environment," *Information Sciences*, vol. 471, pp. 29–51, 2019.

[23] L. Gomes and M. Lima, "Todimi: Basics and application to multicriteria ranking," *Found. Comput. Decis. Sci*, vol. 16, no. 3-4, pp. 1–16, 1991.

[24] D. Xu, X. Lu, and L. Peng, "An extended todim based on cumulative prospect theory for single-valued neutrosophic multi-attribute decision-making." *Engineering Letters*, vol. 30, no. 2, pp. 752–756, 2022.

[25] X. Gong, J. Zhang, and X. Zhao, "A bwm-todim based integrated decision framework for financial technology selection with interval type-2 fuzzy sets." *IAENG International Journal of Applied Mathematics*, vol. 52, no. 4, pp. 826–837, 2022.

[26] H.-y. Zhang, J.-q. Wang, and X.-h. Chen, "Interval neutrosophic sets and their application in multicriteria decision making problems," *The Scientific World Journal*, vol. 2014, no. 3, pp. 645–953, 2014.

[27] P. Majumdar and S. K. Samanta, "On similarity and entropy of neutrosophic sets," *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 3, pp. 1245–1252, 2014.

[28] P. Biswas, S. Pramanik, and B. C. Giri, "Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments," *Neutrosophic Sets and Systems*, vol. 2, no. 1, pp. 102–110, 2014.

Dongsheng Xu, was born on December 20, 1976, in Nanchong County, Sichuan Province, P.R. China. He obtained his Ph.D. degree from Southwest University of Finance and Economics, China. Currently, he holds the position of Professor in the School of Science at Southwestern Petroleum University. Dr. Xu specializes in financial asset pricing. His research endeavors have resulted in the publication of over 30 papers in reputable journals, books, and conference proceedings. Some of the prominent journals where his work has been published include the Journal of Computational and Applied Mathematics, Chaos, Solitons and Fractals, and Journal of Intelligent & Fuzzy Systems.