Robust PID Controller for AQM Based on Linear Quadratic Approach

Ji Hoon Yang and Byung Suhl Suh

Abstract— **This paper proposes a robust PID (Proportional-Integral-Derivative) controller for AQM (Active Queue Management) based on linear quadratic approach. In order to make PID control structure, a new state variable is augmented to AQM system. And, the optimal tuning method of the proposed PID controller is developed by establishing relationships between the design parameters of desired (target) transfer function in order to consider a high buffer occupancy at initial time and a long time to approach the reference queue size, and the weight factors Q and ρ of LQR(Linear Quadratic Regulator) for satisfying the stability-robustness. The performance of the proposed PID controller is verified and compared with PI controller using NS simulations.**

Index Terms— **AQM, Congestion Control, LQR, PID Controller**

I. Introduction

During the past few years, computer networks have been explosively increased by their users, and have confronted severe congestion collapse problems according to growth of computer networks. In the last 80's, Jacobson and Karels[1] propose the end-to-end congestion control algorithm which forms the basic for the TCP(Transmission Control Protocol) congestion control through the widely used. This congestion control algorithm is that a TCP sender keeps a sending window (packets) rate according to the rate of dropped packets when a buffer becomes full in the router queue. In the recently, RED (Random Early Discard) has been proposed for the queue management by Floyd and Jacobson [2]. Its mechanism is that packets are randomly dropped before the buffer of queue overflows. And, Braden et al. [3] has proposed the enhanced end-to-end congestion control for Active Queue Management in the Internet

In an alternative approach using control theories, the dynamic model [4] of the TCP congestion control algorithm was derived, and its linearized model [5] is approximated by small-signal linearization about an operating point to gain insight for the purpose of feedback control. Hollot et al [6] [7] designed the PI (Proportional-Integral) controller based on the linear control theory. Its main contribution is to convert the congestion control algorithm into the controller design problem within the framework of control theory in AQM system. And [8] and [9] have used fundamental control theoies to analyze and develop for AQM. More recently, for the stability and performance issues in AQM, there have developed by the optimal control theory [10], [11]. They design a robust AQM controller based on the Linear Matrix Inequalities (LMI) and the robust μ -analysis technique, respectively.

For these issues, this paper proposes a robust PID controller by linear quadratic approach based on the method of Suh and Yang [12] [13]. The basic ideas are: For transforming the PID controller into an LQ design problem, the optimal state feedback control law for AQM system is expressed as the PID controller by augmenting a new state variable. And, the parameters of PID controller are determined by both the weighting factor Q and ρ of the cost function of LQR and the design parameters of a maximum overshoot and a setting time in time domain in order to consider a high buffer occupancy at a initial time and a long time to approach a reference queue size in AQM control system.

This paper is structured as follows; AQM Control System is presented in Section II. The design methodologies for the tuning method of the proposed controller are discussed in Section III. Simulation results are presented in Section IV.

II. AQM Control System

This section describes an AQM control system representation by augmenting a new state variable. And a PID controller based on Linear Quadratic approach is formulated in order to control AQM system.

A. The Model of AQM Control System

In [4], a non-linear dynamic model for TCP flow control is proposed. A mathematical model of TCP behavior which ignores the TCP timeout mechanism is introduced here as following:

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t-R(t))}p(t-R(t))$$

$$\dot{q}(t) = \frac{W(t)}{R(t)}N(t) - C \tag{1}$$

where $\dot{W}(t)$ denotes the time-derivative of $W(t)$, $\dot{q}(t)$ denotes the time-derivative of $q(t)$, and

$W \doteq$ Expected TCP window size (packets)

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$q \doteq$ Expected queue length (packets)

$R \doteq$ Round-trip time ($= \frac{q}{C} + T_p$) (seconds)

$C \doteq$ Link capacity (packets/second)

$T_p \doteq$ Propagation delay (seconds)

$N \doteq$ Load factor (number of TCP sessions)

$p \doteq$ Probability of packet mark/drop

$t =$ Time

The expected queue length q and the expected TCP window size W are positive value and bounded quantities. And also, the probability of packet mark (drop) p takes value only in $[0, 1]$.

For the control theoretical analysis, (1) was approximated as a linearized constant model by small-signal linearization about an operating point (W_0, q_0, p_0), see [5] for linearization details, which leads to the following (2):

$$\delta\dot{W}(t) = -\frac{2N}{R_0^2 C}\delta W(t) - \frac{R_0 C^2}{2N^2}\delta p(t - R_0)$$

$$\delta\dot{q}(t) = \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t) \tag{2}$$

where $\delta W(t) \doteq W - W_0$, $\delta q(t) \doteq q - q_0$, and $\delta p(t) \doteq p - p_0$.

Thus, the block diagram of linearized AQM control system is shown in Fig. 1. In this diagram $P_{tcp}(s)$ denotes the transfer function from loss probability $\delta p(t)$ to window size $\delta W(t)$, $P_{queue}(s)$ denotes the transfer function from $\delta W(t)$ to queue length $\delta q(t)$, and $C(s)$ denotes the transfer function of controller.

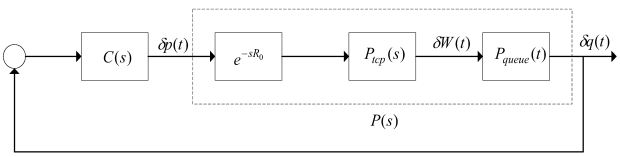

Figure 1. Block diagram of a linearized AQM as feedback control

The transfer functions of $P_{tcp}(s)$ and $P_{queue}(t)$ become as following, respectively,

$$P_{tcp}(s) = \frac{\frac{R_0 C^2}{2N^2}}{s + \frac{2N}{R_0^2 C}} \tag{3}$$

$$P_{queue}(s) = \frac{\frac{N}{R_0}}{s + \frac{1}{R_0}} \tag{4}$$

where $R_0 =$ Round Trip Time (RTT), $N =$ no. of active TCP session, $C =$ link capacity (packet/sec)

And also, the plant transfer function which is denoted as $P(s) = P_{tcp}(s)P_{queue}(s)e^{-sR_0}$ can be expressed

$$P(s) = \frac{\frac{C^2}{2N}e^{-sR_0}}{(s + \frac{2N}{R_0^2 C})(s + \frac{1}{R_0})}$$

$$= \frac{\frac{C^2}{2N}e^{-sR_0}}{s^2 + (\frac{2N}{R_0^2 C} + \frac{1}{R_0})s + \frac{2N}{R_0^3 C}} \tag{5}$$

B. Augmentation of a New State Variable

Let the state variable $x_p(t)$ of (5) be defined as:

$$x_p(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \delta q(t) \\ \frac{d\delta q(t)}{dt} \end{bmatrix} \tag{6}$$

The (5) can also be represented with state-space model:

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t - R_0) \tag{7}$$

$$y_p(t) = C_p x_p(t) \tag{8}$$

where $y_p(t) = \delta q(t)$ is an output variable, $u(t) = \delta p(t)$ is a control variable, $A_p = \begin{bmatrix} 0 & 1 \\ -\frac{2N}{R_0^3 C} & -(\frac{2N}{R_0^2 C} + \frac{1}{R_0}) \end{bmatrix}$, $B_p = \begin{bmatrix} 0 \\ \frac{C^2}{2N} \end{bmatrix}$, and $C_p = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Also, a new integrator state variable $x_0(t)$ which is defined as $\frac{dx_1(t)}{dt} = \delta q(t)$ is augmented to the feed forward loop in order to make PID structure.

The augmented state-variable descriptions are

$$x(t) = \begin{bmatrix} x_0(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \int_0^t \delta q(\tau)d\tau \\ \delta q(t) \\ \frac{d\delta q(t)}{dt} \end{bmatrix} \tag{9}$$

Then (7) and (8) becomes as following (10) and (11), respectively,

$$\dot{x}(t) = A_{aug}x(t) + B_{aug}u(t - R_0) \tag{10}$$

$$y(t) = C_{aug}x(t) \tag{11}$$

where $A_{aug} = \begin{bmatrix} 0 & C_p \\ 0 & A_p \end{bmatrix}$, $B_{aug} = \begin{bmatrix} 0 \\ B_p \end{bmatrix}$, and $C_{aug} = \begin{bmatrix} 0 & C_p \end{bmatrix}$.

It is noted that A_p, B_p and C_p are the constant state-space matrix of (5).

C. The PID Controller Based on Linear Quadratic Approach

For transforming to the LQR, consider a cost function is

$$J = \frac{1}{2}\int_0^{\infty}\left(x(t)^T Q x(t) + \rho u(t)^T u(t) \right) dt \tag{12}$$

with the assumption that a weighting matrix Q is symmetric and positive semi-definite, and a weighting factor ρ is positive value, then the following optimal control law is

$$u(t) = -G\, x(t) \tag{13}$$

where $G = -\rho^{-1} B_{aug}{}^T K$ and $K = K^T$ is a solution matrix of the algebraic Riccatti's equation:

$$K A_{aug} + A_{aug}^T K + Q - \frac{1}{\rho} K B_{aug} B_{aug}^T K = 0 \tag{14}$$

Let G be decomposed into

$$G = \begin{bmatrix} g_0 & g_1 & g_2 \end{bmatrix} \tag{15}$$

then the optimal control law of (13) becomes the following PID control form:

$$u(t) = -\left(g_0 \int_0^t \delta q(\tau) d\tau + g_1 \delta q(t) + g_2 \frac{d\delta q(t)}{dt} \right) \tag{16}$$

Consider the conventional PID control form:

$$u(t) = -K_p \left(y(t) + \frac{1}{T_i}\int_0^t y(\tau) d\tau + T_d \frac{dy(t)}{dt} \right) \tag{17}$$

Relating the gain components g_i to the proportional gain K_p, integral time T_i, and derivative time T_d gives

$$K_p = g_1 \quad , \quad T_i = \frac{g_1}{g_0} \quad , \quad T_d = \frac{g_2}{g_1} \tag{18}$$

Since the optimal control $u(t)$ of (13) can be considered as the PID control form, the conventional PID structure of (17) and the proposed PID controller which is formulated by LQ approach of (16) can be equivalent for the augmented second order system. The parameters of the PID controller K_p, T_i and T_d can be obtained by solving LQR design problem.

Remark 1. It is noted that LQR is able to guarantee the stability robustness, but it is hard to deal with the performance issues because it does not consist of an output feedback. For this, in this section, a special type of LQR structure with an output feedback in the second order system is developed by the augmented state variable description

Remark 2. It is noted that the proposed PID controller by LQ approach can be maintain stability against the tolerable perturbations due to its guaranteed phase margin of 60° and gain margin of infinity.

III. Tuning Method of PID Controller

In this section, an optimal selection procedure of the PID controller parameters is presented by the relationships between the design parameters of desired transfer function and the weighting factors Q and ρ of LQR.

A. The Design Parameters of Desired Transfer Function

Consider the following third-order transfer function equation which is named with the desired (target) transfer function:

$$T(s) = \frac{\omega_n^3}{\left(s + \frac{1}{r}\omega_n \right)\left(s^2 + p\omega_n s + r\omega_n^2 \right)} \tag{19}$$

Based on [13], the parameters p and r of (19) can be utilized as the design parameters to play a role in regulating the overshoots, and the parameter ω_n can same job in regulating the rising time and settling time under the fixed overshoot determined by the parameters p and r. In other words, the design parameters p and r can be utilized to meet the given overshoot requirements and the design parameter ω_n can be utilized to meet the given settling time requirements in time domain.

B. The Relationships between the weighting factors and the design parameters

Let the weighting factor Q be as (20)

$$Q = \begin{bmatrix} Q_{00} & 0 & 0 \\ 0 & Q_{11} & 0 \\ 0 & 0 & Q_{22} \end{bmatrix} \tag{20}$$

By inserting (20) into (14), we can obtain the following expression of Q with respect to G

$$Q_{00} = \rho g_0^2 \tag{21}$$

$$Q_{11} = \rho g_1^2 + 2\rho\left(\frac{2N}{R_0^3 C}\right)\left(\frac{2N}{C^2}\right) g_1 - 2\rho\left(g_2 + \left(\frac{2N}{R_0^2 C} + \frac{1}{R_0}\right)\frac{2N}{C^2}\right) g_0 \tag{22}$$

$$Q_{22} = \rho g_2^2 + 2\rho\left(\frac{2N}{R_0^2 C} + \frac{1}{R_0}\right)\left(\frac{2N}{C^2}\right) g_2 - 2\rho\left(\frac{2N}{C^2}\right) g_1 \tag{23}$$

In order to relate the relationships between the design parameters p, r and ω_n and the weighting factors Q and ρ, it is yielded the characteristic equation by substituting (13) into (10) as:

$$\frac{dx(t)}{dt} = (A_{aug} - B_{aug}G)x(t) \tag{24}$$

and its characteristic equation is

$$\Delta = s^3 + (\frac{2N}{R_0^2 C} + \frac{1}{R_0} + \frac{C^2}{2N}g_0)s^2 + (\frac{2N}{R_0^3 C} + \frac{C^2}{2N}g_1)s + \frac{C^2}{2N}g_2 \tag{25}$$

We consider that (25) should be equal to the characteristic equation of (19) because (19) is considered as the desired transfer function to satisfy the design specifications in time domain.

Equalizing (25) with the characteristic equation of (19) and Rearranging with respect to Q, the equations are derived as following:

$$Q_{00} = \rho\left(\frac{2N}{C^2}\right)^2\left(-\frac{2N}{R_0^2 C} - \frac{1}{R_0} + \left(p + \frac{1}{r}\right)\varpi_n\right)^2 \tag{26}$$

$$\begin{aligned} Q_{11} = {} & \rho\left(\frac{4N^2}{C^4}\right)\left(-\frac{2N}{R_0^3 C} + \left(r + \frac{p}{r}\right)\omega_n^2\right)^2 \\ & + 2\rho\left(\frac{2N}{R_0^3 C}\right)\left(\frac{4N^2}{C^4}\right)\left(-\frac{2N}{R_0^3 C} + \left(r + \frac{p}{r}\right)\omega_n^2\right) \\ & - 2\rho\left(\frac{2N}{C^2}\right)\left(-\frac{2N}{R_0^2 C} - \frac{1}{R_0} + \left(p + \frac{1}{r}\right)\omega_n\right) \\ & \left(\left(\frac{2N}{C^2}\right)\omega_n^3 + \left(\frac{2N}{R_0^2 C} + \frac{1}{R_0}\right)\left(\frac{2N}{C^2}\right)\right) \end{aligned} \tag{27}$$

$$\begin{aligned} Q_{22} = {} & \rho\left(\frac{4N^2}{C^4}\right)\omega_n^6 + 2\rho\left(\frac{2N}{R_0^2 C} + \frac{1}{R_0}\right)\left(\frac{4N^2}{C^4}\right)\omega_n^3 \\ & - 2\rho\left(\frac{4N^2}{C^4}\right)\left(-\frac{2N}{R_0^3 C} + \left(r + \frac{p}{r}\right)\omega_n^2\right) \end{aligned} \tag{28}$$

where N, R_0, and C are given by AQM model.

The Q_{00}, Q_{11} and Q_{22} are able to completely determined by the design parameters p, r and ω_n of desired transfer function (19). In other word, the weight factor Q is only determined by both the design parameters and the given AQM model parameters. Therefore, the proposed PID controller parameter as the gain matrix (15) is achieved by solving the algebraic Riccatti's equation (14).

It is remarked that this paper does not describe the selection procedures of the design parameters p, r and ω_n of desired transfer function (19) in order to satisfy the design specifications as a maximum overshoot and a setting time in time domain. In AQM control system, the maximum overshoot and the setting time can be similar to a high buffer occupancy at an initial time and a long time to near the steady state, respectively. So, for selecting the values of p, r and ω_n, the detailed selection method is referring to the previous research paper [13].

Remark 3. It is noted that we can set $\rho = 1$ for convenience in this paper. And also, the weighting factor Q should become the positive semi-definite matrix to guarantee the robustness property of LQR.

Summary of design procedures

1. Given the values of N, R_0, and C by AQM model.
2. Get the design parameters p, r and ω_n of desired transfer function in order to consider both the high buffer occupancy at the initial time and the long time to near the steady state.
3. Determine the Q_{00}, Q_{11} and Q_{22} by (26)~(28)
4. Calculate the gain value of g_0, g_1 and g_3 by substituting the weighting factor Q into (14)
5. Determine the proposed PID controller parameters by the relation (18).

IV. SIMULATIONS

We evaluate the performance of the proposed PID controller by simulation using the ns2 [14] simulator.

A. Experiment I

This experiment 1 is a mixture of FTP and HTTP sources as following Fig. 2. The parameters of AQM system are set as follows: The link bandwidth is 15Mb/s(3750 packets), and the propagation delays for the flows range uniformly between 160 and 240 ms. The buffer maximum capacity is 800 packets, the reference queue size is 200 packets, and the average packet size is 500 Bytes. And the numbers of ftp flows and http sessions are 60 and 180, respectively.

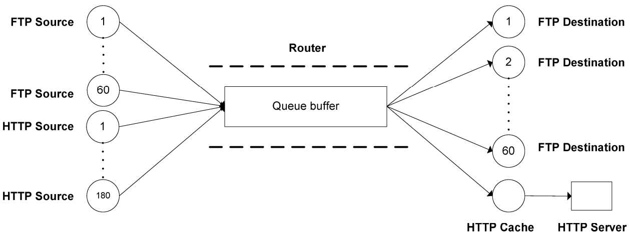

Figure 2. Topology of networks for experiment 1

Figure 3 presents comparisons of PI controller [6] and the proposed PID controller. It shows that the proposed PID controller quickly regulates the queue size to the reference queue size value while the PI controller spends a long time to near the steady state. And also, the proposed PID controller indicates that the high buffer occupancy at initial time, that is a maximum overshoot in time domain, is smaller than the PI controller.

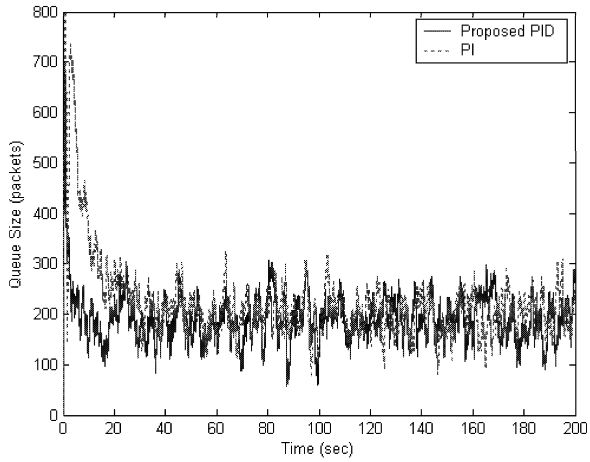

Figure 3. Comparison of the proposed PID controller and PI controller

V. Conclusion

This paper presents the robust PID controller based on linear quadratic approach for AQM control system. The parameters of PID controller are determined by establishing the relationships between the design parameters of desired (target) transfer function to satisfy the design specifications as the maximum overshoot and setting time, that is similar to the high buffer occupancy at initial time and the long time to near the reference queue size, respectively. And, the proposed PID controller can guarantee the phase margin and gain margin because it can be designed by LQR. The simulation result shows that the proposed controller can effectively regulate the queue size to the reference queue size.

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