Proceedings of the International MultiConference of Engineers and Computer Scientists 2008 Vol II
IMECS 2008, 19-21 March, 2008, Hong Kong

On Reconstruction of Signals

H. Fathabadi

***Abstract*—In this paper, we consider a signal which has been sampled synchronously with sampling frequencies equal to bandwidth (B.W) and 1.5 B.W. So, sampling is uniform and sampled signals are undersampled. Then two undersampled signals will be transmitted by two parallel transmission channels that the bandwidth of each channel is only %50 of bandwidth of the signal. Finally, a set of equations will be proposed to reconstruct the original signal from two transmitted signals. Based on these equations, block diagram of the system will be presented.**

***Index Terms*— Signal, reconstruction, aliasing.**

I. Introduction

In many contexts, processing of discrete – time signals is more flexible and is often preferable to processing of continuous – time signals, in part because of increasing availability of inexpensive, light – weight, programmable and easily reproducible digital and discrete – time systems. For example, in communication systems, continuous signals such as speech are sampled and converted to discrete – time signals and then they are quantized in order to convert to digital signals. Then, digital signals are modulated as PCM, PAM and etc. Later, modulated digital signals are transmitted. Conversely, in destination, again digital signals are converted to discrete – time signals and then, to continuous – time signals. Other example is digital control systems which work in similar manner.

A continuous – time signal is converted to discrete – time signal by sampling and under certain conditions original continuous – time signal can be completely recovered from its sampled signal. This somewhat surprising property follows from a basic result which is referred to as the sampling theorem. Much of the importance of the sampling theorem also lies in its role as a bridge between continuous – time signals and discrete – time signals [1]. Recently, reconstruction of a signal from its samples has attracted considerable attention. Most results addressing this problem available in the signal processing literature [2], [3], [4], [5]. Also there are many works in reconstruction and sampling of signals [6], [7]. In this paper, we consider an signal $x(t)$ which can be sampled synchronously with sampling frequencies $\omega_{S_1} = \omega_M$ and $\omega_{S_2} = 1.5\omega_M$. Where ω_M is the bandwidth of $x(t)$. As we see, ω_{S_1} and ω_{S_2} are less than $2\omega_M$. So, we have aliasing in the spectrums of sampled signals. With considering above conditions, a set of essential equations will be proposed for exact reconstruction of $X(\omega)$, where $X(\omega)$ is the spectrum of the original signal $x(t)$.

Manuscript received Dec. 1, 2007. Hassan Fathabadi is with the Electrical Engineering of I. K. I. University, Daneshgah Blv. Qazvin, Iran, P.O.Box: 34149.(tel: 0098-0281-3780038; fax: 0098-281-3780073; e-mail: h4477@hotmail.com). **Preferred address for correspondence:** Hassan Fathabadi- No. 27 - Ghazanfar Khavand Alley - 30 Metry Jey Blover- Tehran-Iran .P.O.Box: 13517-53593.

II. Undersampled Signal

Consider the signal $x(t)$ and the spectrum of it $X(\omega)$ which is shown in Fig. 1. As we see $x(t)$ is band-limited signal with $X(\omega) = 0$ for $|\omega| > \omega_M$. Let us, define the impulse train

$$p(t) = \sum_{m=-\infty}^{+\infty} \delta(t - mT) \tag{1}$$

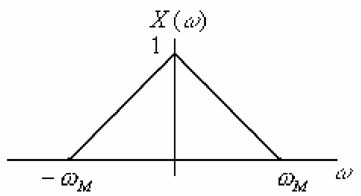

Fig. 1. Spectrum of the original signal.

where $p(t)$ is referred to as the sampling function, T as the sampling period, and the fundamental frequency of $p(t)$, $\omega_S = \frac{2\pi}{T}$, as the sampling frequency. In time domain, we define

$$x_P(t) = x(t)p(t) \tag{2}$$

where $x_P(t)$ is an impulse train with the amplitude of impulses equal to the samples of $x(t)$ at intervals spaced by T, that is,

$$x_P(t) = \sum_{m=-\infty}^{+\infty} x(mT)\delta(t - mT) \tag{3}$$

Since, we have

$$X_P(\omega) = \frac{1}{2\pi}[X(\omega) * P(\omega)] \tag{4}$$

and with using the fact that

$$P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_S) \tag{5}$$

it follows that

$$X_P(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_S)\,. \tag{6}$$

That is, $X_P(\omega)$ is a periodic function of frequency consisting of a sum of shifted replicas of $X(\omega)$ scaled by $\frac{1}{T}$ as illustrated in Fig. 2. In Fig. 2(a), $\omega_S > 2\omega_M$, and thus, there is no overlap between the shifted replicas of $X(\omega)$, whereas in Fig. 2(b) with $\omega_S < 2\omega_M$, there is overlap [1]. For the case illustrated in Fig. 2(a), $X(\omega)$ is faithfully reproduced at integer multiples of sampling frequency. Consequently, if $\omega_S > 2\omega_M$, $x(t)$ can be recovered exactly from $x_P(t)$ by means of a low-pass filter with gain T and a cutoff frequency greater than ω_M and less than $\omega_S - \omega_M$. As we know, this basic result

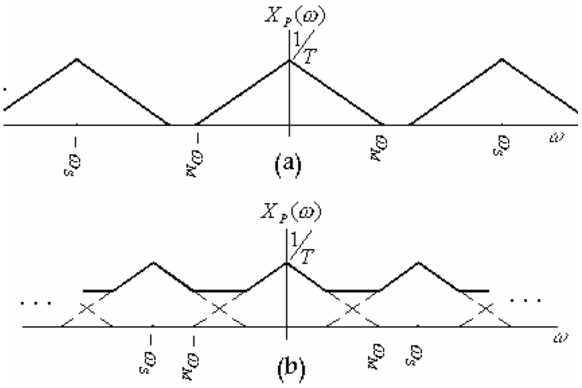

Fig. 2. (a) Spectrum of sampled signal with $\omega_S > 2\omega_M$.
(b) Spectrum of sampled signal with $\omega_S < 2\omega_M$.

, referred to as the sampling theorem [1].

When $\omega_S < 2\omega_M$, $X(\omega)$ is no longer replicated in $X_P(\omega)$ and thus is no longer recoverable by low-pass filtering. This effect, in which the individual terms in equation (6) overlap, is referred to as aliasing, and in this case, $x_P(t)$ is said the undersampled signal of $x(t)$. Undersampled signals are very important. In practice, they are applied in filtering, radar, sonar and etc [8], [9], [10]. For example, blind signal in FM echoes, can be separated with using undersampled signals [11]. Even, in some cases, reconstruction of a signal from other parameters is very important [12].

III. Reconstruction

Again, consider the band-limited signal $x(t)$ and the spectrum of it $X(\omega)$ which is shown in Fig. 1. Suppose that, $x_{P_1}(t)$ and $x_{P_2}(t)$ are samples of $x(t)$ with sampling frequencies $\omega_{S_1} = \omega_M$ and $\omega_{S_2} = 1.5\omega_M$ respectively. So, we have

$$X_{P_1}(\omega) = \frac{1}{T_1}\sum_{k=-\infty}^{+\infty} X(\omega - k\omega_M) \qquad (7)$$

and

$$X_{P_2}(\omega) = \frac{1}{T_2}\sum_{k=-\infty}^{+\infty} X[\omega - k(1.5\omega_M)] \qquad (8)$$

Where $T_1 = \frac{2\pi}{\omega_M}$, $T_2 = \frac{2\pi}{1.5\omega_M}$ and also $X_{P_1}(\omega)$ and $X_{P_2}(\omega)$ are the spectrums of $x_{P_1}(t)$ and $x_{P_2}(t)$ respectively. Equation (8) with using the fact that

$$X(\omega) = 0 \text{ for } |\omega| > \omega_M \qquad (9)$$

gives

$X(\omega) = T_2 X_{P_2}(\omega) \text{ for } |\omega| < 0.5\omega_M$.

(10) Also from (7) with noting (9), we have

$$X(\omega + \omega_M) = T_1 X_{P_1}(\omega) - X(\omega) \text{ for } -0.5\omega_M \le \omega \le 0 \qquad (11)$$

and

$$X(\omega - \omega_M) = T_1 X_{P_1}(\omega) - X(\omega) \text{ for } 0 \le \omega \le 0.5\omega_M \qquad (12)$$

So, $X(\omega)$ can be reconstructed exactly with using (10), (11) and (12).

Consider the block diagram of baseband signal transmission system that is shown in Fig. 3. The block diagram has been made with using above proposed method. The system consists of three sections, the transmitter, two parallel transmission channels, and receiver. Where t_d is the delay time of each transmission channel and the block diagram of the receiver has been made based on equations (10), (11) and (12).

As we see, baseband signal transmission can be done by two parallel transmission channels that the bandwidth of each channel is only one half of the bandwidth of the signal.

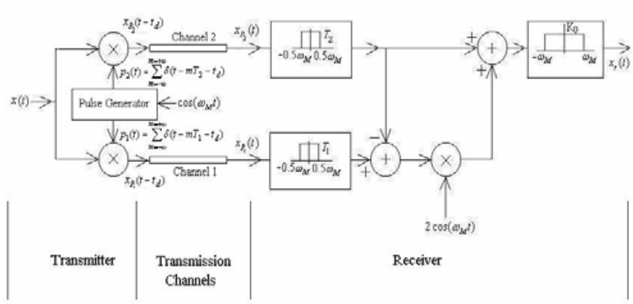

Fig. 3. The block diagram of the system.

IV. Conclusion

In this paper, we considered a signal which was sampled synchronously with sampling frequencies equal to B.W and 1.5 B.W. Two undersampled signals were transmitted by two parallel transmission channels that the bandwidth of each channel was only one half of bandwidth. Equations (10), (11) and (12) proposed to reconstruct the original signal from two transmitted signals.

References

[1] A. V. Oppenheim, A. S. Willsky and I. T. Young , Signals and Systems. Prentice-Hall: Englewood Cliffs, NJ, 1983.
[2] R. Venkataramani and Y. Bresler, "Sampling theorems for uniform and periodic nonuniform MIMO sampling of multiband signals," IEEE Trans. Signal Processing, vol. 51, pp. 3152 - 3163, Dec. 2003.
[3] Tseng Ching-Hsiang, "Bandpass sampling criteria for nonlinear systems," IEEE Trans. Signal Processing, vol. 50, pp. 568 - 577, Mar. 2002.
[4] Choi Kwonhue and Lee Joon-Ho, "Phase discontinuity-free sampling timing control for IF sampling receiver," IEEE Signal Processing Letters, vol. 11, pp. 810 - 812, Oct. 2004.
[5] R.S. Prendergast, B.C. Levy and P.J. Hurst, "Reconstruction of band-limited periodic nonuniformly sampled signals through multirate filter banks," IEEE Trans. Circuit and Systems I, vol. 51, pp. 1612 - 1622, Aug. 2004.
[6] Z. Cvetkovic and J.D. Johnston, "Nonuniform oversampled filter banks for audio signal processing," IEEE Trans. and Audio Processing, vol. 11, pp. 393 - 399, Sep. 2003.
[7] P.A.A. Esquef and L.W.P. Biscainho, "An efficient model-based multirate method for reconstruction of audio signals across long gaps," IEEE Trans. Audio, and Language Processing, vol. 14, pp. 1391 - 1400, Jul. 2006.
[8] A. Santraine, S. Leprince and F. Taylor, "Multiplier-free bandpass channelizer for undersampled applications," IEEE Signal Processing Letters, vol. 11, pp. 904-907, Nov. 2006.

[9] R. S. Prendergast and T. Q. Nguyen, "Optimal filter bank reconstruction of periodically undersampled signals," in Proc. IEEE Conf. Acoustics, , Signal Processing, vol. 4, pp. 201 – 204, Mar. 2005.
[10] J. L. Zolesio and B. Olivier, "Signal reconstruction via under-sampled signals," in Proc. International Conf. Radar 92, pp. 304 – 307, Oct. 1992.
[11] K. Tsuruta, K. Teramoto, K. Kido and H. Mori, "Blind signal separation with undersampled signals for frequency modulated echoes," in Proc. 41st SICE Annual Conf., vol. 5, pp. 3232 – 3235, Aug. 2002.
[12] Sun Wenchang and Zhou Xingwei, "Reconstruction of band-limited signals from local averages," IEEE Trans. Information Theory, vol. 48, pp. 2955 - 2963, Nov. 2002.