

Performance of Rotated Constellation and Trellis Coded Modulation with Channel State Information Errors :MIMO Systems

Sameru Sharma, Derick Engles and S.S. Pathak

Abstract-- Increasing need for high data rate transmission and reliability of communication over fading channels using multiple transmit antennas has drawn attention to high spectral efficiency modulation schemes such as Quadrature Amplitude Modulation (QAM). With the aim of increasing the ‘Diversity Order’ of signal set we consider multidimensional rotated QAM constellation. Increasing diversity order ‘L’ is not the only design parameter, there are some other important parameters like minimum product distance ‘ $d_{p,min}$ ’ and the product kissing number ‘ τ_p ’ which influence the system performance. In this paper, the suitable rotation has been worked out for Multidimensions using the 16QAM & 64QAM constellations and the corresponding kissing numbers have been checked. The combined architecture for MIMO (multiple- input multiple- ouput) Antenna system involving the concept of rotated constellation and concatenated with trellis coded modulation (TCM) is designed and simulated using the low complexity lattice decoder at the receiver. The performance of rotated and non-rotated constellation with Channel State Information (CSI) error has been evaluated.

Keywords -- CSI, Diversity, Multi Antenna, QAM, TCM.

I. Introduction

Recently, the field of multi-antenna processing has attracted large interest in the communication community due to the huge capacity of the multi-antenna environment [10]. Generally MIMO techniques / algorithms aim at data rate maximization or diversity maximization, looking for performance enhancement. There are also limits set for the trade off viz. DMG trade off (Diversity – Multiplexing Gain) [1], [9]. The technique used to get closer to these theoretical capacity limits fall under the realm of space time coding (STC) [8]. To mitigate fading many communications systems use a space diversity.

The error probability of a multidimensional signal set is essentially dominated by four factors. To improve performance it is necessary to:

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- i) minimize the average energy per constellation point.
- ii) Maximize the diversity L
- iii) Maximize the minimum L-product distance.

$$d_{p,min} = \min \prod |x_i - y_i| \quad (1)$$

x_i and y_i being constellation points

- iv) Minimize the product kissing number ‘ τ_p ’ for the L-product distance i.e. the total number of points at the minimum L-Product distance.

Boutros [3] have verified the diversity order ‘L’ & the minimum product distance, ‘ $d_{p,min}$ ’ and found that as diversity increases the bit error rate curves approach the one for the Gaussian channel. But doubling of the diversity and $d_{p,min}$ are not the only important design parameters to increase the performance. In this paper, work has been carried for the third design parameter i.e. product kissing number. The product kissing number ‘ τ_p ’ is a critical design parameter to improve performance.

The primary contribution will be evaluation of new algorithm approach for signal set design that achieves high level of diversity. Utilization of error control coding at the signaling stage of the constellation is apparently an alternative to improve the performance[2][4].

Lattices Code are used in digital transmission as high rate signal constellation. They are obtained by carving a finite number of points from an n-dimensional lattice in the euclidean space. The lattice codes are effective because they present high modulation diversity L i.e. any two code vectors always differ in at least L coordinates[5][6][7]. The effort in this paper is to design a lower complexity lattice decoding algorithm. A new combined architecture for two antennas system involving the concept of rotated constellation and concatenated with Trellis Coded Modulation (TCM) is designed using the low complexity lattice decoder at the receiver and the performance of rotated and non rotated constellations has been evaluated. The essential part of the most of the MIMO Space Time Architecture is the need for the CSI at the receiver, so is the case with the architecture that has been presented in this paper.

II. Diversity Multiplexing Tradeoff

The key point to increase the modulation diversity is to apply a certain rotation to a classical signal constellation in such a way that any two points achieve the maximum number of distinct components. In this paper, the rotated versions of a multi dimensional QAM constellation has been worked out and is as given below.

A. Dimension 2

All two dimensional orthogonal matrices have the following structure:

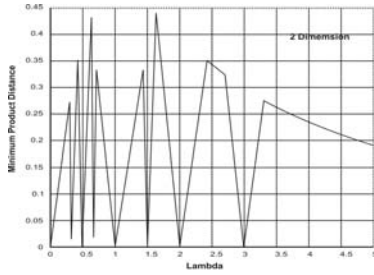


Fig. 1. The minimum product distance vs λ for 16-QAM

$$M_2 = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \text{ with the constraint } a^2 + b^2 = 1$$

Parametrizing this orthogonal matrix as a function of single variable λ gives

$$a = 1 / \sqrt{1 + \lambda^2} \text{ and } b = \lambda a. \quad (2)$$

By varying the λ value in steps of 0.005 and for each value computing all the product distances and picking the least of it, the dp_{\min} has been plotted as a function of λ (fig. 1). From the graph in fig. 1, we pick the value of $\lambda = 1.71$ (maximum case). This leads to the rotation matrix for the 2 dimensional case as

$$M_2 = \begin{pmatrix} 0.50 & 0.86 \\ -0.86 & 0.50 \end{pmatrix}, \text{ the subscript 2 indicating}$$

that it is the 2 dimensional case.

B. Dimension 4

The family of 4- dimensional orthogonal matrices considered here is

$$M_4 = \begin{pmatrix} a & b & -c & -d \\ -b & a & d & -c \\ c & d & a & b \\ -d & c & -b & a \end{pmatrix} = \begin{pmatrix} M_2 & -M_1 \\ M_1 & M_2 \end{pmatrix} \quad (3)$$

where M_2 (submatrix) is fixed as the optimal 2 dimensional matrix and its parameter is used (renamed as λ_2). The other 2x2 submatrix M_1 is dependent on the parameter λ . The orthogonality constraint reduces to $ad - bc = 0$. This gives rise to the following equations:

$$a = \frac{1}{U \sqrt{1 + \lambda_2^2}} \quad b = \frac{\lambda_2}{U \sqrt{1 + \lambda_2^2}}$$

$$c = \frac{\lambda}{U \lambda_2} \quad d = \frac{\lambda}{U}$$

$$\text{Where } U = \frac{\sqrt{\lambda_2^2 + \lambda^2 + \lambda^2 \lambda_2^2}}{\lambda_2}$$

The minimum product distance variation and the kissing number plots are shown in fig. 2 and 3. From the plot, the value $\lambda = 2.05$ is the optimum parameter that gives $dp_{\min} = .0222$. The four dimensional rotation matrix ($M=2$ antennas & $T=2$ symbol periods and MT determines the dimension)

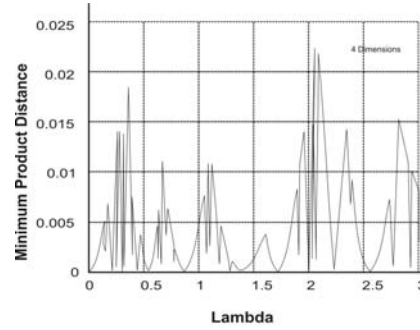


Fig. 2. The minimum product distance vs λ for 16-QAM

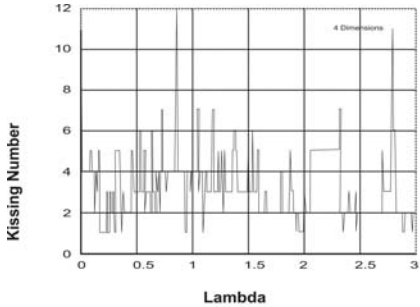


Fig. 3. The Kissing number vs λ for 16-QAM

may be constructed in a Hadamard way. The values of the rotation matrix have hence been computed as:- $a = 0.2012$, $b = 0.33$, $c = 0.49$, $d = 0.79$

Hence the rotation matrix become

$$M_4 = \begin{pmatrix} 0.2 & 0.33 & -0.49 & -0.79 \\ -0.33 & 0.2 & 0.79 & -0.49 \\ 0.49 & 0.79 & 0.2 & 0.33 \\ -0.79 & 0.49 & -0.33 & 0.2 \end{pmatrix} \quad (4)$$

This work involves the utilization of the 4 dimensional rotated constellation in a 2x2 antenna system.

Results

It has been verified that product kissing number ' τ_p ' which is a critical design parameter is equal to 1 for relevant ' dp_{\min} ' value. Simulation of multi dimensions were done to arrive at the optimum rotation to maximize the diversity base on the criteria of maximization of the minimum product distance and minimization of the kissing number. It has been verified that the variation of the minimum product distance are same for higher constellation though the graphs are not included here.

III. Construction of the Rotated Constellation for 2 antenna system

System Model

Consider a system of M transmit N receive antennas. An space time (ST) modulation associates with each $q \times 1$ information symbol vector $s = (s_1, \dots, s_q)$ from a q -dimensional input constellation S^q , an $M \times T$ modulation matrix $B(s) \in \mathbb{C}^{M \times T}$, with \mathbb{C} the field of complex numbers, where M modulated symbols $b_{m,t}$, for $m=1, \dots, M$ are transmitted simultaneously from all transmit antennas at time t for $t = 1, \dots, T$. The modulation B has a

transmission rate of q/T symbols per channel use (PCU). Let X be the $N \times T$ received signal matrix, H be the $N \times M$ channel matrix, and W the $N \times T$ noise matrix. The received signal in matrix form, is :

$$X = HB + W \quad (5)$$

Under quasi-static fading, assuming perfect CSI at the receiver, the pairwise error probability (PEP) of the ML detection of the information vector s' knowing that the information vector $s \neq s'$ has been transmitted is upper bounded by

$$\Pr(s \rightarrow s') \leq \left(\prod_{j=1}^r \lambda_j \right)^{-N} \left(\frac{\bar{E}_s}{8\sigma^2} \right)^{-rN} \quad (6)$$

Where \bar{E}_s is the average energy per symbol, n is the minimum rank of the set of matrices $B(s-s')$ for all pairs of information vectors $s \neq s'$, and $\lambda_j, j = 1 \dots r$, are the non-zero eigenvalues of

$$A(s-s') \underline{\underline{B}} B(s-s') B^H (s-s')$$

with the subscript H denoting the conjugate transpose. Then minimizing the PEP is equivalent to the following criteria

- The Rank Criteria : the minimum rank r of $B(s-s')$ taken over all distinct information vector pairs (s, s') , is the diversity gain.
- The Determinant Criterion : the minimum of geometric mean of the nonzero eigenvalues of $A(s-s')$, taken over all distinct pairs (s, s') is the coding gain and should be maximized.

IV. Low complexity lattice decoder for rotated Constellation with 2 Antenna System

Imposing of a lattice structure necessitates, rewriting of the rotated constellation presented in (5). The equivalent uncoded system is given by

$$\text{Vec}(X^T) = \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ 0 & 0 & h_{11} & h_{12} \\ h_{21} & h_{22} & 0 & 0 \\ 0 & 0 & h_{21} & h_{22} \end{pmatrix} M_{p4} s^T + \text{Vec}(W^T) \quad (7)$$

$$= H_p M_{p4} s^T + \text{Vec}(W^T)$$

where h_{ij} refers to the fading components between the i^{th} receive and j^{th} transmit antennas, $\text{Vec}(X^T)$ is the vector representation of the matrix X^T (superscript T refers to transpose) by putting all its columns one after another in one vector column. M_{p4} is the rotation matrix M_4 with the third line multiplied by -1 , the received signal could be written as

$$X' = \text{Vec}(X^T) = H's^T + W' \quad (8)$$

Where $H' = H_p M_{p4}$ of rank 4, since the rank of H_p is rank 4. Finally the lattice representation of the resulted system is given by

$$X'' = uM_{H'} + W'' \quad (9)$$

The dimension increase is from $M = 2 \rightarrow 2 \times 2 \times 2 = 8$ (2 Time slots and 2 antennas) Fig. 4 presents the flowchart of

low complexity lattice decoder algorithm. The Principle of the algorithm is to search the closest lattice point to the receive signal within a sphere of radius \sqrt{C} centered at the receive signal. This guarantees that only the lattice points within the square distance C from the receive point are considered. This algorithm begins its search near the centre of the sphere. Each time a valid lattice point is found, the search is restricted further by reducing the radius of the sphere to be equal to the distance of the newly discovered lattice point from the sphere centre.

The algorithm uses several specific functions and variable. The function $\text{Chol}(G_M)$ computes a normalized cholesky factorization of the Gram matrix of M_H (i.e. the Gram Matrix is $G_M = M_H M_H^T$). The decomposition is of the form $G_M = RR^T$ with R an upper triangular matrix with entries r_{ij} . When a vector inside the sphere is found, the square distance between this vector and the receive vector (the centre) is given by :

$$\tilde{d}^2 = T_n - T_1 + q_{11}(S_1 - z_{1,x1})^2 \quad (10)$$

This value is compared to the minimum square distance d^2 (initially $d^2 = C$) found so far in the search. If \tilde{d}^2 is smaller than d^2 then we have a new candidate closest point and the search continues until all the vectors inside the sphere are tested. The variable d^2 is the current squared radius of the sphere and \hat{u} is the newly found lattice point. Wherever a valid lattice point x is found within the sphere, in addition to reducing the current squared radius d^2 so that the newly discovered lattice point lies on the surface of the sphere, the algorithm recomputes all the lower and upper bounds according to the new d^2 value.

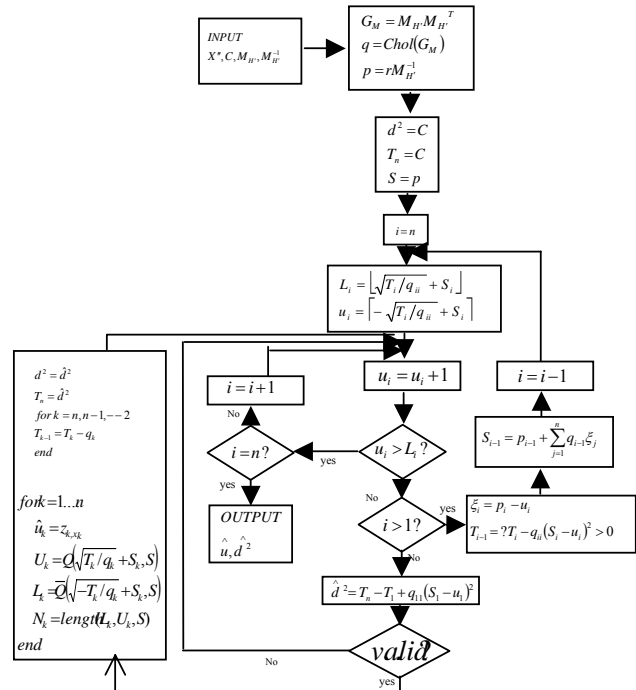


Fig. 4. Flow chart of the low complexity lattice decoder

V. Proposed System architecture

The proposed architecture is given in fig. 5. The input is a binary source, which goes through a TCM stage. This output is fed to the constellation mapper in sets of 4 symbols at a time. The constellation mapper (constellation rotation) provides a set of 4 symbols, which are transmitted through the two transmitters in two time slots. The channel is the Rayleigh fading channel and at the receiver there are two antennas which feed the received signals to the sphere decoder. The output of the sphere decoder is sent to the low complexity lattice decoder with trace back length of 3, whose output is the received bits.

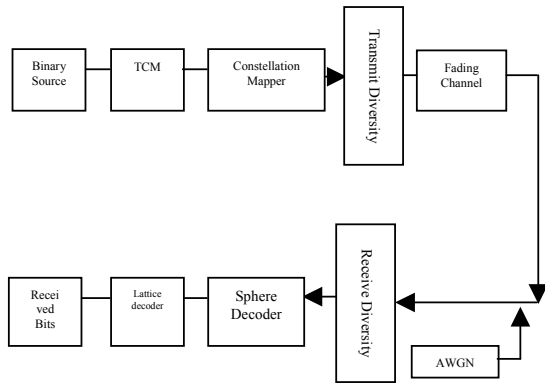


Fig. 5. The proposed system architecture

VI. Simulation Results and Performance Analysis

The proposed architecture was simulated in MATLAB environment and the performance was studied. The channel matrix H is assumed to be available at the receiver. The fading components of the matrix are modeled as complex Gaussain noise with variance of 0.5 per dimension; this gives rise to the Rayleigh fading channel. 10,00,000 input bits have been considered as input to the proposed architecture. Complex AWGN noise, i.e. the W matrix is added at the receiver so that the noise variance per dimension is given by $10^{-SNR/10}$. The choice of the radius for the sphere decoder, \sqrt{C} was varied as a function of SNR and the values were set by trail and error for the various SNR values. The curves in fig 6 represent the average bit error rate (BER) as a function of the SNR in dB at the receiver.

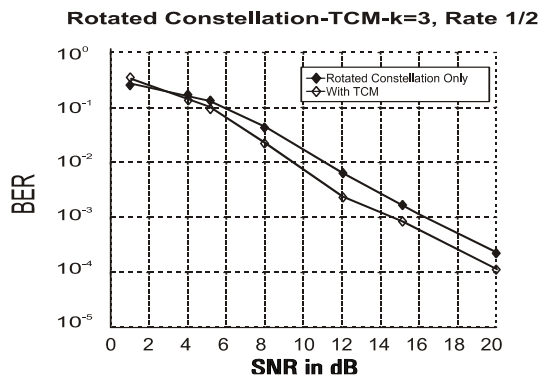


Fig. 6 . Performance of architecture with Rotation TCM stage

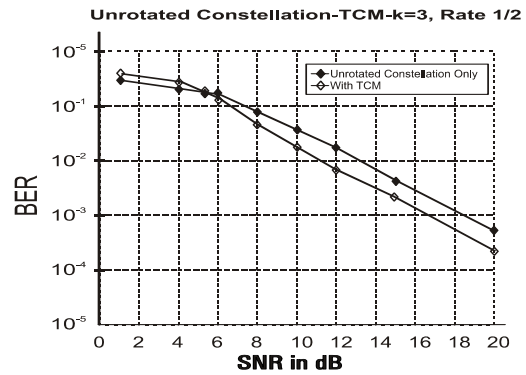


Fig. 7. Performance of architecture without Rotation with and without TCM stage

The curves in fig 6 represent the average bit error rate as a function of the SNR in dB at the receiver. The two plots in the graph correspond to the performance when only the rotation is used and when the rotation is concatenated with the TCM stage. The corresponding data rates of the two cases are 4 bits PCU and 2 bits PCU (Per Channel Use) respectively (since it is rate half convolutional coder). It may be noted that there is a 1.5 to 2 dB improvement in the performance of the system when the TCM stage is introduced. Lowering of BER suitably compensates the loss in capacity.

What has also been considered as a need for comparative study is to look at the performance of a similar architecture if no rotation is given. That is to have the rest of the architecture same and just replace the M_4 matrix unrotated. This can be done by the choice as

$$M_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With this choice, the system shown in fig. 5 becomes an unrotated system. The performance of this system was also carried out on the similar lines and is shown in fig. 7. It is interesting to note that the performance of the unrotated constellation is poorer as compared to the rotated constellation architecture.

VII. Performance of the architecture with CSI errors

There are estimation techniques [11] available to estimate the channel state at the receiver. However, it is always possible that the CSI estimates do have errors inherently and, this necessitates the study of any MIMO architecture under errors in CSI. In this paper the proposed system architecture is evaluated with errors in the estimated CSI.

A Estimation Techniques

Consider N transmit, M receive antenna set up. We can write the received signal samples for the l th symbol within the burst at the optimum sampling instant as

$$y_j(l) = \sqrt{E_s} \tilde{p}(\delta T_s') \sum_{i=1}^N \tilde{\alpha}_{ij}(l) c_i(l) + v_j(l) + \bar{\eta}_j(l) \quad (11)$$

where $(\delta T'_s)$ is the timing error after timing synchronization, $\bar{\eta}_j(l)$ is the AWGN with zero mean and variance $N_o/2$ per dimension, and $v_j(l)$ is the ISI due to the timing error which is modeled as uncorrelated Gaussian noise with zero mean and variance σ_v^2 . $\tilde{\alpha}_{ij}(l)$ are the fading components to be estimated. $\tilde{p}(l)$ is the pulse used for shaping and $c_i(l)$ are the transmitted symbols. This scheme works on pilot symbols being transmitted from the transmitter.

Consider output samples corresponding to the n th pilot sequence within the burst $y((n-1)L_s + 1), y_j((n-1)L_s + 2), \dots, y_j((n-1)L_s + L_p)$. The Channel is assumed to be constant over the duration of the pilot sequence and is equal to the value of the channel in the middle of the pilot sequence. Defining the overall noise term as:

$$z_j(n) = \bar{\eta}_j(n) + v_j(n) \quad (12)$$

We define $Y_j(n)$ as

$$Y_j(n) = [y_j((n-1)L_s + 1) y_j((n-1)L_s + 2) \dots y_j((n-1)L_s + L_p)]^T$$

$$= A_s [P_1 \ P_2 \ \dots \ P_N] \tilde{\alpha}_j(n) + z_j(n) \quad (13)$$

where $A_s = \sqrt{E_s} \tilde{p}(\delta T'_s)$. Using the fact that the pilot pulses P_1, P_2, \dots, P_N are orthogonal, and each of length L_p , we can immediately see that the minimum mean square error (MMSE) estimate of $\tilde{\alpha}_{ij}(l)$ is given by:

$$\hat{\tilde{\alpha}}_{ij}(l) = \frac{P_i^* Y_j(n)}{A_s \cdot \|P_i\|^2}, \quad i = 1, 2, \dots, N \quad (14)$$

Where $\| \cdot \|$ denotes matrix norm. Hence, the estimation of the various fading components is done. It can also be noted that

$$\hat{\tilde{\alpha}}_{ij}(n) = \tilde{\alpha}_{ij}(n) + e_{ij}(n) \quad (15)$$

where $e_{ij}(n)$ is the estimation error due to the noise and ISI given by

$$e_{ij}(n) = \frac{P_i^* z_j(n)}{A_s \cdot \|P_i\|^2}, \quad i = 1, 2, \dots, N \quad (16)$$

Since $z_j(n)$ is assumed to be zero mean Gaussian random vector we have that $e_{ij}(n)$ is also Gaussian with zero mean and variance

$$\sigma_e^2(n) = \frac{\sigma_v^2 + N_o}{A_s^2 \cdot \|P_i\|^2} \quad (17)$$

B Performance of the system with no CSI

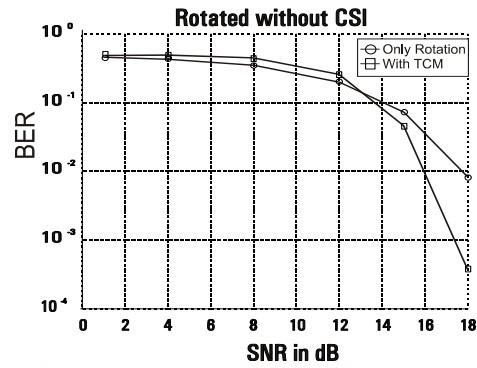


Fig. 8. Performance of proposed architecture with no CSI at receiver

When the CSI is not known at the receiver the channel matrix becomes $H = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ at the receiver. Though, the

fading does take place, since the receiver is not aware of the fading component it is safe to assume as above. However, the problem faced with such a choice of H at the receiver is that this choice is not an orthogonal choice, because of which the Cholesky factorization of the corresponding Gram matrix cannot be performed. The lattice decoding algorithm requires that a Cholesky factorization be performed on the Gram matrix. A matrix has to be positive definite in order that a Cholesky factorization be done on it. In order to make this possible, we force a value other than 1 (but close to it) in one of the components of the H matrix. With this we have the matrix at the receiver as :

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 0.99 \end{pmatrix} \quad (18)$$

This ensures that the corresponding Gram matrix is positive definite and hence Cholesky factorisable. The performance of such a system has been simulated and studied. The performance plot is shown in fig. 8. The performance plot of the nonrotated constellation without CSI is not attached because of its very poor performance. This performance can be compared with the case with perfect CSI to see the degradation in performance; however the TCM stage does come to the rescue of this case and keeps the BER under check at reasonably high SNR atleast.

C. CSI with errors

When looking at the proposed system with CSI errors, we can model the errors in various ways, either as the errors dependent on the SNR or independent of the SNR. We have adopted the way of introducing errors in CSI independent of SNR. The errors in the 2x2 channel CSI, $\begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix}$ are modeled here as zero mean complex

Gaussian variables with variance of 0.01. Hence the overall CSI available at the receiver is $\begin{pmatrix} h_{11} + \epsilon_{11} & h_{12} + \epsilon_{12} \\ h_{11} + \epsilon_{21} & h_{12} + \epsilon_{22} \end{pmatrix}$.

That is, errors have been

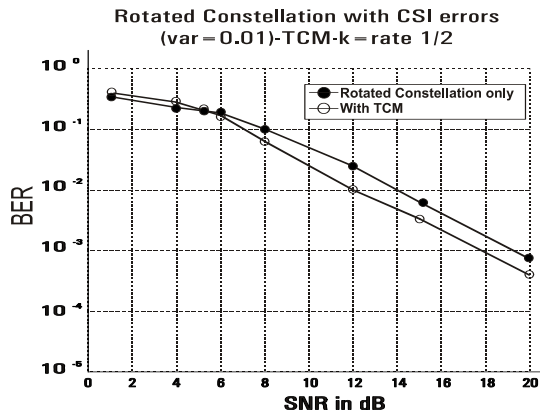


Fig. 9 Performance of proposed architecture with CSI errors

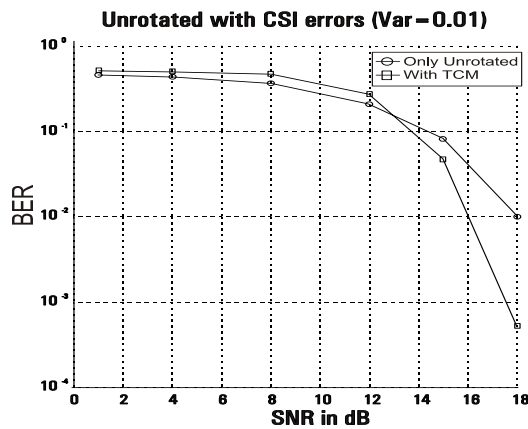


Fig. 10. Performance of unrotated architecture with CSI errors

incorporated in all the 4 channels. Incorporating this error model in the receiver part of the system and carrying out the rest of the simulation as before, the performance plots were obtained. This was done for both the cases of rotated constellation as well as unrotated constellation. The results are in fig. 9 and fig. 10 respectively

VII. CONCLUSION

A new combined architecture for two antenna system involving the concept of rotated constellation and concatenated with TCM using the low complexity lattice decoder at the receiver has shown an improvement of 1 to 1.5 dB in the performance of the rotated constellation as compared to the unrotated constellation. The simulation studies have revealed that this architecture does provide a good performance improvement and better Bandwidth efficiency (2 symbol PCU) as well. Also suitable changes have been incorporated in the existing sphere decoder algorithm by way of adding a feedback mechanism in such a way that provides a huge decrease in complexity and gives rise to systems with high spectral efficiency. To realize the advantage of the proposed system, it was intuitively felt that the system should be evaluated under CSI estimation errors. The architecture was studied under CSI errors as well as the case when no CSI is available at

the receiver, which could happen in case of sudden burst errors wherein the receiver could make a decision that it will assume lack of CSI. This study has shown results wherein the new architecture provides much better performance to that of the nonrotated architecture. The TCM stage comes to show its usefulness in the case of absence of CSI at the receiver. This has justified the use of rotated constellation as well as the use of TCM stage adequately. Intuitively, one can suggest that the TCM stage will find its use in a much more pronounced manner in case of higher (denser) QAM constellation with more number of antennas.

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