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Evaluating Software Quality of Vendors using Fuzzy Analytic Hierarchy Process

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Abstract—**This paper proposes a fuzzy Analytic Hierarchy Process model for evaluating the software quality of vendors. The criteria of software quality adopt the international norm ISO/IEC9126-1:2001 which includes of six criteria. The fuzzy AHP model applies the modified fuzzy Logarithmic Least Squares Method (LLSM) in this software criteria models. The proposed model can help the developers and testers to evaluate the vendors' software applications and select the best alternative under uncertain environment.**

Index Terms—**Fuzzy AHP, Fuzzy Decision Making, Software Quality Assurance, Software Vendor Selection**

I. Introduction

Software systems permeate the modern life. Any failure of the software systems possibly causes many inconvenience or disaster to the people. To meet the standard of software quality, software quality assurance plays an essential role in software development. IEEE [5] defined software quality as (1) the degree to which a system, component, or process meets specified requirements, and (2) the degree to which a system, component, or process meets customer or user needs or expectations. IEEE [5] defined software quality assurance as (1) a planned and systematic pattern of all actions necessary to provide adequate confidence that an item or product conforms to established technical requirements, and (2) a set of activities designed to evaluate the process by which the products are developed or manufactured.

The term software architecture refers to the global structuring of a software system [4]. The design of the architecture should be flexible, extensible, portable and reusable. Usually, when a large scale complex software system is built, third party software components or accessories usually are needed for some functions in the systems with the purpose of reduction of cost, development time, and advantages of the expertise of the third parties. Such incoming components or accessories directly influence the final product. Thus evaluating the incoming components is the vital activity.

For quantitatively and qualitatively evaluating the software quality, a quality metrics model is established. [7,8] reviewed the hierarchical and non-hierarchical models of software quality attributes. This research chooses six attributes in ISO/IEC9126-1: 2001 [6] for discussion.

As the attributes of software quality are identified, the next question is the evaluation and aggregation techniques. Analytic Hierarchy Process [11] and Analytic Network Process [12] are popular models to aggregate multiple criteria for decision making. The limitation is that the measure scale for the value of the utility function, which is basically numerical and probabilistically judgmental, induces evaluation problem. This introduces the studies of fuzzy AHP [e.g. 1,2,3,9,13,14,15] to address the limitation.

The extent analysis method on fuzzy AHP [2] has been used in many studies as it is regarded as less complexity. However, [15] pointed out this method was problematic. [14,15] proposed modified fuzzy Logarithmic Least Squares Method (LLSM) as the appropriate alternative on the basis of [1,13].

The outline of this paper is as follows. Section 2 presents the hierarchical structure using the ISO software quality model which consists of six major attributes. Section 3 presents the computational method using modified fuzzy LLSM model. Section 4 illustrates the numerical example demonstrating the proposed model. Conclusion is in section 5.

II. Hierarchical Models for Software Quality

There are various hierarchical models of software quality attributes such as Factor-Criteria-Metrics Model, McCall's Model, Boehm's Model, FURPS and Dromey's Model [7,8]. This paper typically chooses the ISO/IEC9126-1: 2001 [6] model, which is a well known model, as the measurement criteria to evaluate the software quality. Fig.1 shows the hierarchical model consisting of six criteria, which are defined as follows [6,10]:

1 Functionality (C_1) : the capability of the software product to provide functions which meet stated or implied needs when the software is in use under specified conditions.

2 Reliability (C_2) : the capability of the software product to maintain a specified level of performance when used under specified conditions.

3 Usability (C_3) : the capability of the software product to be understood learned, used, and attractive to the user under specified conditions.

4 Efficiency (C_4) : the capability of the software product to provide appropriate performance, relative to the amount of resources used, under stated conditions.

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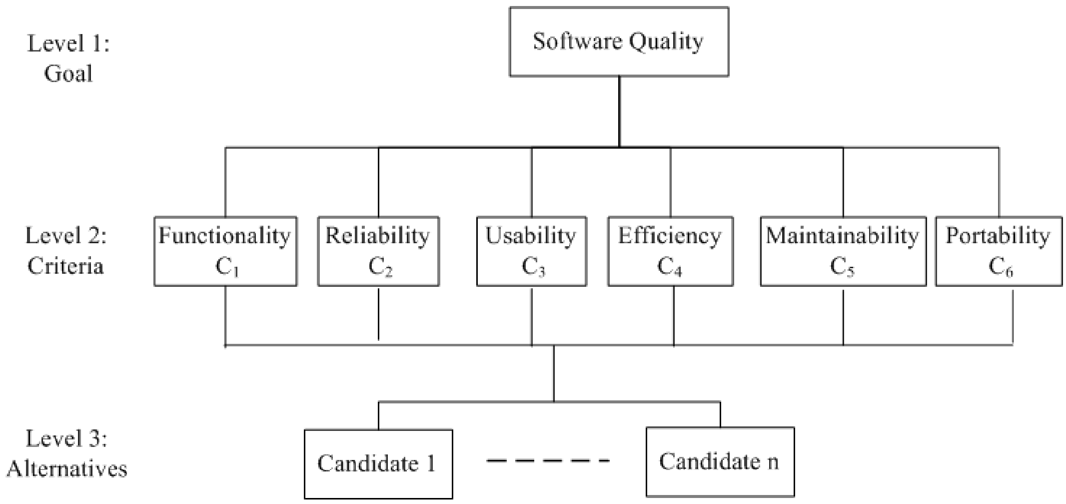

Fig. 1: AHP structure for evaluating Software vendor with respect to ISO six criteria

5 Maintainability (C_5): the capability of the software product to be modified. Modifications may include corrections, improvements or adaptation of the software to changes in environment, and in requirements and functional specifications.

6 Portability (C_6): the capability of the software product to be transferred from one environment to another.

I. Fuzzy AHP

a) Triangular Fuzzy number and its operation

A fuzzy linguistic label can be represented by a fuzzy number which is represented by a fuzzy set. Fuzzy sets capture the ability to handle uncertainty by approximate methods. A triangular fuzzy number (TFN) is applied mostly in the fuzzy theories and application. A TFN is represented by 3-tuple (l,m,u), and its membership has the form:

$$\mu_A(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m}, & m \le x \le u \\ 0, & otherwise \end{cases}$$

where $l \le m \le u$, l and u are the fuzzy boundaries and m is the modal value.

Consider two TFNs $a_1 = (l_1, m_1, u_1)$ and $a_2 = (l_2, m_2, u_2)$. Some of the operational axioms are as follows:

- Addition:
$$\begin{aligned} a_1 + a_2 &= (l_1,m_1,u_1) + (l_2,m_2,u_2) \\ &= (l_1+l_2, m_1+m_2, u_1+u_2) \end{aligned};$$

- Subtraction:
$$\begin{aligned} a_1 - a_2 &= (l_1,m_1,u_1) - (l_2,m_2,u_2) \\ &= (l_1-l_2, m_1-m_2, u_1-u_2) \end{aligned};$$

- Multiplication:
$$\begin{aligned} a_1 \bullet a_2 &= (l_1,m_1,u_1) \bullet (l_2,m_2,u_2) \\ &= (l_1 \bullet l_2, m_1 \bullet m_2, u_1 \bullet u_2) \end{aligned};$$

- Division:
$$\begin{aligned} a_1 / a_2 &= (l_1,m_1,u_1) / (l_2,m_2,u_2) \\ &= (l_1 / l_2, m_1 / m_2, u_1 / u_2) \end{aligned}$$

- Inversion:
$$(l_1,m_1,u_1)^{-1} = (1/(u_1 \cup l_1), 1/m_1, 1/(u_1 \cap l_1)).$$

b) Modified fuzzy LLSM

There are various computational models for fuzzy AHP. This paper chooses the latest research, the modified fuzzy Logarithmic Least Squares Method (LLSM) [14,15], as the fuzzy AHP model. The related comparision with Extent fuzzy AHP model [2] can be referred to [15], and the comparison with LLSM model [1,13] can be referred to [14]. Details of modified fuzzy LLSM are presented as follows:

Consider a fuzzy comparison matrix expressed by

$$\tilde{A} = (a_{ij})_{nxn} = \begin{pmatrix} (1,1,1) & (l_{12},m_{12},u_{12}) & \cdots & (l_{1n},m_{1n},u_{1n}) \\ (l_{21},m_{21},u_{21}) & (1,1,1) & \cdots & (l_{2n},m_{2n},u_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (l_{n1},m_{n1},u_{n1}) & (l_{n2},m_{n2},u_{n2}) & \cdots & (1,1,1) \end{pmatrix} \quad (1)$$

$a_{ij} = (l_{ij}, m_{ij}, u_{ij}) = a_{ji}^{-1} = (1/u_{ji}, 1/m_{ji}, 1/l_{ji})$, and for $i, j = 1,\ldots,n$ and $i \ne j$. $a_{ij} = (1,1,1)$ if $i = j$.

The modified fuzzy LLSM developed in [14,15], which derives the priorities of the triangular fuzzy comparison matrix in (1).

Table 1: Synthesis of local fuzzy weights

Alternatives	Criterion 1	…	Criterion j	…	Criterion m	Global fuzzy weights
	$\left(w_1^L, w_1^M, w_1^U\right)$	…	$\left(w_j^L, w_j^M, w_j^U\right)$	…	$\left(w_m^L, w_m^M, w_m^U\right)$	
A_1	$\left(w_{11}^L, w_{11}^M, w_{11}^U\right)$	…	$\left(w_{1j}^L, w_{1j}^M, w_{1j}^U\right)$	…	$\left(w_{1m}^L, w_{1m}^M, w_{1m}^U\right)$	$\left(w_{A_1}^L, w_{A_1}^M, w_{A_1}^U\right)$
⋮	⋮	…		…	⋮	⋮
A_k	$\left(w_{k1}^L, w_{k1}^M, w_{k1}^U\right)$	…	$\left(w_{kj}^L, w_{kj}^M, w_{kj}^U\right)$	…	$\left(w_{km}^L, w_{km}^M, w_{km}^U\right)$	$\left(w_{A_k}^L, w_{A_k}^M, w_{A_k}^U\right)$
⋮	⋮	…		…	⋮	⋮
A_n	$\left(w_{n1}^L, w_{n1}^M, w_{n1}^U\right)$	…	$\left(w_{nj}^L, w_{nj}^M, w_{nj}^U\right)$	…	$\left(w_{nm}^L, w_{nm}^M, w_{nm}^U\right)$	$\left(w_{A_n}^L, w_{A_n}^M, w_{A_n}^U\right)$

The local weights have following forms:

$$Min\ J = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \left(\left(\ln w_i^L - \ln w_j^U - \ln l_{ij} \right)^2 + \left(\ln w_i^M - \ln w_j^M - \ln m_{ij} \right)^2 + \left(\ln w_i^U - \ln w_j^L - \ln u_{ij} \right)^2 \right)$$

$$\text{Subject to} \begin{cases} w_i^L + \sum_{j=1, j\neq i}^{n} w_j^U \geq 1 \\ w_i^U + \sum_{j=1, j\neq i}^{n} w_j^L \leq 1 \\ \sum_{i=1}^{n} w_i^M = 1 \qquad i = 1,\ldots,n \\ \sum_{i=1}^{n} \left(w_i^L + w_i^U \right) = 2 \\ w_i^U \geq w_i^M \geq w_i^L > 0 \end{cases} \tag{2}$$

The optimum solution to the above model forms normalized triangular fuzzy weights $\tilde{w}_i = \left(w_i^U, w_i^M, w_i^L \right)$, $i = 1,\ldots,n$ to obtain the local fuzzy weights.

After the local fuzzy weights are obtained, then Global fuzzy weight should be calculated with the presentation in table 1. Global fuzzy weights can be obtained by solving the following two linear programming models and an equation for each decision alternative.

$$w_{A_k}^L = \underset{w\in\Omega w}{Min} \sum_{j=1}^{m} w_{kj}^L w_j \ , \quad k = 1,\ldots,K, \tag{3}$$

$$w_{A_k}^U = \underset{w\in\Omega w}{Max} \sum_{j=1}^{m} w_{kj}^U w_j \ , \quad k = 1,\ldots,K, \tag{4}$$

$$w_{A_k}^M = \underset{w\in\Omega w}{Max} \sum_{j=1}^{m} w_{kj}^M w_j^M \ , \quad k = 1,\ldots,K, \tag{5}$$

where $\Omega_w = \left\{ \begin{matrix} W = \left(w_1, \ldots, w_m \right) \mid \\ w_j^U \geq w_j^M \geq w_j^L, \sum_{j=1}^{m} w_j = 1, j = 1,\ldots,m \end{matrix} \right\}$

is the space of weights $\left(w_j^L, w_j^M, w_j^U \right)$ is the normalized triangular fuzzy weight of criterion j $\left(j = 1,\ldots,m \right)$ and $\left(w_{kj}^L, w_{kj}^M, w_{kj}^U \right)$ is the normalized triangular fuzzy weight of alternative A_k with respect to the criterion j $\left(k = 1,\ldots,K;\ j = 1,\ldots,m \right)$.

I. Numerical Example

A company designing and manufacturing Smartphone includes the software and hardware development. Recently the company would like to develop the new model of Smart phone. The company would like to add one accessory application into its product among three candidates A_1, A_2, A_3 with respect to the ISO six criteria in Fig. 1. The following illustrates how modified fuzzy LLSM model is adapted to software quality model.

Firstly, the fuzzy rating scales are defined as:

$Scale = \left\{ H, M, L, I, L^{-1}, M^{-1}, H^{-1} \right\}$, which their triangular fuzzy numbers are defined as follows.

$L = (0.5, 1, 1.5)$,

$M = (1.5, 2, 2.5)$,

$H = (2.5, 3, 3.5)$,

$I = (1, 1, 1)$,

$L^{-1} = (0.5, 1, 1.5)$,

$M^{-1} = (0.4, 0.5, 0.67)$,

$H^{-1} = (0.29, 0.33, 0.4)$.

Secondly, the fuzzy relative importance of the six quality attributes is determined using Eq(2). The input values and the results are shown in table 2.

Thirdly, the experts compare the three candidates A_1, A_2, A_3 under each of six criteria separately. Tables 3-8 show their input comparisons under each criteria, and the comparison results (or local fuzzy weights), which uses Eq(2).

Fourthly, the comparison results under each of six criteria and the fuzzy relative importance are aggregated by Eqs. (3)-(5), and the global fuzzy weights of the three candidates are determined. These details are shown in table 9.

It is clear that A_1 is the best alternative, followed by A_2 and A_3. The result is also illustrated in Fig.2.

Table 2: Fuzzy comparison matrix for the importance of six criteria and their local fuzzy weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	LFW
C_1	(1,1,1)	(0.5,1,1.5)	(1.5,2,2.5)	(0.5,1,1.5)	(1.5,2,2.5)	(1.5,2,2.5)	(0.194,0.231,0.253)
C_2	(0.5,1,1.5)	(1,1,1)	(0.5,1,1.5)	(0.5,1,1.5)	(0.5,1,1.5)	(0.5,1,1.5)	(0.112,0.163,0.227)
C_3	(0.4,0.5,0.67)	(0.5,1,1.5)	(1,1,1)	(1.5,2,2.5)	(0.5,1,1.5)	(0.5,1,1.5)	(0.130,0.163,0.196)
C_4	(0.5,1,1.5)	(0.5,1,1.5)	(0.4,0.5,0.67)	(1,1,1)	(0.5,1,1.5)	(0.5,1,1.5)	(0.108,0.146,0.189)
C_5	(0.4,0.5,0.67)	(0.5,1,1.5)	(0.5,1,1.5)	(0.5,1,1.5)	(1,1,1)	(0.29,0.33,0.4)	(0.099,0.121,0.143)
C_6	(0.4,0.5,0.67)	(0.5,1,1.5)	(0.5,1,1.5)	(0.5,1,1.5)	(2.5,3,3.5)	(1,1,1)	(0.142,0.175,0.206)

Table 3: Fuzzy comparison matrix of the three candidates with respect to functionality C_1 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(2.5,3,3.5)	(2.5,3,3.5)	(0.565,0.594,0.6129)
A_2	(0.29,0.33,0.4)	(1,1,1)	(1.5,2,2.5)	(0.218,0.249,0.282)
A_3	(0.29,0.33,0.4)	(0.4,0.5,0.67)	(1,1,1)	(0.140,0.157,0.181)

Table 4: Fuzzy comparison matrix of the three candidates with respect to reliability C_2 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(2.5,3,3.5)	(1.5,2,2.5)	(0.528,0.548,0.548)
A_2	(0.29,0.33,0.4)	(1,1,1)	(0.5,1,1.5)	(0.164,0.211,0.266)
A_3	(0.4,0.5,0.67)	(0.5,1,1.5)	(1,1,1)	(0.186,0.241,0.308)

Table 5: Fuzzy comparison matrix of the three candidates with respect to usability C_3 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(2.5,3,3.5)	(2.5,3,3.5)	(0.542,0.5842,0.619)
A_2	(0.29,0.33,0.4)	(1,1,1)	(2.5,3,3.5)	(0.256,0.281,0.309)
A_3	(0.29,0.33,0.4)	(0.29,0.33,0.4)	(1,1,1)	(0.125,0.135,0.149)

Table 6: Fuzzy comparison matrix of the three candidates with respect to efficiency C_4 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(2.5,3,3.5)	(2.5,3,3.5)	(0.565,0.594,0.613)
A_2	(0.29,0.33,0.4)	(1,1,1)	(1.5,2,2.5)	(0.218,0.249,0.282)
A_3	(0.29,0.33,0.4)	(0.4,0.5,0.67)	(1,1,1)	(0.141,0.157,0.181)

Table 7: Fuzzy comparison matrix of the three candidates with respect to maintainability C_5 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(0.29,0.33,0.4)	(0.29,0.33,0.4)	(0.142,0.142,0.142)
A_2	(2.5,3,3.5)	(1,1,1)	(0.5,1,1.5)	(0.329,0.429,0.530)
A_3	(2.5,3,3.5)	(0.5,1,1.5)	(1,1,1)	(0.329,0.429,0.530)

Table 8: Fuzzy comparison matrix of the three candidates with respect to portability C_6 and their local fuzzy weights

Candidates	A_1	A_2	A_3	LFW
A_1	(1,1,1)	(0.4,0.5,0.67)	(0.29,0.33,0.4)	(0.168,0.168,0.168)
A_2	(1.5,2,2.5)	(1,1,1)	(0.5,1,1.5)	(0.285,0.388,0.490)
A_3	(2.5,3,3.5)	(0.5,1,1.5)	(1,1,1)	(0.341,0.444,0.547)

Table 9: Synthesis of local fuzzy weights of three candidates and their Global Fuzzy Weight

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion6	Global fuzzy weights
	(0.194,0.231,0.253)	(0.112,0.163,0.227)	(0.130,0.163,0.196)	(0.108,0.146,0.189)	(0.099,0.121,0.143)	(0.142,0.175,0.206)	
A_1	(0.565,0.594,0.6129)	(0.528,0.548,0.548)	(0.542,0.5842,0.619)	(0.565,0.594,0.613)	(0.142,0.142,0.142)	(0.168,0.168,0.168)	(0.412,0.455,0.496)
A_2	(0.218,0.249,0.282)	(0.164,0.211,0.266)	(0.256,0.281,0.309)	(0.218,0.249,0.282)	(0.329,0.429,0.530)	(0.285,0.388,0.490)	(0.231,0.294,0.364)
A_3	(0.140,0.157,0.181)	(0.186,0.241,0.308)	(0.125,0.135,0.149)	(0.141,0.157,0.181)	(0.329,0.429,0.530)	(0.341,0.444,0.547)	(0.190,0.250,0.330)

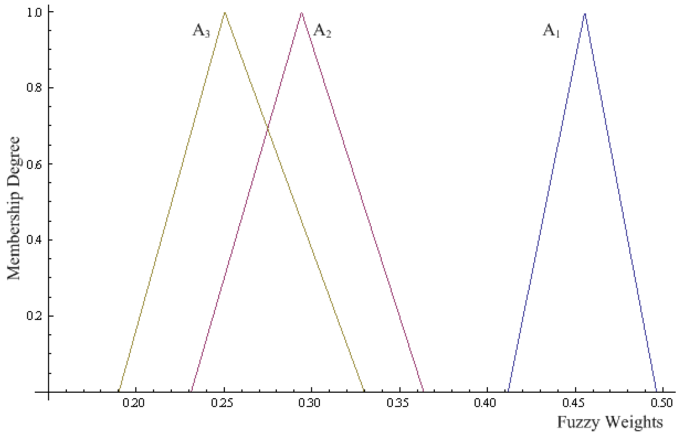

Fig. 2 Global Fuzzy weights of the three candidates

I. Discussion and Conclusion

This research proposes a fuzzy AHP model for software quality evaluation and software vendor selection under uncertainty. The model uses modified fuzzy Logarithmic Least Squares Method. Six attributes of software quality are chosen from ISO/IEC9126. A numerical example illustrates the usability and validity of this model. The limitation of this method is the computational efficiency and complexity which will be discussed and improved in the future research.

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