

# Chaotic Binary Particle Swarm Optimization for Feature Selection using Logistic Map

Li-Yeh Chuang, Jung-Chike Li, and Cheng-Hong Yang, *Member, IAENG*

**Abstract**—Feature selection is a useful technique for increasing classification accuracy. The primary objective is to remove irrelevant features in the feature space and identify relevant features. Binary particle swarm optimization (BPSO) has been applied successfully in solving feature selection problem. In this paper, chaotic binary particle swarm optimization (CBPSO) with logistic map for determining the inertia weight is used. The K-nearest neighbor (K-NN) method with leave-one-out cross-validation (LOOCV) serves as a classifier for evaluating classification accuracies. Experimental results indicate that the proposed method not only reduces the number of features, but also achieves higher classification accuracy than other methods.

**Index Terms**—Feature selection, binary particle swarm optimization, logistic map, K-nearest neighbor, leave-one-out cross-validation.

## I. INTRODUCTION

Feature selection is the process of choosing a subset of features from the original feature set and thus can be viewed as a principal pre-processing tool prior to solving classification problems [1]. Feature selection is a NP-hard problem. The goal is to select a subset of  $d$  features from a set of  $D$  features ( $d < D$ ) in a given data set [2].  $D$  is comprised of all features in a given data set; it may include noisy, redundant, and misleading features. Therefore, an exhaustive search performed in the solution space, which usually takes a long time, often does not work in practice [3]. To resolve these feature selection problems, we aimed at retaining only  $d$  relevant features. Irrelevant features are not only useless for classification, but could also potentially reduce the classification performance. By deleting irrelevant features computational efficiency can be improved and classification accuracy increased.

Three different categories of evaluation criteria can be employed when deciding which features are relevant and which ones should be removed: filters, wrapper and hybrid models [4]. The filter model relies on general characteristics

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of the data to evaluate and select feature subsets without involving any learning algorithm [4]. Wrapper models apply an unsupervised learning algorithm to each subset of features and then evaluate the subset of features by criterion functions [5]. Wrapper models usually have a higher computational cost than the filter modes. Hybrid models attempt to take advantage of the two models by exploiting their different evaluation criteria at different search stages, and have recently been proposed to handle large data sets [4].

We adopted a wrapper model to work on the feature selection problem. In recent years, different evolutionary algorithms have been proposed to obtain near-optimal subsets of solutions. These approaches include genetic algorithms (GA) designed by imitating natural evolution [2], ant colony optimization (ACO)[6], particle swarm optimization (PSO) [1] with swarm intelligence and tabu search (TS) [7] with intermediate memory.

Particle swarm optimization is a search process based on the idea of swarm intelligence in biological populations. In PSO, an information sharing algorithm randomly generates an initial population for the search process. The position and velocity of each particle is adjusted based on its individual experience and the experience of its neighbors. The information is updated due to social interactions between particles.

In this paper, chaotic binary particle swarm optimization (CBPSO) with logistic map for determining the inertia weight is used. The K-nearest neighbor (K-NN) method based on Euclidean distance calculations serves as a classifier for five data sets taken from the literature [2]. Experimental results show that CBPSO can not only reduce the number of features, but also achieves higher classification accuracies.

## II. METHOD

### A. Binary Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population-based evolutionary computation technique developed by Kennedy and Eberhart in 1995 [8]. PSO simulates the social behavior of animals, i.e. birds in a flock or fish in a school. This behavior can be described by a swarm intelligence system. In PSO, each solution can be considered an individual particle in a given search space, which has its own position and velocity. During movement, each particle adjusts its position by changing its velocity based on its own experience, as well as the experience of its neighboring particles, until an optimum position is reached by itself and its neighbor [9]. All of the particles have fitness values based on the calculation of a

fitness function. Particles are updated by following two parameters called  $pbest$  and  $gbest$  at each iteration. Each particle is associated with the best solution (fitness) the particle has achieved so far in the search space. This fitness value is stored, and represents the position called  $pbest$ . The value  $gbest$  is a global optimum value for the entire population.

PSO was originally developed to solve real-value optimization problems. Many optimization problems occur in a space featuring discrete, qualitative distinctions between variables and levels of variables. To extend the real-value version of PSO to a binary/discrete space, Kennedy and Eberhart proposed a binary PSO (BPSO) method. In a binary search space, a particle may move to near corners of a hypercube by flipping various numbers of bits; thus, the overall particle velocity may be described by the number of bits changed per iteration [10].

The position of each particle is represented by  $X_p = \{X_{p1}, X_{p2}, \dots, X_{pd}\}$  and the position of each particle is represented in binary string form and is randomly generated; the bit value {0} and {1} represent a non-selected and selected feature, respectively. The velocity of each particle is represented by  $V_p = \{V_{p1}, V_{p2}, \dots, V_{pd}\}$  (d is number of particles) and the initial velocities in particles are probability limited to a range of {0.0~1.0}. In BPSO, once the adaptive values  $pbest$  and  $gbest$  are obtained, the features of the  $pbest$  and  $gbest$  particles can be tracked with regard to their position and velocity. Each particle is updated according to the following equations [10].

$$v_{pd}^{new} = w \times v_{pd}^{old} + c_1 \times rand \times (pbest_{pd} - x_{pd}^{old}) + c_2 \times rand \times (gbest_{pd} - x_{pd}^{old}) \quad (1)$$

if  $v_{pd}^{new} \notin (V_{min}, V_{max})$  then

$$v_{pd}^{new} = \max(\min(V_{max}, v_{pd}^{new}), V_{min}) \quad (2)$$

$$S(v_{pd}^{new}) = \frac{1}{1 + e^{-v_{pd}^{new}}} \quad (3)$$

If  $(rand < S(v_{pd}^{new}))$  then  $x_{pd}^{new} = 1$ ; else  $x_{pd}^{new} = 0$  (4)

(3)

In Eq.(1),  $w$  is the inertia weight,  $c_1$  and  $c_2$  are acceleration parameters, and  $rand$ ,  $rand_1$  and  $rand_2$  are three independent random numbers between [0, 1]. Velocities  $v_{pd}^{new}$  and  $v_{pd}^{old}$  are those of the updated particle and the particle before being updated, respectively,  $x_{pd}^{old}$  is the original particle position (solution), and  $x_{pd}^{new}$  is the updated particle position (solution). In Eq. (2), particle velocities of each dimension are tried to a maximum velocity  $V_{max}$ . If the sum of accelerations causes the velocity of that dimension to exceed  $V_{max}$ , then the velocity of that dimension is limited to  $V_{max} \cdot V_{max}$  and  $V_{min}$  are user-specified parameters (in our case  $V_{max} = 6$ ,  $V_{min} = -6$ ). The updated features are calculated by the function  $S(v_{pd}^{new})$  (Eq. (3)), in which  $v_{pd}^{new}$  is the updated velocity value. If  $S(v_{pd}^{new})$  is larger than a randomly produced disorder number that is within {0.0~1.0}, then its position value  $S_n$ ,  $n = 1, 2, \dots, m$  is represented by {1} (meaning this feature is selected as a required feature for the next

update). If  $S(v_{pd}^{new})$  is smaller than a randomly produced disorder number that is within {0.0~1.0}, then its position value  $F_n$ ,  $n = 1, 2, \dots, m$  is represented by {0} (meaning this feature is not selected as a required feature for the next update).

Two independent numbers,  $rand_1$  and  $rand_2$  in Eq. (1), affect the velocity of each particle. The proper adjustment of the BPSO parameters  $w$  (inertia weight) and the acceleration factors  $c_1$  and  $c_2$  is an important task. The inertia weight  $w$  controls the balance between the global exploration and local search ability.  $c_1$  and  $c_2$  control the movement of particles. To avoid premature BPSO convergence, the adjustment can not be too excessive, since this might cause extreme particle movements, which it makes impossible to obtain optimized features. Hence, suitable parameter adjustment is paramount.

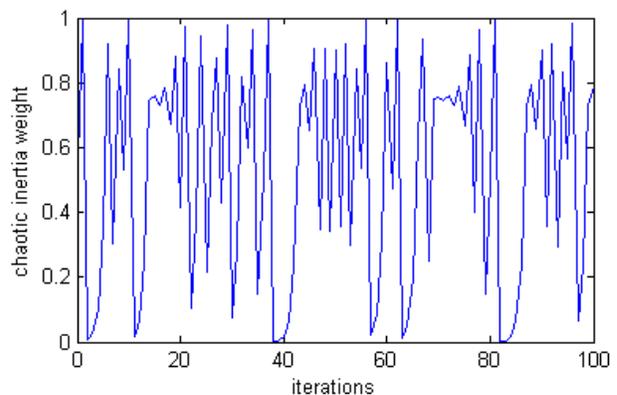
### B. Chaotic Sequences for Inertia Weight

The inertia weight  $w$  controls the balance between the global exploration and local search ability. A large inertia weight facilitates the global search, while a small inertia weight facilitates the local search. How to adjust the inertia weight value is important; it will affect the BPSO search process and through it the classification accuracy.

The BPSO process suffers from getting trapped in a local optimum which results in premature convergence. In order to prevent this from happening, we used chaotic binary particle swarm optimization to overcome the disadvantage. Chaos is a deterministic dynamic system and is very sensitive to initial values. A chaotic map is used to determine the inertia weight value in each iteration. Logistic map is the most frequently used chaotic behavior and is a bounded unstable dynamic behavior. In this paper, we used logistic map to determine the inertia weight value [11]. The inertia weight value is modified according to

$$w(t+1) = 4.0 \times w(t) \times (1 - w(t)) \quad w(t) \in (0, 1) \quad (5)$$

The inertia weight value depends on the chaotic logistic map for total iterations shown in **Figure 1**



**Figure 1** Chaotic inertia weight using logistic map

### C. K-Nearest Neighbor

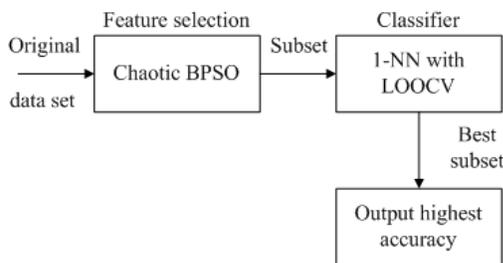
The K-nearest neighbor (K-NN) method is a supervised

learning algorithm introduced by Fix and Hodges in 1951, and is still one of the most popular nonparametric methods [12][13]. The K-NN method is easy to implement since only the parameter K (number of nearest neighbors) needs to be determined. The parameter K is the most important factor affecting the performance of the classification process. In a multidimensional feature space, the data is divided into testing and training samples. K-NN classifies a new object based on the minimum distance from the test samples to the training samples based either their Euclidean or Mahalanobis distance. We used the Euclidean distance in this paper. If an object is near to the number of K nearest neighbors, the object is classified into the K-object category. In order to increase the classification accuracies, the parameter K has to be adjusted based on the different data set characteristics.

In K-NN, a large category tends to have a small classification error, while the classification error for minority classes is usually rather large, a fact that lowers the performance of K-NN [14]. Cross-validation is a useful technique for choosing the parameter K. In this paper, the leave-one-out cross-validation (LOOCV) method was implemented. When there are  $n$  data to be classified, the data is divided into one testing sample and  $n-1$  training samples at each iteration of the evaluation process, and finally a classifier is constructed by training the  $n-1$  training samples. The testing sample category can be judged by the classifier. In this paper, the 1-NN with leave-one-out cross-validation method serves as a classifier to calculate the classification accuracies.

*D. CBPSO -KNN procedure*

Initially, the position of each particle is represented by a binary (0/1) string  $S = F_1 F_2 \dots F_D$ .  $D$  is the dimension of a given data set where 1 represents a selected feature, while 0 represents a non-selected feature. For example, for  $D=10$ , we obtain a random binary string  $S=1000100010$ , which means that only features  $F_1$ ,  $F_5$  and  $F_9$  are selected. The initial velocities in particles are probability limited to a range of  $\{0.0 \sim 1.0\}$ . **Figure 2** shows the three stages employed in this study.



**Figure 2** Simple three stages for feature selection

The classification accuracy of a 1-nearest neighbor (1-NN) determined by the leave-one-out cross-validation method is used to measure the fitness of each particles. CBPSO procedure can be described as follows:

- Step 1** Randomly generate an initial population for CBPSO.
- Step 2** Evaluate fitness values of all particles.
- Step 3** Update the  $pbest$  and  $gbest$  values. Each particle

updates its position and velocity by CBPSO through updates Eqs. (1) (2) (3) (4) (5).

**Step 4** Check the termination criterion. If satisfied, output final solution. Otherwise go to **Step 2**.

The CBPSO was configured to contain 20 particles which the number of particles is equal to the populations of HGAs from the literature. The termination criterion is 100 iterations. The acceleration factors  $c_1$  and  $c_2$  were both set to 2 [15]. The inertia weight  $w(0)$  was 0.48 [11]

III. RESULTS AND DISCUSSION

The data sets in this study were obtained from the UCI Repository [16]. Table 1 illustrates the format of the five classification problems. Three different feature selection problems were classified. If the number of features is between 10 and 19, the sample groups can be considered small; these data sets included the Wine, Vehicle and Segmentation data sets. If the number of features is between 20 and 49, the sample test groups are medium scale problems; these include the WDBC problems. If the number of features is greater than 50, the test problems are large scale problems; this group included the Sonar problem. The classifier used was the 1-NN method with leave-one-out cross-validation for all five data sets.

**Table 1** Format of classification test problems

Data sets	Number of samples	Number of classes	Number of features
Wine	178	3	13
Vehicle	846	4	18
Segmentation	210/2100	7	19
WDBC	569	2	30
Sonar	208	2	60

**Legend:** x/y indicates the number of testing and training samples, respectively

Table 2 compares experimental results obtained by other methods from the literature [2] to the proposed method. The proposed method obtained the highest classification accuracy for the Wine, Vehicle, Segmentation and WDBC classification problems. The classification accuracies of the Wine, Vehicle, WDBC and Segmentation classification problems obtained by the proposed method are 99.44%, 74.35%, 97.54% and 97.92%, respectively. The proposed method obtained a lower classification accuracy for the Sonar classification problem. The classification accuracy for the Sonar classification problem obtained by the proposed method is 93.27%. Figures 3.1-3.5 show the Iterations vs. Classification accuracy for five test data sets.

Classification accuracies were measured by four values ( $D/5$ ,  $2D/5$ ,  $3D/5$ , and  $4D/5$ ) of the total number of features ( $D$ ) from the literature. Optimal classification accuracy was obtained by exhaustive search for various numbers of features. In fact, the optimal number of features for each test problem is unknown and was determined through an extremely cumbersome and time-consuming process compared to the method we propose. In most of the case, the methods from the

**Table 3** Classification accuracies for the test data sets

Datasets	d*	SFS	PTA	SFFS	SGA	HGA (1)	HGA (2)	HGA (3)	HGA (4)	CBPSO-KNN	
										d**	A(%)
Wine (D=13)	3	93.82	93.82	93.82	93.82	93.82	93.82	93.82	93.82	8	<b>99.44</b>
	5	94.38	94.38	94.94	95.51	95.51	95.51	95.51	95.51		
	8	95.51	95.51	95.51	95.51	95.51	95.51	95.51	95.51		
	10	92.13	92.13	92.70	92.70	92.70	92.70	92.70	92.70		
Vehicle (D=18)	4	62.77	64.78	69.15	69.50	69.74	69.74	69.74	69.74	10	<b>74.35</b>
	7	69.15	70.09	73.52	72.97	73.52	73.52	73.52	73.52		
	11	69.50	71.75	71.87	71.84	72.46	72.46	72.46	72.46		
	14	68.20	70.80	70.80	70.80	70.80	70.80	70.80	70.80		
Segmentation (D=19)	4	92.81	92.81	92.81	92.81	92.81	92.81	92.81	92.81	10	<b>97.92</b>
	8	92.95	92.95	92.95	92.95	92.95	92.95	92.95	92.95		
	11	92.95	92.95	92.95	92.95	92.95	92.95	92.95	92.95		
	15	92.57	92.57	92.57	92.57	92.57	92.57	92.57	92.57		
WDBC (D=30)	6	93.15	93.15	94.20	93.67	94.90	94.90	93.99	93.99	8	<b>97.54</b>
	12	92.62	92.97	94.20	94.38	94.38	94.38	94.38	94.38		
	18	94.02	94.20	94.20	93.85	94.20	94.20	94.20	94.20		
	24	92.44	93.50	93.85	93.85	93.85	93.85	93.85	93.85		
Sonar (D=60)	12	87.02	89.42	92.31	93.75	94.71	95.67	95.19	95.67	27	93.27
	24	89.90	90.87	93.75	95.67	96.63	96.63	<b>97.12</b>	<b>97.12</b>		
	36	88.46	91.83	93.27	95.67	96.15	96.15	96.15	96.15		
	48	91.82	92.31	91.35	92.79	92.79	93.27	93.27	93.27		

**Legend:** D: total number of features, d\*: selected number of features, SFS: sequential forward search, PTA: plus and take away, SFFS: sequential forward floating search, SGA: sequential Genetic Algorithm, HGA: hybrid Genetic Algorithm, d\*\*: optimal selected number of features, CBPSO-KNN: Chaotic BPSO with K-NN, A(%): classification accuracy in %, Highest values are in bold-type.

literature, obtained lower classification accuracies, a fact clearly illustrated by the Wine test problem, for which the classification accuracy obtained by the proposed method was higher than the results from the literature, while the number of features selected was the same. From Table 3, GA has been shown to outperform SFS (sequential forward search), PTA (plus and take away) and SFFS (sequential forward floating search) from the literature [2]. In this study, we used three differently sized categories of test problems, namely small, medium and large size problems. If the feature dimensionality is between 10 and 19, the sample groups can be considered small size problems. If the feature dimensionality is between 20 and 49, the sample group test problems are of medium size problems. If the feature dimensionality is over 50, the test problems are large size problems. Results in Table 2 indicate that, CBPSO works well for small and medium size problems, although for high-dimension problems, like the Sonar test problem, CBPSO may become trapped in a local optimal region. Overall, CBPSO has competitive performance in feature selection problems.

A chaos system has certain, ergodic and stochastic properties. Using chaotic behavior in the inertia weight value with logistic map would cause the inertia weight values to fluctuate between [0, 1]. The changed inertia weight values affect the velocities and positions of particles at each iteration. Introducing a fluctuating inertia weight value, the particles to move on to new search regions. Extending the search space region is an important task. Chaotic behavior increases the local and global search ability. A group of particles may search new regions of the solution space and congregate

toward a global or near-global optimum. This extended search in the solution space is equivalent to different subset combinations of features which lead to superior classification.

#### IV. CONCLUSION

In recent years, different evolutionary algorithms have been developed for feature selection. In this paper, a new feature selection method based on chaotic binary particle swarm optimization (CBPSO) is proposed. The classification accuracy is calculated by a 1-NN classifier with LOOCV. The proposed method saves computing time compared to other methods from the literature. CBPSO with chaotic sequences for the inertia weight is applied to feature selection process. Experimental results indicate that the proposed method not only effectively reduces the number of features, but also achieves higher classification accuracy. CBPSO is very competitive compared to other methods and can serve as an ideal pre-processing tool to help optimize the feature selection process.

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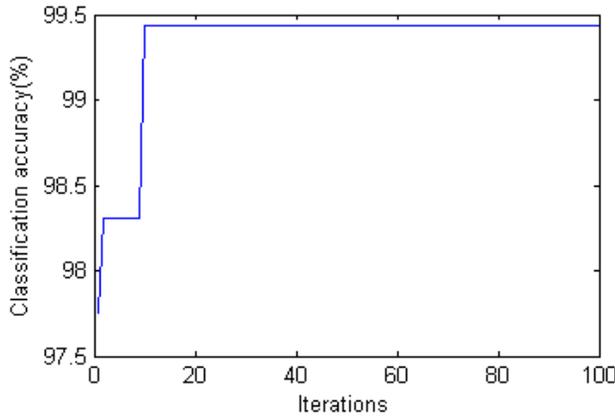


Figure 3.1 Iterations vs. Classification accuracy in Wine

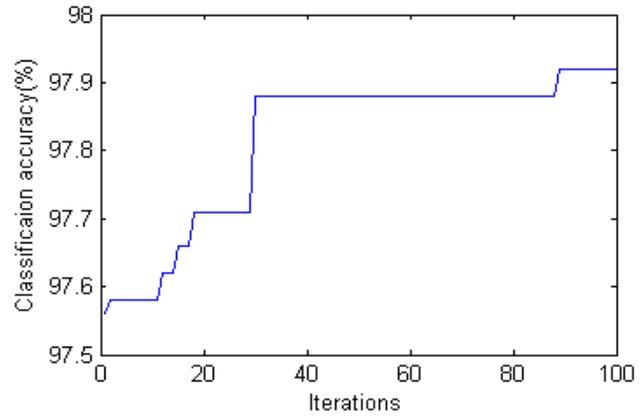


Figure 3.3 Iterations vs. Classification accuracy in Segmentation

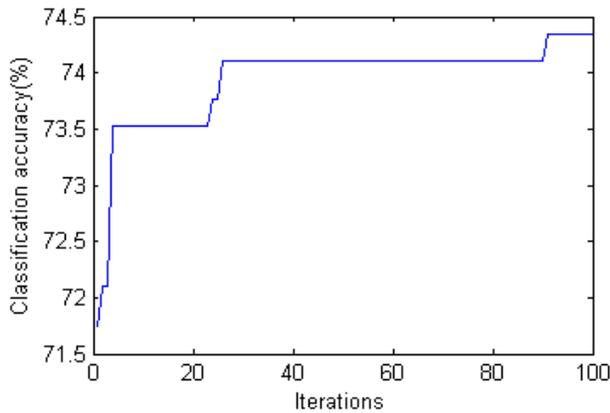


Figure 3.2 Iterations vs. Classification accuracy in Vehicle

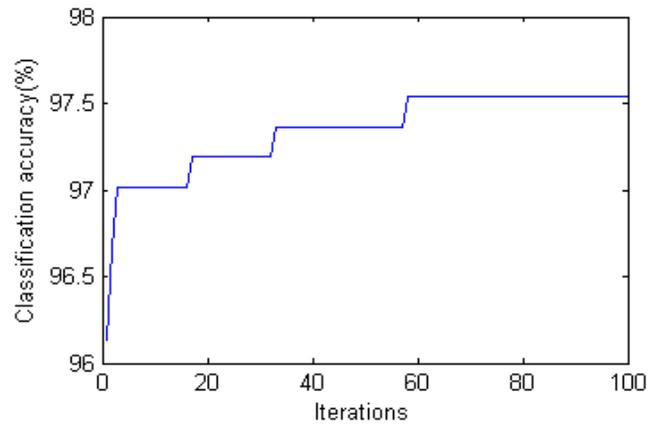
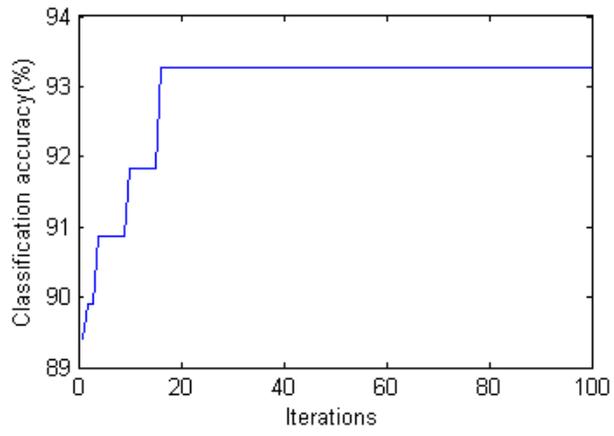


Figure 3.4 Iterations vs. Classification accuracy in WDBC



**Figure 3.5** Iterations vs. Classification accuracy in Sonar