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Robust Stabilization Design of the Uncertain Duffing-Holmes Chaotic System

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***Abstract*—This paper proposes a robust and simple controller design for the Duffing-Holmes chaotic system. The control system is robust against parametric uncertainties and external disturbances. The control input consists of a continuous nominal control part and a discontinuous switching control input. Illustrative example is given. Simulation results show the promise of the proposed method. Input chattering is remarkably eliminated. Trajectory tracking is effectively achieved.**

***Index Terms*—Robust control, Sliding mode control, Stability, Tracking.**

I. Introduction

Robust stabilization of uncertain systems is an important topic in the field of control. Many approaches account for the uncertainties under various hypotheses. Sliding mode control (SMC) is one of the popular strategies to deal with uncertain control systems [1-5]. The main feature of SMC is the robustness against parameter variations and external disturbances. Various applications of SMC have been found, such as robotic manipulators, aircrafts, DC motors, chaotic systems, and so on.

Chaos exists in many engineering systems such as electronic circuits, power converters, chemical systems, and so on [6]. A fundamental characteristic of a chaotic system is its extreme sensitivity to initial conditions; that is, small differences in the initial state can lead to extraordinary differences in the system state. Chaos control has been of broad interest since the early 1990s. Since the pioneering work of Ott, *et al*. proposed the well-know OGY control method [7]. The OGY method was modified by Shinbort *et al*. to reduce the length of necessary time for stabilizing the target orbit [8]. Later, the control of chaos in a Bonhoeffer-van de Pol oscillator using a feed-forward backpropagating neural network trained on two different control schemes, via., the OGY control algorithm, and the Pyragas method of delayed continuous feedback control was demonstrated [9]. Recently, various methods have been proposed to control chaotic systems, such as neural network, fuzzy control, adaptive control, sliding mode control, etc [9-12].

In this paper, a robust sliding mode control for Duffing-Holmes chaotic system is presented. The goal is to achieve system robustness against parameter variations and external disturbances. The control input consists of a continuous nominal control part and a discontinuous switching control part. The former is the equivalent control for the nominal system and latter deals with the parametric variation and disturbance. To reduce the high frequency chattering in the controller, the boundary layer technique was used [13]. Theoretical analysis and numerical simulations verify the effectiveness of the proposed method. Another advantage of proposed method is that the input chattering does not appear.

This paper is organized as follows. Section 2 describes the robust controller design for Duffing-Holmes chaotic system. Section 3 shows simulation results of proposed method. Finally, conclusion is given.

II. Robust Stabilization Design

The Duffing-Holmes chaotic system is considered. In 1918, Duffing introduced a nonlinear oscillator [14], with a cubic stiffness term, to describe the hardening spring effect observed in many mechanical problems. Duffing's equation has been modified in different manners afterwards such as Moon and Holmes. In the present paper, to be more general we consider a modified Duffing equation of the form is named Duffing-Holmes. Consider the Duffing-Holmes chaotic system [15] described as

$$\ddot{x} + p_1\dot{x} + p_2 x + x^3 - q\cos(w_1 t) = 0 \,, \qquad (1)$$

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where $p_1 = 0.25$, $p_2 = -1$, $q = 0.3$, and $w_1 = 1$. The sampling time is equal to 0.001 sec. This Duffing-Holmes chaotic system displays chaotic behavior without control input as shown in Fig. 1 for the initial condition $x(0) = 2$ and $\dot{x}(0) = 2$.

In order to solve this problem, consider a chaotic system described by the following time-varying second-order differential equation with uncertainties and disturbances:

$$\ddot{x} + a_1(t)\dot{x} + a_2(t)x = b(t)(u + d(t,x)), \quad (2)$$

where $x \in R$ denotes the system state, $u \in R$ is the system input, and $d(t,x)$ is the disturbance or unmodeled dynamics. Assume that the upper and lower bounds of the uncertain system parameters $a_1(t)$, $a_2(t)$ and $b(t)$, and the disturbance $d(t)$ are specified as

$$\begin{cases} \beta_{\min} \le b^{-1}(t) \le \beta_{\max}, \\ \alpha_{1\min} \le b^{-1}(t)a_1(t) \le \alpha_{1\max}, \\ \alpha_{2\min} \le b^{-1}(t)a_2(t) \le \alpha_{2\max}, \\ \max\left|d(t,t)\right| < D|x|. \end{cases} \quad (3)$$

In the following, the robust suppression method is developed. The design procedure is divided into two steps. The first step is to define a sliding surface function such that in the sliding mode the system behaves equivalently as a linear system. The second step is to determine a control law such that the system will reach and stay on the sliding surface $s = 0$.

First, define the sliding surface function as

$$s = \dot{e} + ce, \quad (4)$$

where

$$e = x - r. \quad (5)$$

The symbol e is the tracking error, r is the desired path, and c is a positive constant.

In order to satisfy the sliding condition, $s\dot{s} < 0$, let the control input u be

$$u = u_o + u_s, \quad (6)$$

where u_o is the continuous nominal control, and u_s is the discontinuous switching control. The former is the equivalent control for the nominal system and the latter deals with the parametric variation and disturbances. Let $b^{-1}(t)$, $b^{-1}(t)a_1(t)$, and $b^{-1}(t)a_2(t)$ be divided into two parts: nominal part $(\hat{\beta}, \hat{\alpha}_1, \hat{\alpha}_2)$ and uncertain part $(\Delta\beta, \Delta\alpha_1, \Delta\alpha_2)$, and

$$\begin{cases} \hat{\beta} = \dfrac{\beta_{\max} + \beta_{\min}}{2}, \Delta\beta = \dfrac{\beta_{\max} - \beta_{\min}}{2}, \\ \hat{\alpha}_1 = \dfrac{\alpha_{1\max} + \alpha_{1\min}}{2}, \Delta\alpha_1 = \dfrac{\alpha_{1\max} - \alpha_{1\min}}{2}, \\ \hat{\alpha}_2 = \dfrac{\alpha_{2\max} + \alpha_{2\min}}{2}, \Delta\alpha_1 = \dfrac{\alpha_{2\max} - \alpha_{2\min}}{2}. \end{cases} \quad (7)$$

The control law u_o and u_s are formulated as

$$u_o = (\hat{\alpha}_1 - \hat{\beta}c)\dot{x} + \hat{\alpha}_2 x + \hat{\beta}\ddot{r} + \hat{\beta}c\dot{r}. \quad (8)$$

$$u_s = -(\Delta\alpha_1|\dot{x}| + \Delta\alpha_2|x| + \Delta\beta|\ddot{r} + c\dot{r} - c\dot{x}| + D|x|)\operatorname{sgn}(s) \quad (9)$$

where

$$\operatorname{sgn}(s) = \begin{cases} +1, & s > 0, \\ -1, & s < 0. \end{cases} \quad (10)$$

Taking derivative of (4) yields

$$\dot{s} = b(t)u + (c - a_1(t))\dot{x} - a_2(t)x - \ddot{r} - c\dot{r} + b(t)d(t,x). \quad (11)$$

Substituting (8)-(10) into the equation of (11) and multiplying with s yields,

$$\begin{aligned} s\dot{s} &= b(t)[s(\hat{\alpha}_1 - \hat{\beta}c)\dot{x} + s\hat{\alpha}_2 x + s\hat{\beta}\ddot{r} + s\hat{\beta}c\dot{r} \\ &\quad - (\Delta\alpha_1|\dot{x}| + \Delta\alpha_2|x| + \Delta\beta|\ddot{r} + c\dot{r} - c\dot{x}| + D|x|)|s|] \\ &\quad + s[(c - a_1(t))\dot{x} - a_2(t)x - \ddot{r} - c\dot{r} + b(t)d(t,x)] \\ &\le |b(t)||s|\left[|\hat{\alpha}_1 - \Delta\alpha_1||\dot{x}| + |\hat{\alpha}_2 - \Delta\alpha_2||x| \right. \\ &\quad \left. + |\hat{\beta} - \Delta\beta||\ddot{r} + c\dot{r} - c\dot{x}| - D|x|\right] + |s|\left[-|a_1(t)||\dot{x}| \right. \\ &\quad \left. - |a_2(t)||x| - |\ddot{r} + c\dot{r} - c\dot{x}| + |b(t)||d(t,x)|\right] \\ &= |b(t)||s|(|d(t,x)| - D|x|) \\ &< 0. \end{aligned} \quad (12)$$

Thus, the control law given by (8)-(10) guarantees the reaching and sustaining of the sliding mode.

In general, the inherent high-frequency chattering of the control input may limit the practical application of developed method. We further replace sgn(s) in (9) by the function $\mathrm{sat}(\frac{s}{\delta})$, i.e.,

$$\mathrm{sat}(\frac{s}{\delta}) = \begin{cases} 1, & \frac{s}{\delta} \geq 1, \\ \frac{s}{\delta}, & -1 < \frac{s}{\delta} < 1, \\ -1, & \frac{s}{\delta} \leq -1, \end{cases} \tag{13}$$

where δ is the width of the boundary layer. With this replacement, the sliding surface function s with an arbitrary initial value will reach and stay within the boundary layer $|s| \leq \delta$.

III. Simulation Results

In order to verify the proposed method, the following uncertain Duffing-Holmes chaotic system [15] is considered,

$$\ddot{x} = -p_1\dot{x} - p_2 x + x^3 - q\cos(w_1 t) + d + f + u\,. \tag{14}$$

Assume that the parameter uncertainty f and disturbance d satisfy $|f| \leq 0.1|x|$ and $|d| \leq 0.2$, respectively. The sampling time is equal to 0.001 sec. The initial condition is $x(0) = \dot{x}(0) = 2$. The aim is to control the uncertain Duffing-Holmes chaotic system to follow the trajectory $r = \sin(1.1t)$. According to (8), (9), and (13), the control law is chosen to be

$$u = -2.75\dot{x} - x + \ddot{r} + 3\dot{r} - 15sat(s/0.01)\,, \tag{15}$$

where the sliding surface function is $s = \dot{e} + 3e$.

Simulation results show that the proposed robust sliding mode control can effectively reduce input chattering as plotted in Fig. 2. The trajectory of the system in the phase-plane is shown in Fig. 3. The state tracking response is shown in Fig. 4. The error time response of the proposed sliding mode control converges to zero as shown in Fig. 5. As shown in Fig. 6, the sliding surface function using the proposed robust suppression SMC does not chatter in the sliding mode.

IV. Conclusions

In this paper, a schematic robust suppression sliding mode control design for chaotic systems is proposed. The control law consists of a continuous nominal control part and a discontinuous switching control input. The high frequency chattering in the control input is eliminated. System stability is assured. The uncertain Duffing-Holmes chaotic system is examined. The advantages of the proposed method are the simple design procedure, good tracking performance, insensitive to uncertainties, and effectiveness in eliminating the input chattering. Therefore, this method can be easily applied to many mechanical systems.

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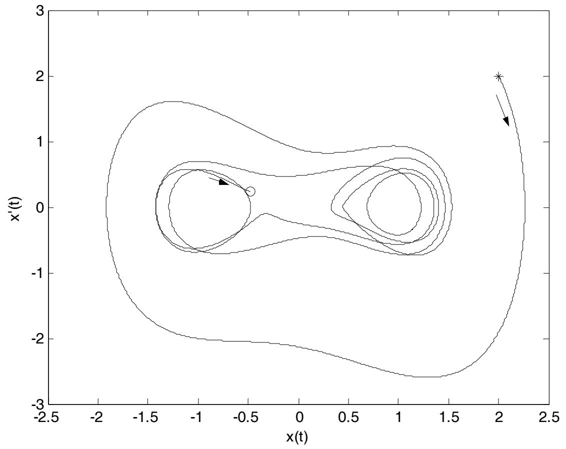

Fig. 1. The phase-plane plot of unforced Duffing-Holmes chaotic system with $x(0)=2$ and $\dot{x}(0)=2$.

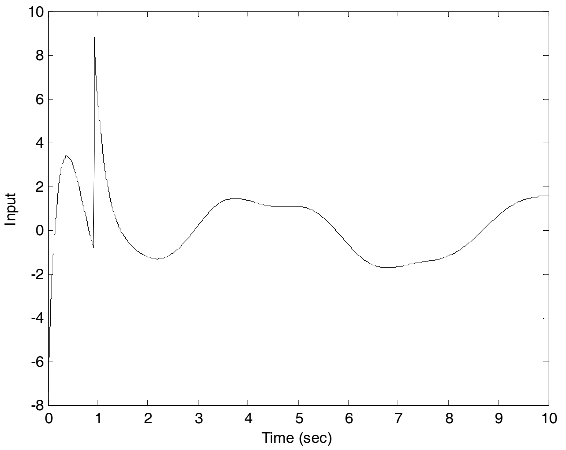

Fig. 2. The time response of control input.

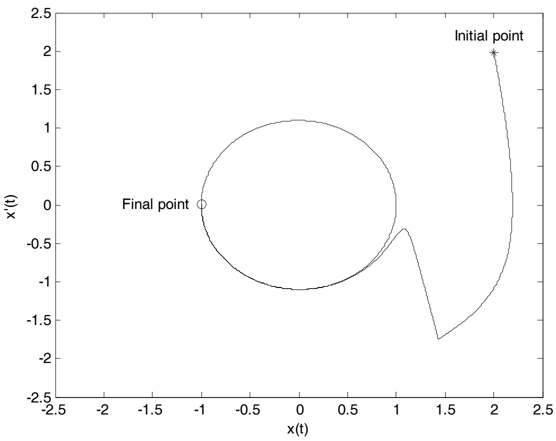

Fig. 3. The phase-plane plot of controlled Duffing-Holmes chaotic system.

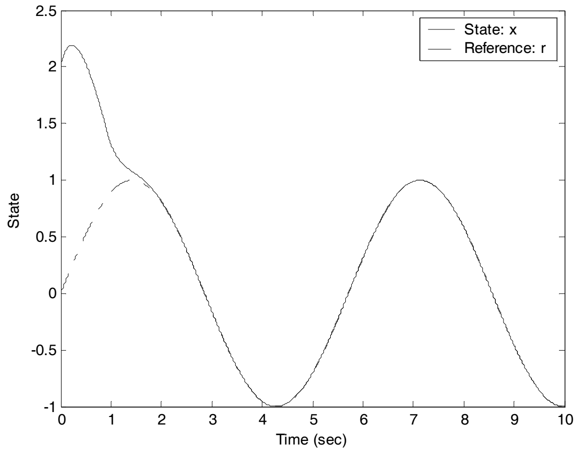

Fig. 4. The time response of the state.

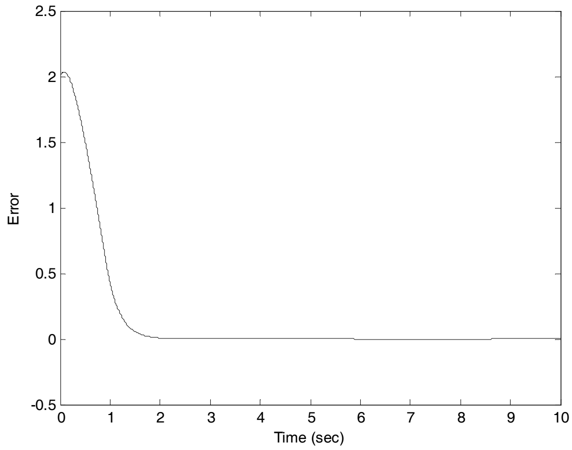

Fig. 5. The time response of the trajectory error.

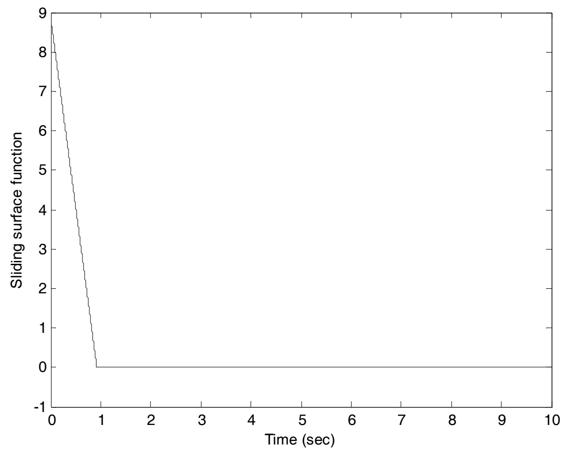

Fig. 6. The time response of the sliding surface function.

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