A Programming Design for Calculating Suitable Insulation Thickness And Heat Transfer Characteristics of an Insulated Rectangular Duct With One To Three Insulated Layers

King-Leung Wong (*Member, IAENG)*, Wen-Lih Chen, Tsung-Lieh Hsien, Hsueh-Chieh Yu

***Abstract*—This study is on programming a user-friendly code to calculate suitable insulation thickness and relative heat transfer characteristics for an insulated hot/cold rectangular duct involving one to three different insulation layers. Users simply input the required data, the inner fluid and the ambient conditions of the insulated duct, the sizes of duct, the properties of insulation materials under the specified conditions (uncondensed surface for the cold duct or maximum safe surface temperature for the hot duct), step by step, then the optimum insulation thickness, reliable heat transfer rate, minimum surface temperature of a cold duct/maximum surface temperature of a hot duct and insulation effect can be easily computed. Comparing with the inaccurate insulated thickness and relative unreliable heat transfer characteristics calculated from the PTR model, the employment of this code can greatly enhance the quality and stability of a system design involving insulated rectangular ducts.**

***Index Terms*—insulation thickness, insulated effect, programming, rectangular duct.**

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I. INTRODUCTION

In many engineering applications, constant surface area Plate Thermal Resistance (PTR) model [such as 1-5] is conventionally used to analyze insulated rectangular ducts. PTR model, as illustrated in Fig. 1, assumes insulated surface area is the same as that of a bare duct, thus resulting in some area of the insulation layer being neglected. The negligence is inappropriate and leads to the reduction of the insulation effect due to the some surface convective heat transfer being unaccounted for from the ignored area; hence an insufficient insulation thickness is returned. A system design based on this faulty method could have serious consequence. For example, in the case of insulating an air duct for very cold air in a high humidity environment, an under-estimated insulation thickness could cause a great deal of water condensation on the surface of the wrapped insulation material. The infiltration of condensed water into the insulation layer then causes the decay of the insulation effect, and the wet floor due to dripping of condensed water further makes our living environment hazardous. Therefore, the conventional constant surface area PTR model should not be used to analyze the heat transfer characteristics of insulated rectangular ducts.

Chou and Wong [6] developed a Plane Wedge Thermal Resistance (PWRT) model to investigate the heat transfer characteristics of an insulated regular polygonal duct. In this model, it is assumed that the isothermal contours within the duct material and the insulation layer are regular polygons, but the heat transfer area increases through the material thickness. In addition, the entire outer surface is assumed to be subjected to external convective heat transfer with the ambient fluid. As the thermal resistance of the inner convection term and the duct conduction term are neglected, they found that the dimensionless heat transfer characteristics of the insulated duct with a PWTR model are the same as those of an insulated circular duct even though their actual heat transfer rates are different. If the one-dimensional PWTR model [6] is used to analyze the two-dimensional heat transfer problem of an insulated regular polygonal duct, better accuracy is obtained with increasing the number of sides. Thus, it is interesting to know how accurate the method is when it is applied to an insulated square or rectangular duct with only four edges. Recently, Wong et al. [7] studied the heat-transfer characteristics of an insulated long rectangular or square duct by using the one-dimensional PWTR model and PTR model. It is found that the errors generated by PWTR model are all positive, and the errors generated by PTR model are all negative. Then the Combined Plate Wedge Thermal Resistance (CPWTR) model which combines the results of PWTR and PTR models with the proportion factors of 0.6 vs. 0.4 (64-CPWTR model) can effectively neutralize the positive and negative errors and return very accurate results, versified by two-dimensional numerical solutions computed by FLUENT CFD package. The errors of the one-dimensional 64-CPWTR model are within 1% for practical duct sizes and insulated thickness in air conditioning and refrigeration systems. Consequently, engineers should be able to obtain very reliable heat transfer results when applying 64-CPWTR model to insulated rectangular ducts.

The purpose of this study is to summarize the results of Wong et al. [7] and design a computer code to calculate suitable insulation thickness, reliable heat transfer rat, and the maximum surface temperature for hot duct or the minimum surface temperature for cold duct of an insulated rectangular duct with one to three insulated layers. A user-friendly interface has also been developed which allows the required data to be input with ease.

II. THE FLOW CHART OF THE PROGRAMMING

The flow chart of programming is shown in Appendix.

III. THE KEY DETINITIONS IN THE PROGRAMMING

III-1. DEW POINT

If the surface temperature of the outer surface of an insulated duct is lower than the dew point of ambient air, condensed water will occur on the insulated surface. Some accurate equations, which have been widely used in air conditioning engineering, to obtain dew point are shown below:

$$\ln P_g = \ln 100 + 14.4351 - \frac{5333.3}{T + 273.15} \tag{1}$$

$$T_{dp} = \frac{5333.3}{\ln 100 - \ln \phi P_g + 14.4351} - 273.15 \tag{2}$$

Where T is ambient air temperature, P_g is saturated water pressure, ϕ is relative humidity, and T_{dp} is dew point.

By inputting ambient air temperature T and relative humidity ϕ into equations (1) and (2), the dew point can be obtained.

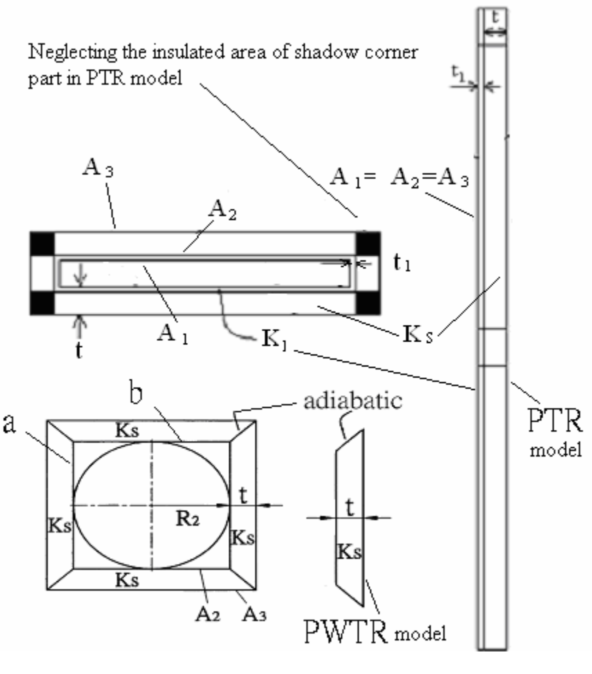

Fig. 1. An insulated rectangular duct with one insulated layer

III-2. HEAT TRANSFER CHARACTERISTICS

To show the structure of the code, an insulated rectangular duct with one insulation layer is illustrated in Fig. 1. Here, L is duct length, t_1 and t

are respectively the thicknesses of duct and insulated layer, A_1 is the inner surface area of rectangular duct, A_2 is the outer surface area of bare rectangular duct, A_3 is the outer surface area of insulation, h_i and h_o are respectively the inner and outer fluids' heat convective coefficients of duct, K_1 and K_s are respectively the heat conductivities of duct and insulated layer, and finally T_i and T_o are the temperatures of inner fluid and outer ambient air respectively. Let b and a represent the outer long-edge and short-edge lengths, the following relations can be written:

$$A_1 = 2(a + b - 4t_1)L \tag{3}$$

$$A_2 = 2(a + b)L \tag{4}$$

$$A_3 = 2(a + b + 4t)L \tag{5}$$

It can be seen from Fig. 1 that the PTR model treats the insulated rectangular duct as an expanded insulated flat plate with equivalent surface area A_2 then the total thermal resistance of an insulated rectangular duct with PTR model is:

$$\Sigma R_{th\,P} = \frac{1}{h_i A_2} + \frac{t_1}{KA_2} + \frac{t}{K_s A_2} + \frac{1}{h_o A_2} \tag{6}$$

Therefore, the heat transfer rate becomes:

$$q_p = \frac{T_i - T_o}{\sum R_{thP}} = \frac{T_s - T_0}{\frac{1}{h_o A_2}} \tag{7}$$

Where T_s is surface insulation temperature.

Wong et al. [7] proved that the maximum insulated surface temperature of a hot duct or the minimum insulated surface temperature of a cold duct can be obtained from equation (7). If T_s is greater than T_{dp}, there will be no condensed water on the insulated surface.

The total thermal resistance of a bare rectangular duct with PTR model is:

$$\Sigma R_{th\,P0} = \frac{1}{h_i A_2} + \frac{t_1}{KA_2} + \frac{1}{h_o A_2} \tag{8}$$

Then the associated heat transfer rate can be written as:

$$q_{p0} = \frac{T_i - T_o}{\sum R_{th\,P0}} \tag{9}$$

The total thermal resistance equation of an insulated rectangular duct with PWTR model is:

$$\Sigma R_{th\,w} = \frac{1}{h_i A_1} + \frac{t_1 \ln \frac{A_2}{A_1}}{K(A_2 - A_1)} + \frac{t \ln \frac{A_3}{A_2}}{K_s (A_3 - A_2)} + \frac{1}{h_o A_3} \tag{10}$$

Then its heat transfer rate is:

$$q_w = \frac{T_i - T_o}{\sum R_{thW}} \tag{11}$$

The total thermal resistance equation of a bare rectangular duct with PWTR model is:

$$\Sigma R_{th\,w0} = \frac{1}{h_i A_1} + \frac{t_1 \ln \frac{A_2}{A_1}}{K(A_2 - A_1)} + \frac{1}{h_o A_2} \tag{12}$$

This results in a heat transfer rate:

$$q_{w0} = \frac{T_i - T_o}{\sum R_{thW\,0}} \tag{13}$$

Subsequently, the reliable heat transfer rate of an insulated rectangular duct can be obtained from CPWTR model with a proportion factor α for PWTR model, and a proportion factor β for PTR model from Wong et al. [7] is :

$$q_C = \alpha \times q_W + \beta \times q_P \tag{14}$$

In this paper, q_{64} represented q_C with α=0.6 and β =0.4 (64-CPWTR model) and q_{73} represented q_C with α=0.7 and β=0.3 (73-CPWTR model).

And the reliable heat transfer rate of a bare rectangular duct with CPWTR from is:

$$q_{C0} = \alpha \times q_{W0} + \beta \times q_{P0} \tag{15}$$

The insulation effect is defined as the reduction of heat-transfer rate from a bare duct to an insulated duct as follows:

$$\varepsilon = (1 - \frac{q_C}{q_{C0}}) * 100\% \tag{16}$$

In the cases of two different insulated layers, the conductivities of the two layers are K_s and K_{s2}, respectively; and their thicknesses are t and d_2, respectively. Then the new surface areas become:

$$A_3 = 2(a + b + 4t)L \tag{17}$$

$$A_4 = 2(a + b + 4t + 4d_2)L \tag{18}$$

Then the total thermal resistance of PTR model can be expressed as:

$$\Sigma R_{th\,P} = \frac{1}{h_i A_2} + \frac{t_1}{K A_2} + \frac{t}{K_s\ A_2} + \frac{d_2}{K_{s2} A_2} + \frac{1}{h_o A_2} \quad (19)$$

And the total thermal resistance of PWTR model is:

$$\Sigma R_{th\,w} = \frac{1}{h_i A_1} + \frac{t_1 \ln \frac{A_2}{A_1}}{K(A_2 - A_1)} + \frac{t \ln \frac{A_3}{A_2}}{K_s\ (A_3 - A_2)} + \frac{d_2 \ln \frac{A_4}{A_3}}{K_{s2}(A_4 - A_3)} + \frac{1}{h_o A_4} \quad (20)$$

To this end, the heat transfer rate, outer maximum insulated surface temperature of a hot duct or the minimum insulated surface temperature of a cold duct, and the insulation effect can be obtained by substituting equations (19) and (20) into equations (7), (11), and (14).

In the case of insulated rectangular duct with three different insulated layers, the conductivities of these three layers are K_s, K_{s2} and K_{s3}, respectively; and their thicknesses are t, d_2, and d_3, respectively. Then the surface areas become:

$$A_5 = 2(a + b + 4t + 4d_2 + 4d_3)L \quad (21)$$

The total thermal resistance of PTR model is:

$$\Sigma R_{thP} = \frac{1}{h_i A_2} + \frac{t_1}{K A_2} + \frac{t}{K_s A_2} + \frac{d_2}{K_{s2} A_2} + \frac{d_3}{K_{s3} A_2} + \frac{1}{h_o A_2} \quad (22)$$

and the total thermal resistance of PWTR model is:

$$\Sigma R_{th\,w} = \frac{1}{h_i A_1} + \frac{t_1 \ln \frac{A_2}{A_1}}{K(A_2 - A_1)} + \frac{t \ln \frac{A_3}{A_2}}{K_s (A_3 - A_2)} + \frac{d_2 \ln \frac{A_4}{A_3}}{K_{s2}(A_4 - A_3)} + \frac{d_3 \frac{A_5}{A_4}}{K_{s3}(A_5 - A_4)} + \frac{1}{h_o A_5} \quad (23)$$

Again, the heat transfer rate, outer maximum insulated surface temperature of a hot duct or the minimum insulated surface temperature of a cold duct, and the insulation effect can be obtained by substituting equations (22) and (23) into equations (7), (11) and (14).

Wong et al. [7] used accurate two-dimensional numerical solutions, q_F, of an insulated rectangular duct to assess the accuracies of results obtained with PWTR, PTR and CPWTR models. In the practice, the error generated by PWTR model is:

$$E_W = \frac{q_W - q_F}{q_F} \times 100\ \% \quad (24)$$

The error generated by PTR model is:

$$E_P = \frac{q_P - q_F}{q_F} \times 100\ \% \quad (25)$$

The error generated by CPWTR model is:

$$E_{64} = \frac{q_{64} - q_F}{q_F} \times 100\ \% \quad (26)$$

$$E_{73} = \frac{q_{73} - q_F}{q_F} \times 100\ \% \quad (27)$$

Because the equivalent radius of a circular duct intersected the four sides of a bare outer surface of an equivalent square duct of a rectangular duct, it can be defined as:

$$R_2 = (a+b)/4 \quad (28)$$

And the dimensionless insulated thickness are t/R_2, $(t+d_2)/R_2$ *and* $(t+d_2+d_3)/R_2$ for one-layer, two-layer and three-layer insulation, respectively. In practice, the dimensionless insulated thickness, t/R_2 ,$(t+d_2)/R_2$ *and* $(t+d_2+d_3)/R_2$, is usually less than 0.5

IV. RESULTS AND DISCUSSIONS

This code is written in LabVIEW of NI. The Front Panel and Block Diagram of an insulated rectangular duct with one to three insulation layers are shown in Figs. 2~4, respectively. Four typical cases, simulating practical situations of the rectangular and square ducts with one insulated layer and various boundary conditions, are given as examples in Tables 1 and 2 to demonstrate their heat transfer characteristics, including heat transfer rates, q_W, q_P and q_F , respectively, calculated by PWTR model, PTR model and numerical method, and the heat transfer rate errors, E_W, E_P, E_{64} and E_{73}, respectively generated by PWTR, PTR, 64-CPWTR

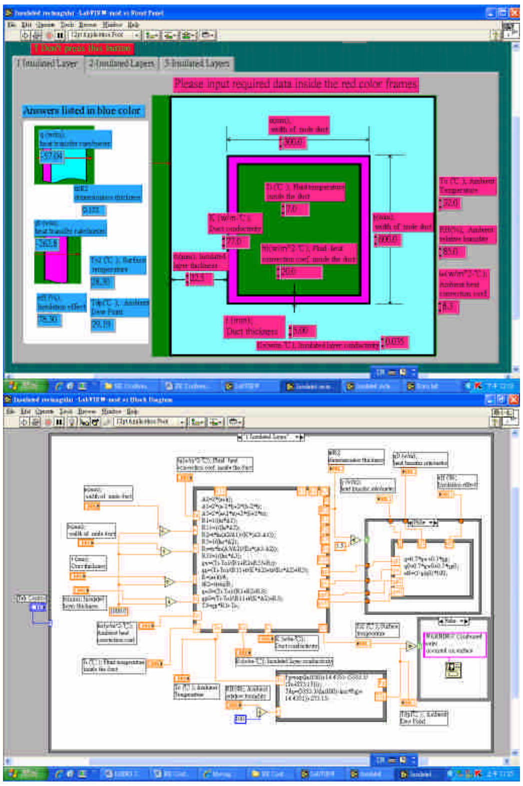

Fig. 2 The Front Panel and Block Diagram of an insulated rectangular duct with one insulated layer

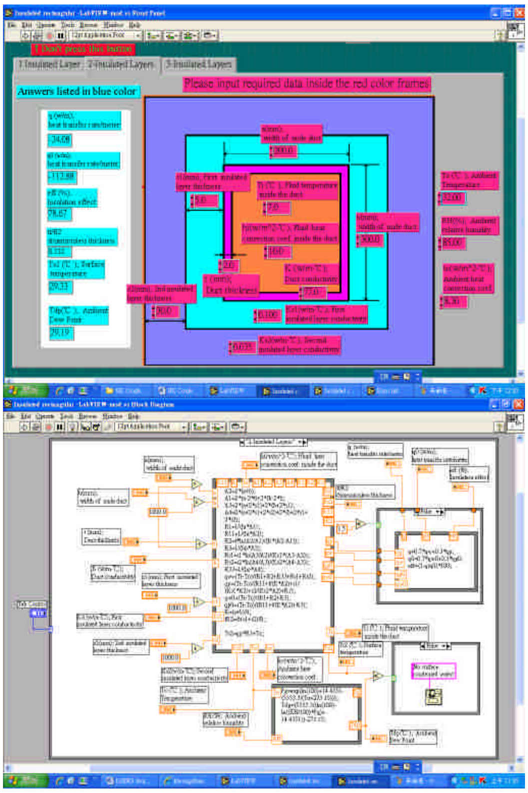

Fig. 3 The Front Panel and Block Diagram of an insulated rectangular duct with two insulated layers

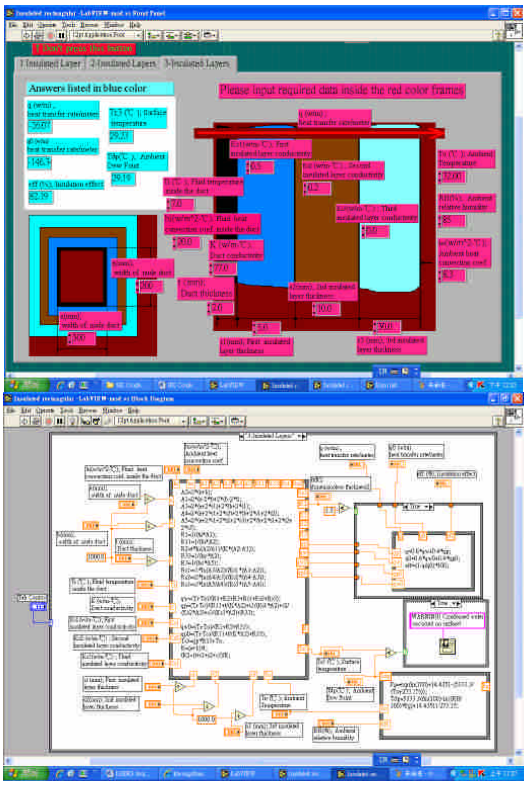

Fig. 4 The Front Panel and Block Diagram of an insulated rectangular duct with three insulated layers

(α=0.6 and β=0.4), and 73-CPWTR (α=0.7 and β=0.3) models.

Tables 1 and 2 show that all E_P are negative, and all E_W are positive. The greater the dimensionless insulation thickness t/R_2, the larger the E_W and E_P are. Also, all the absolute values of E_P are greater than those of E_W at the same conditions. The data showed that PWTR model is more accurate than the PTR model for insulated rectangular ducts although its predictive accuracy is not very satisfactory. Between two different combinations of values of α and β, the errors by E_{64} (α=0.6 and β=0.4) are less than 1% and also less than those of E_{73} (α=0.7 and β=0.3) when $t/R_2 < 1$ (meets most practical situations) but not for the cases of $t/R_2 \geqq 1.5$ (rarely encountered). Typically, dimensionless insulation thickness t/R_2 is smaller than 0.2, and in that sense, E_{64} is smaller than 0.6%. Therefore, 64-CPWTR model is strongly recommended for practical applications. This proves that 64-CPWTR model is a reliable one-dimensional model, returning very accurate heat transfer characteristics of insulated rectangular ducts in practical situations ($t/R_2 < 1.5$). Because most values of E_{73} (α=0.7 and β=0.3) are less than those of E_{64} (α=0.6 and β=0.4) when $1.5 \leqq t/R_2 \leqq 2$, it is sensible to apply 73-CPWTR model when $t/R_2 \geqq 1.5$. Thick insulation such as $t/R_2 \geqq 1.5$ is seldom encountered in practical situations, except in some particular cases of insulated rectangular ducts containing very cold/hot fluid. Meanwhile, Wong et al. [7] proved that the maximum insulated surface temperature of a hot duct or the minimum insulated surface temperature of a cold duct can be obtained from PTR model as shown in equation (7).

V. CONCLUSIONS

In this current computer code, 64-CPWTR model for situations of $t/R_2 < 1.5$ and 73-CPWTR model for situations of $t/R_2 \geqq 1.5$ are used to calculate the heat transfer rate and the insulation thickness of a insulated rectangular duct, while, PTR model is employed to calculate the maximum insulated surface temperature of a hot duct or the minimum insulated surface temperature of a cold duct. The current computer code can be applied to insulated hot/cold rectangular ducts with one to three insulated layers and it is able to return results of suitable insulation thickness, reliable heat transfer rate, outer maximum surface temperature of a hot duct or the minimum surface temperature of a cold duct. In contrast to inaccurate insulation thickness obtained by experience or calculated from the conventional PTR model, the use of this code can greatly enhance the quality and reliability of a system design which involves insulated rectangular duct.

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REFERENCES

[1]. Dossat RJ, Principles of refrigeration, 3rd edition, Prentice Hall, (1991), pp.167-178.

[2].Chatenever R, Air conditioning and refrigeration for the professional, John Wiley & Sons Inc., (1988), pp.445-448.

[3]. Pita EG, Air conditioning principles and system, 3rd edition, Prentice Hall, (1998), pp.252-256.

[4]. Stoecker WF and Jones JW, Refrigeration and air conditioning, 2nd edition, McGraw-Hill Inc., (1982), pp.24-35

[5]. Soylemez MS and Unsal M. Optimum insulation thickness for refrigeration applications. Energy Conversion and Management 40, (1999), pp.13-21.

[6]. Wong KL, Chou HM and Li YH, Complete Heat Transfer Solutions of an Insulated Regular Polygonal duct by Using a PWTR Model, Energy Conversion and Management, 45, (2004), pp.1705~1724.

[7]. Wong KL, Hsien TL, Richards P, Her BS, The reliable simple one-dimensional 64-CPWTR model applied to the two-dimensional heat transfer problem of an insulated rectangular duct in an air conditioning or refrigeration system, International Journal of Refrigeration, 28, (2005), pp.1029-1039.

TABLES

Table 1. Insulated rectangular cold fluid duct with a=0.6m, b=0.3m, R_2=0.225m, t_1=0.005m；K_1=77 W m^{-1}℃$^{-1}$, K_s=0.035 W m^{-1}℃$^{-1}$, h_o=8.3 W m^{-2}℃$^{-1}$, R_{cr}=0.00422m, R_2/R_{cr}=53.3; T_i=7℃, T_o=32℃

(a) h_i=100,000 W m^{-2}℃$^{-1}$

t (m)	$\frac{t}{R_2}$	q_p (W m^{-1})	q_w (W m^{-1})	q_F (W m^{-1})	E_w (%)	E_p (%)	E_{73} (%)	E_{64} (%)
0	0	-373.27	-373.26	-373.26	0	0	0	0
0.0225	0.1	-58.95	-62.30	-60.89	2.32	-3.19	0.66	0.11
0.0450	0.2	-32.00	-35.36	-33.95	4.16	-5.75	1.19	0.2
0.0675	0.3	-21.96	-25.29	-23.91	5.77	-8.15	1.59	0.2
0.09	0.4	-16.72	-20.01	-18.67	7.16	-10.46	1.87	0.11
0.225	1	-6.87	-9.96	-8.82	12.97	-22.1	2.45	-1.06
0.45	2	-3.47	-6.34	-5.4	17.33	-35.79	1.39	-3.92

(b) h_i=20 W m^{-2}℃$^{-1}$

t (m)	$\frac{t}{R_2}$	q_p (W m^{-1})	q_w (W m^{-1})	q_F (W m^{-1})	E_w (%)	E_p (%)	E_{73} (%)	E_{64} (%)
0	0	-263.86	-262.11	-262.08	0.01	0.68	0.21	0.28
0.0225	0.1	-55.32	-58.18	-56.94	2.18	-2.84	0.67	0.17
0.045	0.2	-30.90	-34.00	-32.68	4.03	-5.44	1.19	0.24
0.0675	0.3	-21.44	-24.58	-23.28	5.60	-7.91	1.54	0.19
0.09	0.4	-16.41	-19.56	-18.28	7.02	-10.22	1.85	0.12
0.1125	0.5	-13.29	-16.43	-15.18	8.27	-12.42	2.06	-0.01
0.225	1	-6.82	-9.85	-8.73	12.86	-21.89	2.43	-1.04
0.45	2	-3.45	-6.29	-5.36	17.36	-35.56	1.48	-3.81

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Table 2. Insulated square hot boiling liquid duct with a+b=2m, R_2=0.5m, t_1=0.01m ; K_1=350 W m^{-1}℃$^{-1}$, K_s=0.035 W m^{-1}℃$^{-1}$, h_i=100000 Wm^{-2}℃$^{-1}$, h_o=5 W m^{-2}-℃$^{-1}$, T_i=100℃, T_o=0℃

（a）Square duct with a=1m, b=1m, R_2=0.5m

t (m)	$\frac{t}{R_2}$	q_p (W m^{-1})	q_w (W m^{-1})	q_F (W m^{-1})	E_w (%)	E_p (%)	E_{73} (%)	E_{64} (%)
0	0	1999	1999	1996	0.0	0.0	0.0	0.0
0.15	0.3	89.17	102.5	97.35	5.29	-8.40	1.18	-0.19
0.2	0.4	67.63	80.81	75.60	6.89	-10.54	1.66	-0.08
0.25	0.5	54.47	67.49	62.24	8.44	-12.48	2.16	0.07
0.5	1	27.61	39.99	35.39	13.00	-21.98	2.50	-0.99
0.75	1.5	18.49	30.37	26.30	15.48	-29.70	1.92	-2.59
1	2	13.90	25.37	21.72	16.80	-36.00	0.96	-4.32

（b）Rectangular duct with a=0.5m, b=1.5m, R_2=0.5m

t (m)	$\frac{t}{R_2}$	q_p (W m^{-1})	q_w (Wm-1)	q_F (W m^{-1})	E_w (%)	E_p (%)	E_{73} (%)	E_{64} (%)
0	0	1999	1999	1999	0	0	0	0
0.25	0.5	54.47	67.50	62.29	8.37	-12.55	2.09	0
0.5	1	27.61	39.91	35.40	12.97	-22	2.48	-1.02
0.75	1.5	18.49	30.37	26.21	15.88	-29.44	2.28	-2.25
1	2	13.90	25.38	21.55	17.77	-35.49	1.79	-3.53

APPENDIX

THE FLOW CHART OF PROGRAMMING