Advanced Mathematical Model of the Material Point Relative Motion Dynamics

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Abstract—**The authors have proposed to show the advantages of different specialized software that can used in solving the algebraic and transcendental differential equations applied in mechanics, as problems of material point relative motion dynamics.**

***Index Terms*—dynamics, equations, software**

I. Introduction

One considers the following problem:
Inside a tube OA a material point M of mass m slides with friction (fig. 1). In the same time the tube is rotating with the angular velocity ω, around a vertical axis which intersects its ends.

In this example the absolute and relative motion of the material point will be studied considering the following cases:

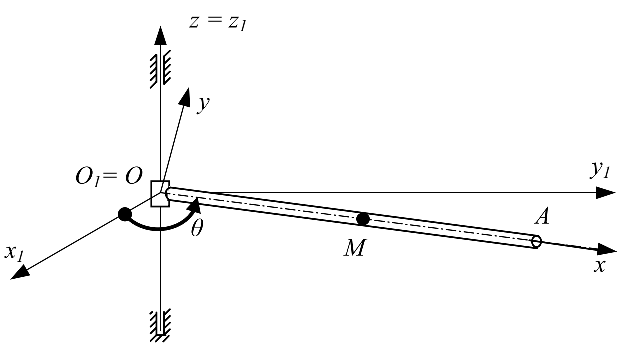

Fig. 1 Material point M in relative motion

Case I: $F_r = k\, f(\dot{x})$

a) $\dot{x} < 1$ *m/s,* $Fr = k\dot{x}$ *, k=12.5 kg/s*

b) 1 *m/s* $< \dot{x} < 250$ *m/s,* $F_r = k\dot{x}^2$*, k=12.5 kg/m*

c) 250 *m/s* $< \dot{x} < 300$ *m/s,* $F_r = k\dot{x}^3$*, k=0.05 kg·s/m²*

Case II: $F_r = \mu\sqrt{N_1^2 + N_2^2}$

where

- F_r is the resistant force which acts on the material point;
- k is a constant which depends on the values domains of material point velocity;
- μ is the friction coefficient between the material point and the tube's wall;
- N_1 , N_2 are the normal reactions of the tube's wall on the material point

In order to be able to compare the motion of the point in the two cases, using numerical methods [2], the following particular situations will be considered:
m = 0,2 kg, OA = 250 m, $\omega = 8$ *rad/s.*

II. Computing method

One considers the fixed reference frame $x_1y_1z_1$, and the moving reference frame, bonded to the tube.
The differential equation of the material point motion is [1]:

$$m\vec{a}_r = \vec{F}_r + \vec{G} + \vec{N}_1 + \vec{N}_2 + \vec{F}_t + \vec{F}_C \qquad (1)$$

where G is the weight of the material point and F_t, respectively F_C are the transport and Coriolis forces which act on the material point (fig. 2). The terms of the equation (1) are expressed by (2) and (3).

$$m\,\overrightarrow{a_r} = m\,\ddot{x}\,\vec{i}\,,\ \overrightarrow{F_r} = -k\,f(\dot{x})\,\vec{i}\,,$$
$$\vec{G} = -m\,g\,\vec{k}\,,\ \overrightarrow{N_1} = N_1\,\vec{j} \qquad (2)$$

$$\overrightarrow{N_2} = N_2\,\vec{k}\,,$$
$$\overrightarrow{F_C} = -2\,m\,\omega\,\dot{x}\,\vec{j}\,,\ \overrightarrow{F_t} = m\,\omega^2\,x\,\vec{i} \qquad (3)$$

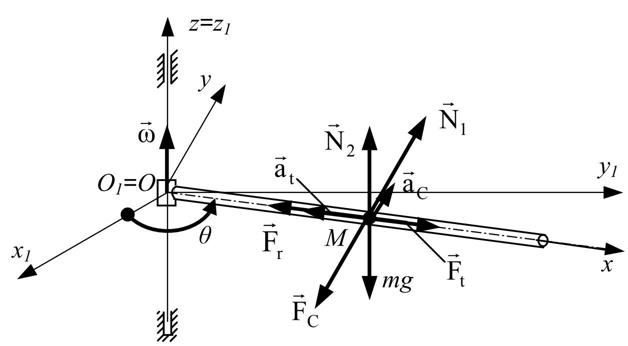

Fig. 2 The system of forces acting on the material point

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Case I a

In the case of velocity values in the domain *[0,1] m/s,* the resistant force F_r is proportional to the velocity: $F_r = k\,\dot{x}$, where $k = 12,5$ *kg/s.*

If one projects the equation (1) on the *x* axis of the moving reference system, one obtains:

$$m\ddot{x} = -k\,\dot{x} + m\,x\,\omega^2 \qquad (4)$$

The solution of this equation is given by relation (5).

$$x[t] = C_1\, e^{\frac{t\left(-k_1-\sqrt{k_1^2+4m^2\omega^2}\right)}{2m}} + C_2\, e^{\frac{t\left(-k_1+\sqrt{k_1^2+4m^2\omega^2}\right)}{2m}} \qquad (5)$$

where C_1 and C_2 are the integration constants.

Considering the particular numerical data [3] and the initial conditions $x[0] = 0,25$ *m*, $\dot{x}[0] = 0\ m/s$ one obtains the solution (6), plotted in fig.3.

$$x[t] = 0,00390507\, e^{-63,5078\, t} + 0,46095\, e^{1,00775\, t} \qquad (6)$$

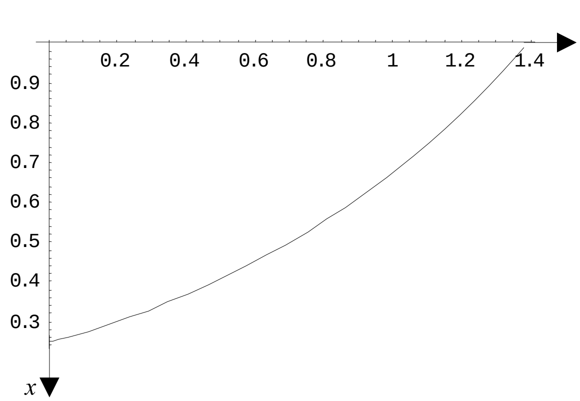

Fig. 3 Relative motion law for case I a

Considering that the tube's law of the motion is $\theta(t) = 8\,t$, the parametric equations of the material point with respect to the fixed reference system are:

$$x_1[t] = x[t]\cos 8t = (0.00390507\ e^{-63..5078\ t} + 0.246095\ e^{1.00775\ t})\cos 8t$$

$$y_1[t] = x[t]\sin 8t = (0.00390507\ e^{-63..5078\ t} + 0.246095\ e^{1.00775\ t})\sin 8t \qquad (7)$$

The trajectory of the material point for the time span *[0,1.38]s* can be seen in fig. 4.

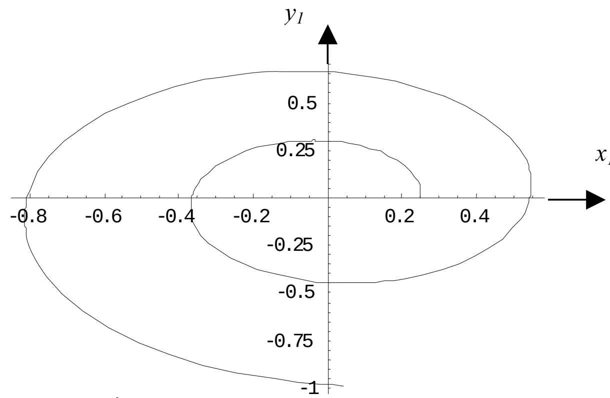

Fig. 4 Absolute motion trajectory for case I a

At moment $t = 1,38$ *s,* the velocity value reaches $\dot{x} = 1$ *m/s*, so the material point enters the values domain where the resistant force is proportional to the squared velocity.

At instant $t = 1,38$ *s*, the material point has the coordinate $x = 0,98$ *m*.

Due to the fact that the point is now in the values domain *[1, 250] m/s*, the resistant force is $F_r = k\,\dot{x}^2$, where $k = 12,5$ *kg/m (case I b).*

Case I b

The projection of the motion equation (1) on the x axis of the mobile reference system is:

$$m\ddot{x} + k\,\dot{x}^2 - m\,x\,\omega^2 = 0 \qquad (8)$$

This equation is solved for the following initial conditions: $x[1,38] = 0,98$ *m*, $\dot{x}[1,38] = 1\ m/s$.

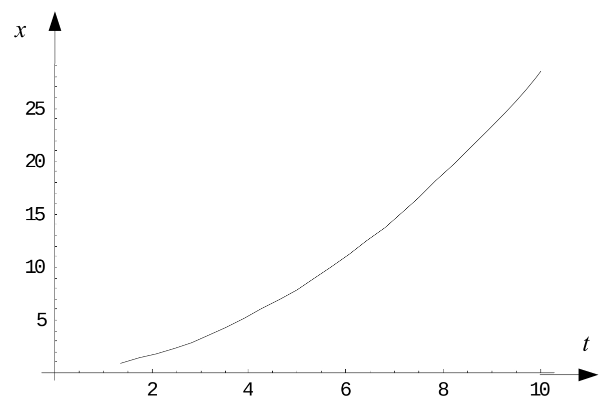

Fig. 5 Relative motion law for case I b

Due to the non-linearity of this differential equation, the solution may be obtained only using advanced numerical methods. An analytical solution is not available, just a numerical one can be obtained. This solution's graph is plotted in fig. 5.

This numerical solution may be approximated using interpolation methods involving the polynomial function shown by relation (9).

$$\begin{aligned} x = {} & 0.98 + (1.14516 + (0.263249 + (-0.00200546 + \\ & 0.000765443 + (-0.000579773 + 0.000311021(-0.0001123 + \\ & (0.0000299515 + (6.30748 \times 10^{-6} + 1.10011 \times 10^{-6} \\ & (-9+t))(-8+t))(-7+t))(-6+t))(-5+t)) \\ & (-4+t))(-3+t))(-2.5+t))(-2+t))(-1.38+t)) \end{aligned} \qquad (9)$$

At instant $t = 12.5218$ *s,* the material point's velocity reaches the value $\dot{x} = 250\ m/s$ which is in another domain of values. In this domain the resistant force is proportional with the velocity's power of 3.

Considering the parametric equations of the material point with respect to the fixed reference frame, the trajectory of the material point for the time span [1.38,12.5218] s can be plotted in fig.6.

At instant $t = 12.5218\ s$, the material point has the coordinate $x = 182.713\ m$. The resistant force is $F_r = k\ \dot{x}^3$, where $k = 0{,}05\ kg\ s / m^2$ *(case I c)*

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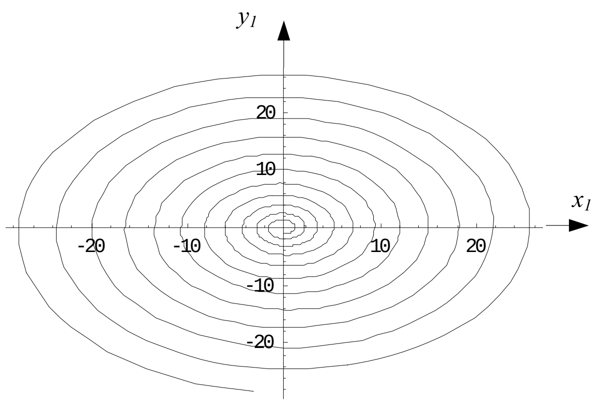

Fig. 6 Absolute motion trajectory for case I b

Case I c

The projection of the motion equation (1) on the x axis of the moving reference frame will be:

$$m\ddot{x} + k_3\dot{x}^3 - m x\omega^2 = 0 \qquad (10)$$

This equation is solved considering the initial conditions $x[12.5218]=182.713m,\ \dot{x}[12.5218] = 250\,m/s$

The obtained numerical solution is plotted in fig 7.

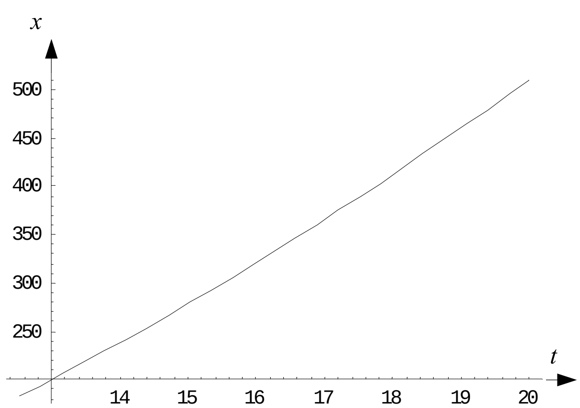

Fig. 7 Relative motion law for case I c

The numerical solution can be approximated with the polynomial relation (11)

$$\begin{aligned} x = {} & 182.713 + (36.7001 + (1.06405 + (0.0082531 + \\ & (-0.00769165 + (0.00186645 + (-0.00034831 + \\ & (0.0000550529 - 7.7067 \times 10^{-6}(-19+t)) \\ & (-18+t))(-17+t))(-16+t))(-15+t)) \\ & (-14+t))(-13+t))(-12.5+t)) \end{aligned} \qquad (11)$$

At instant $t = 26.46\ s$, the material point leaves the tube.

Taking into account the parametric equations of the material point with respect to the fixed reference frame, the trajectory of the point for the time span [12.5218 , 26.46] will be fig. 8:

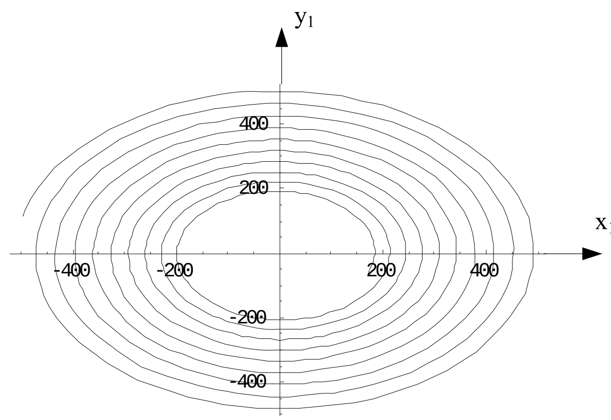

Fig. 8 Absolute motion trajectory for case I c

Case II

One considers the case when the resistant force, which acts on the material point, is a Coulombian force $F_r = \mu\sqrt{N_1^2 + N_2^2}$.

The projections of the motion equation (1) on the axis of the moving reference system are:

$$\begin{cases} m\ddot{x} = -\mu\sqrt{N_1^2 + N_2^2} + m x\omega^2 \\ N_1 = 2m\omega\dot{x} \\ N_2 = G \end{cases} \qquad (12)$$

This leads to the differential equation of motion on the x axis:

$$m\ddot{x} + \mu\sqrt{(2m\omega\dot{x})^2 + (m\,g)^2} - m x\omega^2 = 0 \qquad (13)$$

Solving this equation with the initial conditions $x[0] = 0.25\ m,\ \dot{x}[0] = 0\,m/s$ one obtains the numerical solution plotted by fig. 9.

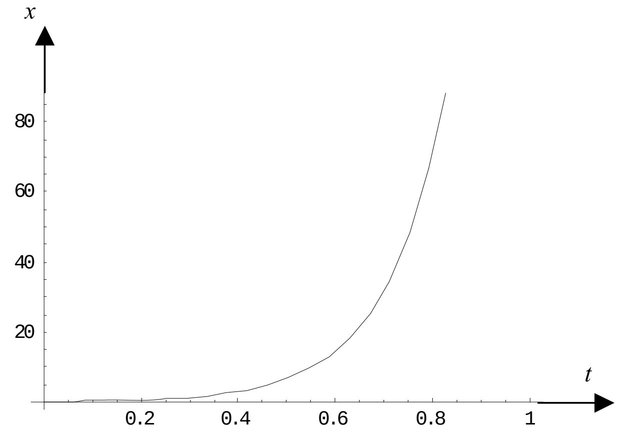

Fig. 9 Relative motion law for case II

The material point exists the tube at instant $t = 0{,}955\ s$ and up to this point the numerical solution may be approximated by the polynomial relation (14).

$$x = 0.25+(1.945+(24.6484+(392.125+(-462.338+5874.51(-0.8+t))(-0.6+t))(-0.4+t))(-0.2+t))t \tag{14}$$

Considering the parametric equations of the material point with respect to the fixed reference frame, the trajectory of the point for the time span [0, 0.955] s, has the shape shown by fig. 10:

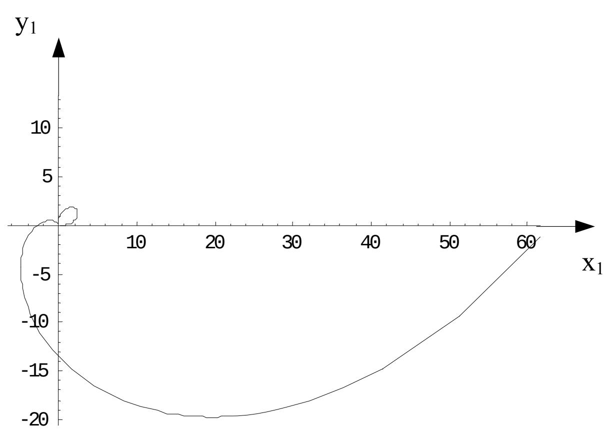

Fig. 10 Absolute motion trajectory for case II

III. Conclusion

- In the given conditions from case I a, the differential equation is linear and admits an analytical solution. The other cases though exhibit non-linear equations with numerical solutions that can be obtained only by means of advanced mathematical software. They can be approximated by polynomial functions using interpolation methods.

- This paper contributes to the study of the material point relative motion dynamics using the advanced mathematical techniques. These advanced techniques allow the trajectory determination, the numerical solutions determination and also the motion significant moments calculation.

IV. References

[1] Deciu E., Rădoi M., "*Mecanică*", Editura Didactică şi Pedagogică, R.A., Bucureşti, 1993, pag. 334;

[2] Zaharia S., Filip V., Mateoiu C., " e-Mechanics", 5th International Conference on Information Technology Based Higher Education and Training: ITHET '04" , Proceedings, IEEE Catalog Number 04EX898C, p. 674-676, ISBN 0-7803-8597-7, 2004, Istanbul, Turkey