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A New Study on Optimal Calculation of Partial Transmission Ratios of Helical Gearboxes with First-step Double Gear-sets

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***Abstract*—This paper introduces a study on the applications of the optimization and computer techniques for optimal calculation of partial ratios of helical gearboxes with first-step double gear-sets for minimal gearbox length. In the study, basing on moment equilibrium condition of a mechanic system including three gear units and their regular resistance condition, models for determining the partial ratios of the gearboxes were proposed. Specially, by using regression analysis, explicit models for prediction of the partial ratios are introduced. These models allow calculating the partial ratios accurately and simply.**

***Index Terms*—Gearbox design; optimal design; helical gearbox, transmission ratio.**

I. INTRODUCTION

It is known that in gearbox design the partial ratios are main factors affecting the size, dimension, mass, and the cost of the gearboxes. Consequently, optimal prediction of the partial ratios of gearboxes has been subjected to many studies.

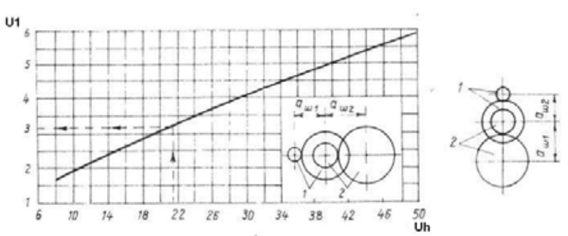

Fig. 1: *Transmission ratio of step 1 versus the total transmission ratio [1]*

So far, there have been many studies on the prediction of two-step helical gearboxes. Kudreavtev V.N. [1] presented a graph method (see Fig. 1) for determining the partial ratio of the first-step of the gearbox in order to get the minimal sum of the center distances. Based on a "practical method", G. Milou et al. [2] found that the gearbox weight will be minimal if the ratio a_{w2}/a_{w1} is from 1.4 to 1.6 (a_{w1}, a_{w2} are the center distances of the first and the second step, respectively). From this, the authors suggested tabulated optimal values for the partial ratios. Recently, Vu Ngoc Pi [3] introduced models for optimal calculation of the partial ratios in order to get the minimal across section of the gearbox.

From previous studies, it is clear that there have been many studies on the determination of the partial ratios for two step helical gearboxes. Nevertheless, there have not been studies on optimal calculation of the partial ratios for gearboxes with first-step double gear-sets. This paper introduces a new result for optimal determination of partial ratios for helical gearboxes with first-step double gear-sets for getting the minimal gearbox length.

II. DETERMINATION OF GEARBOX LENGTH

In practice, the length of a helical gearbox with first-step double gear sets is decided by the dimension of L which is determined by the following equation (see Fig. 2):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} \quad (1)$$

The center distance of the first step can be calculated by:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right) \quad (2)$$

Or

$$a_{w1} = \frac{d_{w21}}{2} \left(\frac{1}{u_1} + 1 \right) \quad (3)$$

Using the same way for the second step we can get:

$$a_{w2} = \frac{d_{w22}}{2} \left(\frac{1}{u_2} + 1 \right) \quad (4)$$

Substituting (3) and (4) into (1) with the note that $d_{w11} = d_{w21}/u_1$ we have

$$L = \frac{d_{w21}}{2} \cdot \left(\frac{2}{u_1} + 1 \right) + \frac{d_{w22}}{2} \cdot \left(\frac{1}{u_2} + 2 \right) \quad (5)$$

In the above equations, u_1, u_2 are transmission ratios, d_{w11}, d_{w12}, d_{w21}, d_{w22} are pitch diameters (mm) and a_{w1}, a_{w2} are center distances (mm) of helical gear units 1 and 2, respectively.

The following equation can be written as the design equation for the pitting resistance of the first step (a helical gear unit) [4]:

$$\sigma_{H1} = Z_{M1} Z_{H1} Z_{\varepsilon 1} \sqrt{\frac{2T_{11} K_{H1} \sqrt{u_1 + 1}}{b_{w1} d_{w11}^2 u_1}} \le [\sigma_{H1}] \quad (6)$$

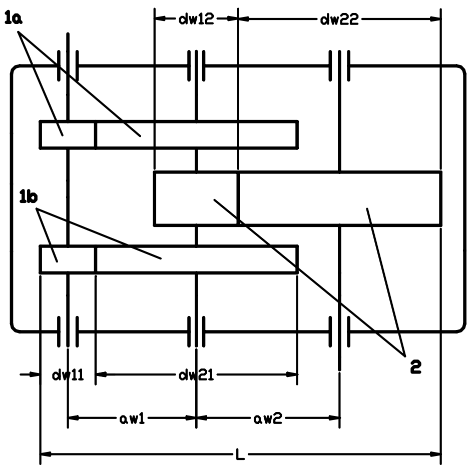

Fig. 2: *Calculating schema for helical gearbox with first-step double gear-sets*

From (6) we have:

$$[T_{11}] = \frac{b_{w1} \cdot d_{w11}^2 \cdot u_1}{2 \cdot (u_1 + 1)} \frac{[\sigma_{H1}]^2}{K_{H1} \cdot (Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1})^2} \quad (7)$$

In which, b_{w1} and d_{w11} are determined as follows:

$$b_{w1} = \psi_{ba1} \cdot a_{w1} = \frac{\psi_{ba1} \cdot d_{w11} \cdot (u_1 + 1)}{2} \quad (8)$$

$$d_{w11} = \frac{d_{w21}}{u_1} \quad (9)$$

Substituting (8) and (9) into (7) we get:

$$[T_{11}] = \frac{\psi_{ba1} \cdot d_{w21}^3 \cdot [K_{01}]}{4 \cdot u_1^2} \quad (10)$$

Where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1} \cdot (Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1})^2} \quad (11)$$

From (10) the pitch diameter d_{w21} can be calculated by:

$$d_{w21} = \left(\frac{4[T_{11}] u_1^2}{\psi_{ba1} [K_{01}]} \right)^{1/3} \quad (12)$$

Calculating in the same way, the following equation for the second gear unit is found:

$$d_{w22} = \left(\frac{4[T_{12}] u_2^2}{\psi_{ba2} [K_{02}]} \right)^{1/3} \quad (13)$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $[\sigma_{H1}]$ is allowable contact stresses of the first helical gear unit; ψ_{ba1} and ψ_{ba2} are coefficients of helical gear face width of steps 1 and 2, respectively.

From the condition of moment equilibrium of the mechanic system which includes three gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{2T_{11}} = \frac{[T_r]}{2[T_{11}]} = u_1 \cdot u_2 \cdot \eta_{brt}^2 \cdot \eta_o^2 \quad (14)$$

Where, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [4]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [4]).

Choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into (14) we have

$$[T_{11}] = \frac{0.54 \cdot [T_r]}{u_1 \cdot u_2} \quad (15)$$

Substituting (15) into (12) with the note that $u_1 = u_h / u_2$ we have

$$d_{w21} = \left(\frac{2.16 \cdot [T_r] \cdot u_h}{\psi_{ba1} \cdot [K_{01}] \cdot u_2^2} \right)^{1/3} \quad (16)$$

For the second helical gear unit we also have:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot \eta_{brt} \cdot \eta_o \quad (17)$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ (17) becomes

$$[T_{12}] = \frac{[T_r]}{0.9622 \cdot u_2} \quad (18)$$

Substituting (18) into (13) we got

$$d_{w22} = \left(\frac{4.1571 \cdot [T_r] \cdot u_2}{\psi_{ba2} \cdot [K_{02}]} \right)^{1/3} \quad (19)$$

Substituting (16) and (19) into (5), we have a new equation for the length of the gearbox:

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$$L=\frac{1}{2}\left(\frac{[T_r]}{[K_{01}]}\right)^{1/3}\cdot\left[\left(\frac{2.16\cdot u_h}{\psi_{ba1}\cdot u_2^2}\right)^{1/3}\cdot\left(\frac{2}{u_1}+1\right)+\right.$$
$$\left.+\left(\frac{4.1571\cdot u_2}{\psi_{ba2}\cdot K_{C2}}\right)^{1/3}\cdot\left(\frac{1}{u_2}+2\right)\right] \qquad (20)$$

Where, $K_{C2}=[K_{02}]/[K_{01}]$.

III. Optimization problem and results

Based on (20), the optimal problem for finding the minimal length of the gearbox can be expressed as follows:
The objective function:

$$\min L = f(u_h;u_2;u_3) \qquad (21)$$

With the constraints:

$$u_{h\min}\le u_h\le u_{h\max}$$

$$u_{2\min}\le u_2\le u_{2\max}$$

$$K_{C2\min}\le K_{C2}\le K_{C2\max} \qquad (22)$$

$$\psi_{ba1\min}\le\psi_{ba1}\le\psi_{ba1\max}$$

$$\psi_{ba2\min}\le\psi_{ba2}\le\psi_{ba2\max}$$

A computer program was built for performing the above optimization problem. The following data were used in the program: K_{C2} was from 1 to 1.3, ψ_{ba1}, ψ_{ba2} were from 0.25 to 0.4 [4], u_2 was from 1 to 9 [1]; u_h was from 5 to 40.

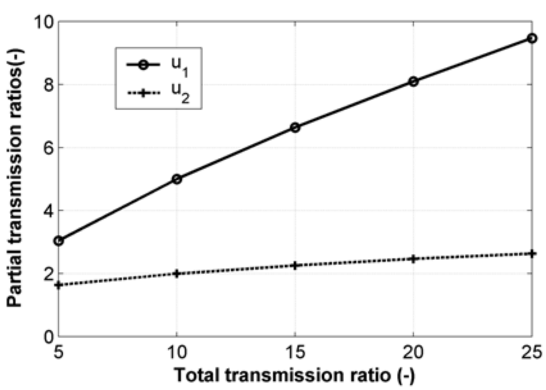

Fig. 3: *Partial transmission ratios versus the total transmission ratio*

It is observed that with the increase of the total ratio u_h the partial ratios increase (see Figure 3 - calculated with ψ_{ba1}=0.3, ψ_{ba2}=0.35 and K_{c2}=1.1). Also, the increase of partial ratio of the first step u_1 is much larger than that of the second step u_2 when the total ratio increases. The reason of that is with the increase of the total ratio u_h, the operating torque on the output shaft T_r is much larger than that on the driving shaft of the first gear unit T_{11}. Therefore, the partial ratio u_2 has to increase slowly in order to reduce the gearbox length.

From the results of the optimization program, the following regression model was determined to calculate the optimal values of the partial ratio of the second step u_2:

$$u_2\approx 0.9196\cdot\left(\frac{K_{C2}\cdot\psi_{ba2}}{\psi_{ba1}}\right)^{0.347}\cdot u_h^{0.3018} \qquad (23)$$

The regression model fit quite well with the data. The coefficient of determination was $R^2=0.9996$.

Equation 23 is used for determining the transmission ratio u_2 of the second helical gear unit. After calculating u_2, the transmission ratio of the first gear unit u_1 can be calculated by the following equation:

$$u_1=\frac{u_h}{u_2} \qquad (24)$$

IV. Conclusion

It can be concluded that the minimal length helical gearboxes with first-step double gear-sets can be obtained by optimal splitting the total transmission ratio of the gearboxes.

Models for determination of the optimal partial ratios of helical gearboxes with first-step double gear-sets in order to get the minimal length of the gearboxes have been proposed.

By introducing explicit models, the partial ratios of the gearboxes can be calculated accurately and simply.

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