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Optimal Calculation of Partial Transmission Ratios of Four-step Helical Gearboxes for Getting Minimal Cross Section Dimension

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***Abstract*—In this paper, a new study on the applications of the optimization and computer techniques for optimal determination of partial ratios of four-step helical gearboxes for getting their minimal cross section dimension is presented. In the paper, basing on the moment equilibrium condition of a mechanic system including four gear units and their regular resistance condition, models for calculation of the partial ratios of the gearboxes are proposed. In particular, explicit models for prediction of the partial ratios are introduced by using regression analysis technique. These models allow determining the partial ratios accurately and simply.**

***Index Terms*—Gearbox design; optimal design; helical gearbox, transmission ratio.**

I. Introduction

In optimal gearbox design, the optimal determination of partial transmission ratios has a decisive role. From both theory and manufacturing practice, it is known that the partial ratios are main effected factors on the size, the dimension, the mass, and the cost of the gearboxes. Therefore, optimal calculation of the partial ratios has been subjected to various researches.

Until now, many studies have been carried out on the determination of the partial ratios of helical gearboxes. For this type of gearboxes, the partial ratios can be predicted by the following methods:

-By graph method: for two, three and four-step helical gearboxes [1], [2], [3] (see an example in Figure 1).

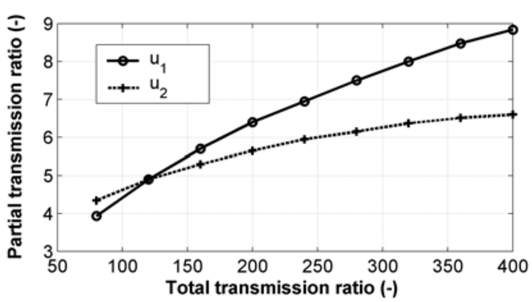

Fig. 1: *Determination of partial ratios of three-step helical gearboxes [1]*

-By "practical method" (or based on practical data): for two-step helical gearboxes [4].

-By models: for getting minimal cross section dimension of two-step gearboxes [5], for minimal gearbox mass of two and three-step gearboxes [6], or for minimal gear mass of three-step gearboxes [7].

It is clear that until now there have been many researches on the prediction of the partial ratios for two and three-step helical gearboxes. However, there have not been many studies on that for four-step gearboxes. Also, for four-step gearboxes, there has been only graph method for the determination of the partial ratios. This paper introduces a new result for optimal determination of partial ratios for helical gearboxes with first-step double gear-sets for getting the minimal cross section dimension.

II. Determination of the Dimension of the Gearbox Cross Section

The cross section dimension of a four-step helical gearbox is decided by the dimension of A which is determined by the following equation (see Figure 2):

$$A = L \cdot h \tag{1}$$

In which, h and L are determined by the following equations:

$$h = \max(d_{w21}, d_{w22}, d_{w23}, d_{w24}) \tag{2}$$

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + a_{w3} + a_{w4} + \frac{d_{w24}}{2} \tag{3}$$

For the first step (a helical gear unit), the center distance can be expressed as follows:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1\right) \tag{4}$$

Or we have

$$a_{w1} = \frac{d_{w21}}{2}\left(\frac{1}{u_1} + 1\right) \tag{5}$$

For the second, the third and the fourth step, using the same way we get:

$$a_{w2} = \frac{d_{w22}}{2}\left(\frac{1}{u_2} + 1\right) \tag{6}$$

$$a_{w3} = \frac{d_{w23}}{2}\left(\frac{1}{u_3}+1\right) \tag{7}$$

$$a_{w4} = \frac{d_{w24}}{2}\left(\frac{1}{u_4}+1\right) \tag{8}$$

Substituting (5), (6), (7) and (8) into (3) with the note that $d_{w11} = d_{w21}/u_1$ we have

$$L = \frac{d_{w21}}{2}\cdot\left(\frac{2}{u_1}+1\right)+\frac{d_{w22}}{2}\cdot\left(\frac{1}{u_2}+1\right)+ \\ +\frac{d_{w23}}{2}\cdot\left(\frac{1}{u_3}+1\right)+\frac{d_{w24}}{2}\cdot\left(\frac{1}{u_4}+2\right) \tag{9}$$

In the above equations, u_1, u_2, u_3, u_4 are transmission ratios, d_{w11}, d_{w12}, d_{w21}, d_{w22}, d_{w23}, d_{w24} are pitch diameters (mm) and a_{w1}, a_{w2}, a_{w3}, a_{w4} are center distances (mm) of helical gear units 1, 2, 3 and 4, respectively.

For the first step, the following equation can be used as the design equation for the pitting resistance [8]:

$$\sigma_{H1} = Z_{M1}Z_{H1}Z_{\varepsilon 1}\sqrt{\frac{2T_{11}K_{H1}\sqrt{u_1+1}}{b_{w1}d_{w11}^2 u_1}} \le [\sigma_{H1}] \tag{10}$$

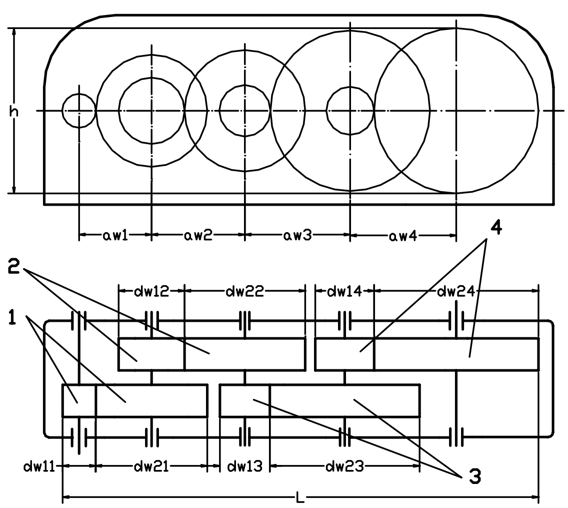

Fig. 2: *Calculating schema for four-step helical gearbox*

From (10) we have:

$$[T_{11}] = \frac{b_{w1}\cdot d_{w11}^2\cdot u_1}{2\cdot(u_1+1)}\frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon 1})^2} \tag{11}$$

In which, b_{w1} and d_{w11} are determined as follows:

$$b_{w1} = \psi_{ba1}\cdot a_{w1} = \frac{\psi_{ba1}\cdot d_{w11}\cdot(u_1+1)}{2} \tag{12}$$

$$d_{w11} = \frac{d_{w21}}{u_1} \tag{13}$$

Substituting (12) and (13) into (11) we get:

$$[T_{11}] = \frac{\psi_{ba1}\cdot d_{w21}^3\cdot[K_{01}]}{4\cdot u_1^2} \tag{14}$$

Where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon 1})^2} \tag{15}$$

From (14) the pitch diameter d_{w21} can be calculated by:

$$d_{w21} = \left(\frac{4[T_{11}]u_1^2}{\psi_{ba1}[K_{01}]}\right)^{1/3} \tag{16}$$

Calculating in the same way, the following equations can be found for the second, the third and the four step:

$$d_{w22} = \left(\frac{4[T_{12}]u_2^2}{\psi_{ba2}[K_{02}]}\right)^{1/3} \tag{17}$$

$$d_{w23} = \left(\frac{4[T_{13}]u_3^2}{\psi_{ba3}[K_{03}]}\right)^{1/3} \tag{18}$$

$$d_{w24} = \left(\frac{4[T_{14}]u_4^2}{\psi_{ba4}[K_{04}]}\right)^{1/3} \tag{19}$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $[\sigma_{H1}]$ is allowable contact stresses of the first helical gear unit; ψ_{ba1}, ψ_{ba2}, ψ_{ba3} and ψ_{ba4} are coefficients of helical gear face width of steps 1, 2, 3 and 4, respectively.

The following equation can be obtained from the condition of moment equilibrium of the mechanic system which includes four gear units and the regular resistance condition of the system:

$$\frac{T_r}{T_{11}} = \frac{[T_r]}{[T_{11}]} = u_1\cdot u_2\cdot u_3\cdot u_4\cdot\eta_{brt}^4\cdot\eta_o^4 \tag{20}$$

Where, η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [8]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [8]).

Choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into (20) we have

$$[T_{11}] = \frac{[T_r]}{0.8573 \cdot u_1 \cdot u_2 \cdot u_3 \cdot u_4} \quad (21)$$

Substituting (21) into (16) with the note that $u_1 = u_h / u_2$ we have

$$d_{w21} = \left(\frac{4.6658 \cdot [T_r] \cdot u_h}{\psi_{ba1} \cdot [K_{01}] \cdot u_2^2 \cdot u_3^2 \cdot u_4^2} \right)^{1/3} \quad (22)$$

For the second helical gear unit we also have:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^3 \cdot \eta_o^3 \quad (23)$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ (23) becomes

$$[T_{12}] = \frac{[T_r]}{0.8909 \cdot u_2 \cdot u_3 \cdot u_4} \quad (24)$$

Substituting (24) into (17) we got

$$d_{w22} = \left(\frac{4.4898 \cdot [T_r] \cdot u_2}{\psi_{ba2} \cdot [K_{02}] \cdot u_3 \cdot u_4} \right)^{1/3} \quad (25)$$

In exactly similar manner, we can derive the following equations for the third and the fourth steps:

$$d_{w23} = \left(\frac{4.3201 \cdot [T_r] \cdot u_3}{\psi_{ba3} \cdot [K_{03}] \cdot u_4} \right)^{1/3} \quad (26)$$

$$d_{w24} = \left(\frac{4.1571 \cdot [T_r] \cdot u_4}{\psi_{ba4} \cdot [K_{04}]} \right)^{1/3} \quad (27)$$

Substituting (22), (25), (26) and (27) into (9), the length of the gearbox can be calculated by:

$$L = \frac{1}{2} \left(\frac{[T_r]}{[K_{01}]} \right)^{1/3} \cdot \left[\left(\frac{4.6658 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2 \cdot u_4^2} \right)^{1/3} \cdot \left(\frac{2}{u_1} + 1 \right) + \right.$$
$$+ \left(\frac{4.4898 \cdot u_2}{\psi_{ba2} \cdot K_{C2} \cdot u_3 \cdot u_4} \right)^{1/3} + \left(\frac{4.3201 \cdot u_3}{\psi_{ba3} \cdot K_{C3} \cdot u_4} \right)^{1/3} + \quad (28)$$
$$\left. + \left(\frac{4.1571 \cdot u_4}{\psi_{ba4} \cdot K_{C2}} \right)^{1/3} \cdot \left(\frac{1}{u_4} + 2 \right) \right]$$

Where, $K_{C2} = [K_{02}] / [K_{01}]$, $K_{C3} = [K_{03}] / [K_{01}]$, and $K_{C4} = [K_{04}] / [K_{01}]$.

From (22), (25), (26) and (27), (2) can be rewritten as follows:

$$h = \max \left(\frac{4.6658 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2 \cdot u_4^2}; \frac{4.4898 \cdot u_2}{\psi_{ba2} \cdot k_{c2} \cdot u_3 \cdot u_4}; \right.$$
$$\left. \frac{4.3201 \cdot u_3}{\psi_{ba3} \cdot k_{c3} \cdot u_4}; \frac{4.1571 \cdot u_4}{\psi_{ba4} \cdot k_{c4}} \right) \quad (29)$$

III. OPTIMIZATION PROBLEM AND RESULTS

Based on (1), (28) and (29), the optimal problem for finding the minimal cross section dimension can be expressed as follows:
The objective function is:

$$\min A = f(u_h; u_2; u_3; u_4) \quad (30)$$

Where, A is determined by (1) and L and h (in Equation 1 are calculated by (28) and (29), respectively.

With the following constraints:

$$\begin{aligned}
& u_{h\min} \le u_h \le u_{h\max} \\
& u_{2\min} \le u_2 \le u_{2\max} \\
& u_{3\min} \le u_3 \le u_{3\max} \\
& u_{4\min} \le u_4 \le u_{4\max} \\
& K_{C2\min} \le K_{C2} \le K_{C2\max} \\
& K_{C3\min} \le K_{C3} \le K_{C3\max} \\
& K_{C4\min} \le K_{C4} \le K_{C4\max} \\
& \psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max} \\
& \psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max} \\
& \psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max} \\
& \psi_{ba4\min} \le \psi_{ba4} \le \psi_{ba4\max}
\end{aligned} \quad (31)$$

To perform the above optimization problem a computer program was built. The data used in the program as follows: K_{C2}, K_{C3}, K_{C4} were from 1 to 1.3, ψ_{ba1}, ψ_{ba2}, ψ_{ba3}, ψ_{ba4} were from 0.25 to 0.4 [8], u_2, u_3, u_4 were from 1 to 9 [1]; u_h was from 50 to 400.

From the results of the optimization program, the following regression models were determined to calculate the optimal values of the partial ratios of the second, third and fourth steps:

$$u_2 \approx \frac{1.2053 \cdot K_{C2}^{0.4492} \cdot \psi_{ba2}^{0.4638} \cdot u_h^{0.257}}{K_{C3}^{0.0419} \cdot K_{C4}^{0.1233} \cdot \psi_{ba1}^{0.2757} \cdot \psi_{ba3}^{0.0617} \cdot \psi_{ba4}^{0.1279}} \quad (32)$$

$$u_3 \approx \frac{1.1846 \cdot K_{C3}^{0.4596} \cdot \psi_{ba3}^{0.4417} \cdot u_h^{0.1197}}{K_{C2}^{0.2339} \cdot K_{C4}^{0.0622} \cdot \psi_{ba1}^{0.1211} \cdot \psi_{ba2}^{0.2445} \cdot \psi_{ba4}^{0.0741}} \quad (33)$$

$$u_4 \approx \frac{1.107 \cdot K_{C4}^{0.4683} \cdot \psi_{ba4}^{0.462} \cdot u_h^{0.0601}}{K_{C2}^{0.1165} \cdot K_{C3}^{0.2696} \cdot \psi_{ba1}^{0.0603} \cdot \psi_{ba2}^{0.1225} \cdot \psi_{ba3}^{0.2793}} \quad (34)$$

The above regression models fit quite well with the data. The coefficients of determination were $R^2 = 0.9895$, $R^2 = 0.9689$, and $R^2 = 0.9663$ for Equations 32, 33 and 34, respectively.

Equations 32, 33 and 34 are used to calculate the transmission ratio u_2, u_3, and u_4 of steps 2, 3 and 4 of the gearbox. After determining u_2, u_3, and u_4, the transmission ratio of the first step u_1 can be determined as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3 \cdot u_4} \quad (35)$$

IV. CONCLUSION

The minimal across section dimension of four-step helical gearboxes can be obtained by optimal splitting the total transmission ratio of the gearboxes.

Models for determination of the optimal partial ratios of four-step helical gearboxes for getting the minimal across section dimension of the gearboxes have been proposed.

The partial ratios of the gearboxes can be determined accurately and simply by explicit models.

REFERENCES

[1] V.N. Kudreavtev; I.A. Gierzaves; E.G. Glukharev, *Design and calculus of gearboxes* (in Russian), Mashinostroenie Publishing, Sankt Petersburg, 1971.

[2] Trinh Chat, Optimal calculation the total transmission ratio of helical gear units (in Vietnamese), *Scientific Conference of Hanoi University of Technology,* 1996, pp. 74-79.

[3] A. N. Petrovski, B.A. Sapiro, N.K. Saphonova, About optimal problem for multi-step gearboxes (in Russian), *Vestnik Mashinostroenie,* No. 10, 1987, pp. 13-14.

[4] G. Milou; G. Dobre; F. Visa; H. Vitila, Optimal Design of Two Step Gear Units, regarding the Main Parameters, *VDI Berichte* No 1230, 1996, pp. 227-244.

[5] Vu Ngoc Pi, A new method for optimal calculation of total transmission ratio of two step helical gearboxes, *The Nation Conference on Engineering Mechanics*, Hanoi, 2001, pp. 133- 136.

[6] Romhild I. , Linke H., Gezielte Auslegung Von Zahnradgetrieben mit minimaler Masse auf der Basis neuer Berechnungsverfahren, *Konstruktion* 44, 1992, pp. 229- 236.

[7] Vu Ngoc Pi, Nguyen Dang Binh, Vu Quy Dac, Phan Quang The, Optimal calculation of total transmission ratio of three-step helical gearboxes for minimum mass of gears *(In Vietnamese), Journal of Science and Technology of Six Engineering Universities,* Vol. 55, 2006, pp. 91-93.

[8] Trinh Chat, Le Van Uyen, *Design and calculus of Mechanical Transmissions* (in Vietnamese), Educational Republishing House, Hanoi , 1998.