On the Global Uniform Asymptotic Stability of Nonlinear Dynamic System

Jiemin Zhao

Abstract—We give a concise result of global uniform asymptotic stability for nonlinear dynamical system

\[
\begin{align*}
\dot{x}(t) &= y(t), \\
\dot{y}(t) &= [1 + \sin^2(Ay(t))] \int_0^t y(t+s) K(x(t+s)) \, ds \\
&- By(t) + Cy(t) [1 + \sin^2(Ay(t))]
\end{align*}
\]

where \( a \) is an arbitrary constant. Thus, if

\[
(x(t), y(t)) = (x(t, t_0, x_0), y(t, t_0, y_0))
\]

is a solution of dynamical system (1), then the derivative \( \dot{V} \) of \( V \) along \((x(t), y(t))\) satisfies

\[
\dot{V}(t) = 2Cx(t) \dot{x}(t) + \frac{2y(t)}{1 + \sin^2(Ay(t))} \dot{y}(t) + a \int_0^t [y^2(t) - y^2(t+s)] \, ds.
\]

by means of the method of Liapunov functional.

Index Terms—dynamical system, finite delay, model, Stability.

I. INTRODUCTION

Consider the mathematical model

\[
\begin{align*}
\dot{x}(t) &= y(t), \\
\dot{y}(t) &= [1 + \sin^2(Ay(t))] \int_0^t y(t+s) K(x(t+s)) \, ds \\
&- By(t) + Cy(t) [1 + \sin^2(Ay(t))]
\end{align*}
\]

where \( A, B, C = \text{const.} \), the finite delay \( r = \text{const.} > 0 \), \( K(x) \) is a continuous function, \( B, C > 0 \). The nonlinear dynamical system (1) can be used to describe many practical engineering problems [1—10]. The problem of global uniform asymptotic stability of dynamical system (1) is not only the considerable significance in theory, but also of important background in application [1, 2, 5—11]. In this paper, a convenient and efficient result is given to solve the problem above.

II. ANALYSIS AND COMPUTING

Let

\[
V = Cx^2 + \int_0^t \frac{2z}{1 + \sin^2(Az)} \, dz + a \int_0^t \int_{-r}^0 y(u) \, duds
\]

Using the inequality

\[
2y(t) \int_0^t y(t+s) K(x(t+s)) \, ds \leq 2 \int_0^t |y(t)||y(t+s)||K(x(t+s))| \, ds,
\]

we have

\[
\dot{V}(t) = 2y(t) \int_0^t y(t+s) K(x(t+s)) \, ds -
\]

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\[
\frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))} + a \int_{0}^{t} [ y^{2}(s) - y^{2}(t + s) ] ds.
\]

\[
\leq 2 \int_{0}^{t} \left| y(t) \right| \left| y(t + s) \right| K(x(t + s)) ds - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))} + a \int_{0}^{t} [ y^{2}(s) - y^{2}(t + s) ] ds.
\]

If there is a constant \( \mu > 0 \) such that \( |K(s)| \leq \mu \), then the derivative \( \dot{V}(t) \) of \( V \) satisfies

\[
\dot{V}(t) \leq 2 \mu \int_{0}^{t} \left| y(s) \right| \left| y(t + s) \right| ds - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))} + a \int_{0}^{t} [ y^{2}(s) - y^{2}(t + s) ] ds.
\]

Taking \( a = \mu \), we have

\[
\dot{V}(t) \leq 2 \mu \int_{0}^{t} \left| y(s) \right| \left| y(t + s) \right| ds - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))} + \mu \int_{0}^{t} [ y^{2}(s) - y^{2}(t + s) ] ds.
\]

Using the inequality

\[
2 \alpha \beta \leq \alpha^{2} + \beta^{2},
\]

we have

\[
\dot{V}(t) \leq \mu \int_{0}^{t} [ y^{2}(s) + y^{2}(t + s) ] ds - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))} + \mu \int_{0}^{t} [ y^{2}(s) - y^{2}(t + s) ] ds
\]

\[
= 2 \mu \int_{0}^{t} y^{2}(s) ds - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))}
\]

\[
= \mu \int_{0}^{t} y^{2}(t) - \frac{2B y^{2}(t)}{1 + \sin^{2}(A y(t))}
\]

\[
\leq 2 \mu y^{2}(t) - B y^{2}(t).
\]

If \( 2 \mu r < B \), then \( \dot{V}(t) \leq (2 \mu r - B) y^{2}(t) \leq 0 \).

Thus,

\[
\dot{V}(t) \leq 0 \quad \text{and} \quad V(t) = 0 \quad \text{only if} \quad (x(t), y(t)) = (0, 0).
\]

In fact, since \( y(t) = 0 \), we have

\[
\dot{x}(t) = y(t) = 0,
\]

\[
\dot{y}(t) = [1 + \sin^{2}(A y(t))] \int_{0}^{t} y(t + s) K(x(t + s)) ds - B y(t) - C \dot{x}(t) = -C \dot{x}(t).
\]

Thus, \( -C \dot{x}(t) = 0 \), since \( C = \text{const.} > 0 \) we have \( x(t) = 0 \).

On the other hand,

\[
C \dot{x}^{2} \rightarrow +\infty \left( |x| \rightarrow +\infty \right)
\]

And

\[
\int_{0}^{t} \frac{2 \xi}{1 + \sin^{2}(A \xi)} d\xi \rightarrow +\infty \left( |\xi| \rightarrow +\infty \right)
\]

Thus, the nonlinear dynamical system (1) is globally uniformly asymptotically stable.

III. MAIN RESULT

From analysis and computing above, we have result as follow:

Suppose \( K(x) \) is a continuous function. If there is a constant \( \mu > 0 \) such that

\[
(i) \quad |K(s)| \leq \mu,
(ii) \quad 2 \mu r < B.
\]

then the system (1) is globally uniformly asymptotically stable.

REFERENCES


