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A Treatise on Mathematical Color Inpainting of Japanese Old Statues

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***Abstract*—There are many photos of statues having lost their coloring in the databases containing Japanese old statues. This article is a treatise on color inpainting of these photos of old statues. In most cases, before color inpainting of historical precious statues, we must pay tremendous attention to the details of the statues and conduct in-depth research on them. On the contrary, in this treatise, we argue how we can contribute to the color inpainting of the photos, not just of the statues themselves, but also by using mathematical image analysis techniques assuming no previous professional profound knowledge of the statue inpainting.**

***Index Terms*—Color inpainting, Photos of old Japanese statues, Poisson equation, Color transfer method, Sparse coding inpainting.**

I. Introduction

Japan has many famous old statues that are very precious historically. However, in almost all cases, they are damaged to some degree and the color is imperfect. It is widely known that it is very difficult to repair the statues themselves, however it may be a little easier to inpaint photos of such damaged statues rather than repairing them. This approach is thought to be of value, and above all, it is stimulating to see what their true photo images were like. In this paper we try inpainting the lost colors of Japanese old statues purely on the photos by four mathematical inpainting techniques, compare the results, and argue about the possibility of applying mathematical techniques in dealing with inpainting problems of precious statues. In fact, we adopted as our final objective, to inpaint the lost colors of four big soldiers called "Shitenno" (see Figure 1) who guard the *Vairocana*, *Sahasrabhuja* and *Bhaisajyaguru* in the main hall of *Tosho* temple, because they are very attractive statues and no one has tried to repair or inpaint them.

However, we again need to realize that such inpainting work is essentially difficult because the soldiers have lost almost all their color. Due to the difficulty of our final goal, and as a first step, in this paper, we mainly try to inpaint the lost colors of the photo of "Ganjin Wajo", which has been less damaged in its color structure, consequently, less difficult to conduct inpainting(see Figure 2(a) and (b)). In this paper, we argue about the possibility of four kinds of color inpainting techniques and the results applied to "Ganjin Wajo" (Priest Ganjin) or "Zocho-ten" are checked in detail, followed by arguments on how we can improve the results in the future.

In Section II, we deal with a color inpainting method by Sapiro([9]) by solving Poisson equations. This method is lined along the recent many works of the mathematical inpainting based on partial equations (see, for example, Chan-Shen[2], Sapiro[8], Tschumperie-Deriche [10], etc.). The method can recover the lost colors based on characteristics of luminance Y of natural images in $YCrCb$ color space instead of RGB color space.

In Section III.A we deal with color transfer method proposed by Horiuchi [6], which transfers given seed colors at initially chosen spots to the whole image. Next in Section III.B we develop an easier method that uses detailed information of color structure of the image. In Section IV, we deal with sparse coding inpainting method, which was dealt with by Mairal-Elad-Sapiro [7]. In the sparse coding method,

(a) Komoku-ten (viruupaakSa)

(b) Zocho-ten (viruudhaka)

(c) Jikoku-ten (dhRtaraaSTra)

(d) Tamon-ten (vaizravaNa)

Fig.1 Shitenno (Four Heavenly Kings)

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(a) Seated Ganjin Wajo (b) Face of Ganjin Wajo

Fig.2 Ganjin Wajo

a photo is decomposed into small-sized patches, and the whole image is recovered by estimating each small patch by using an appropriate ample dictionary. In an algorithmic aspect it is realized by K-SVD method. It is viewed as one of example-based recovery methods. In each technique we show the inpainted images and argue about their possibility and improvements in the future.

II. Color Inpainting by Solving Poisson Equation

A coordinate system based on opponent color space, such as *YCbCr*, is widely used in the image processing. In Sapiro[9], chrominance components *Cb* and *Cr* were estimated from luminance component *Y* based on the basic geometry of these channels observed in natural images. In fact Sapiro[9] proposed to find Cb component by solving the following optimization equation,

$$\min_{Cb} \int_{\Omega} \rho\,(\|\nabla Y - \nabla Cb\|)\, d\Omega \tag{1}$$

Here, $\nabla = (\partial/\partial x, \partial/\partial y)$ and & is the whole region of the image. The norm || || is the Euclidian norm, and ρ can be chosen as a various positive function. When it is a quadratic function, the equation (1) becomes a famous Poisson equation (2). That is, the method recovers the color by solving Poisson equation with the Laplacian of luminance in the right hand side of the equation.

$$\Delta Cb = \Delta Y \tag{2}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and for *Cr* the same equation should be solved simultaneously. To solve the Poisson equation (2), we need an appropriate boundary condition, and for this to occur, usually some appropriately chosen colors are put by the operator in the appropriate boundary spots of the grayscale image.

In Figures from 3(a) to 3(f), the color recovery results by this method are given for the original photo image of “Ganjin Wajo” in Figure 2(b). It should be noted that for obtaining *Cr* and *Cb* values numerically the following discretization (3) was adopted for the Laplacian, and the equation (2) was solved by iteratively solving the following equation (4) till the convergence was reached,

$$\Delta Y = \frac{Y_{i+1j} + Y_{i-1j} + Y_{ij+1} + Y_{ij-1} - 4Y_{ij}}{h^2} \tag{3}$$

$$Cr_{ij} = \left(\frac{Cr_{i+1j} + Cr_{i-1j} + Cr_{ij+1} + Cr_{ij-1}}{h^2} - \Delta Y_{ij}\right)\frac{h^2}{4} \tag{4}$$

where suffix *i* and *j* denotes the position of pixel on the restored image, defined as increasing downward and rightward, respectively and *h* is the distance between the two adjacent pixels’ centers. Now the pixel indices (*i*,*j*) are taken as the original coordinate on the restored image, and then *h* is taken to 1 in (3) and (4).

Figure 3(a) denotes the *Y* values of the original Figure 2(b) and this corresponds to the grayscale image of the original color images. Applying the operation given by equation (3) we have ΔY in Figure 3(b). It is clear that the boundary of “the face of Ganjin Wajo” with a large change of luminance is depicted. Additionally, more closely checking it in detail, it is seen that around the boundary some pixels with large positive ΔY(white), and with large negative ΔY(black) are adjacent, (though this might happen due to the coarseness of the photo images with 106x141 pixels), therefore, the adjustment of enhancing the edges seems to be done by this operation (3). On the other hand, as is shown in Figure 3(c), the *Cr* values does not show the clear edges of the face and seems even to be random, and it does not seem to be easy to recover clearly the *Cr* values of the original image from this image. This condition is similar to the other chrominance *Cb*. These results correspond to the face that the saturation of the photo of “Ganjin Wajo” is low and in some spots the black lacquer comes out.

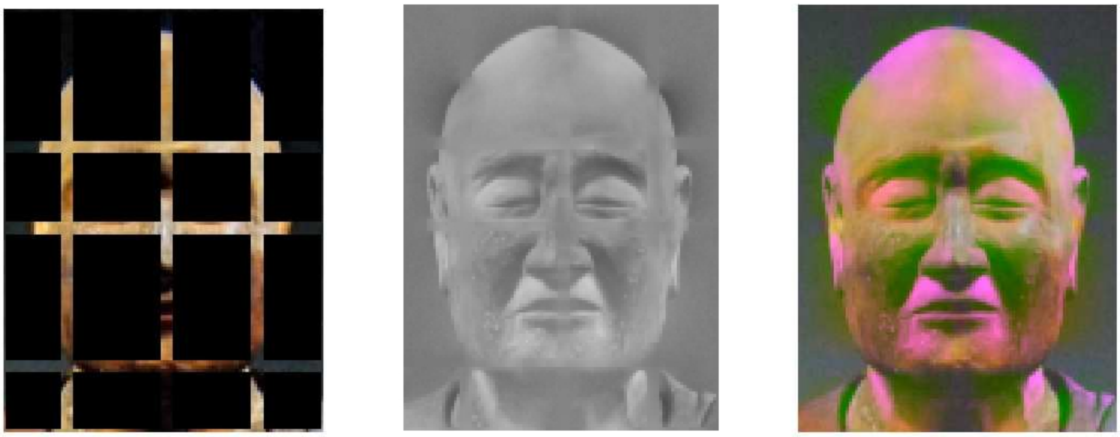
Figure 3(d) indicates the position at which the target colors were set. The target color was established as the skin color, and the colors in the original image were enhanced and adjusted slightly to a brighter one. It is clear that the position and scale of the target colors were very influential to the inpainted result, but as a first trial, they were put on the lattice points. The inpainted images are listed in Figures 3(e) and 3(f).

The estimated *Cr* values of Figure 3(e) iteratively obtained by the equation (3) has greater information compared to those of Figure 3(c), that is, clear edges of the face; the final result in Figure 3(f) has brighter colors. In some spots at the boundary of the head, near to the area with a distributed reference color, the recovered color and the reference color are not sufficiently smoothly connected. Thus, it seems that, as stated above, more keen attention should be paid on the appropriate boundary conditions and the selection of target colors.

Consequently, the final result of Figure 3(f) was obtained by superimposing the *Cr* image (Figure 3(e)), and the C_b image was obtained by solving the equation (4) based on *Y* values. From the figure it is seen that at the neighbor of the sites with seed colors the target skin colors are diffused but at the sites apart from the spots with target color the estimated *Cr* and *Cb* values become dominant.

At around the region where the absolute value of ΔY is large (the black or white region of Figure 3(b)), the green and magenta color have imaged unexpectedly. Since *Y* values are invariant from the original image, this comes from the

(a) Y image (b) ΔY image (c) Cr image



(d) Reference region (e) Estimated Cr image (f) Inpainted image

Fig.3 Inpainting by Poisson equation method

estimating process of Cr and Cb.

Considering the result from the obtained Cr image similar to Y values, the color restoration method has a tendency to exaggerate green and magenta colors under a certain condition. In the transformation from (Y, Cr, Cb) to RGB

$$\begin{cases} R = Y + 1.402\,Cr \\ G = Y - 0.714\,Cr - 0.344\,Cb \\ B = Y \qquad\qquad\quad + 1.772\,Cb \end{cases} \tag{5}$$

If the distribution of Cr and Cb are similar, then R and B change in synchronization. On the other hand, G has the reverse phase, and so in the recovered image, either R+B or G becomes stronger in some spots.

The result of Figure 3(e) can be understood because the target image is composed only of the two parts, the face (head) and the background, and has a few colors. The change of the luminance possibly becomes dominant, the distribution of Y was transferred to the one of Cr and Cb directly, and these distributions become similar. If this allocation is true, the selection and the distribution of the reference color are vitally important for this recovery method.

More essentially, in order that the target color is felt natural compared to the color of the original image, what is needed is that the Y values of the target color must be same with those of the original image. Therefore, after setting the target color as an initial one and having processed the recovery of the color, some readjustment or change of the target color might be needed. These aspects of the post adjustment will be explored in the future.

III. Color Recovery Based on Detail Color Speculation

A. *Color seed transfer method*

In this section we apply the color blend method proposed by Horiuchi[6] to recover the color of "Zocho-ten". In the original image, there remained a slightly golden color in some parts of the head and the costume. We selected 20 spots with bright red brown colors like parts of the face, and the images in Figure 4 were restored. It seems that the simple application of the method is not sufficient because the luminance information is incomplete, and moreover, there is much noise due to the white mist and this makes the recovery harder. A more elaborate method is needed for the case of the recovery of old statues.

Consequently in the next section we utilize more detailed information of color structure. In order to make the situation

(a) Gold seeds case

(b) Red brown seeds case

Fig.4 Color recovery with color transform method

simpler we choose "Ganjin Wajo" as the target image because the structure of color damage is simpler than "Zocho-ten".

B. Color recovery based on detailed color information

The *RGB* primary color system uses the primary colors, red, green and blue and each color has brightness level of

$$0 \le R \le 255, \quad 0 \le G \le 255, \quad 0 \le B \le 255$$

This colorimetric system can express 16.7million(=2^{24}) colors. Unfortunately, it is difficult for users to specify subjective color concepts in the system of *RGB* color model. *RGB* color model is hardware-oriented. By contrast, *HSV* color model is user-oriented, because it is based on "tint", "shade", and "tone" which artists use intuitively. The *HSV* system is shown in Figure 5. This coordinate system is cylindrical where *H* denotes "Hue", *S* denotes "Saturation", and *V* denotes "Value".

H is measured by the angle around the vertical axis, with "red" at 0°, "yellow" at 60°, "green" at 120°, "cyan" at 180°, "blue" at 240°, "magenta" at 300°. *H* takes a value of

$$0° \le H < 360°$$

"Saturation" is given by the distance from the vertical axis and the value of "Saturation" ranges from 0 to 1.

$$0 \le S \le 1$$

Fully saturated colors (the purest colors) occur at S=1. If S=0, *H* is undefined and the color is achromatic i.e., some shade of gray.

"Value" or "brightness" increases along the vertical axis from 0 to 1. *V*=1 is at the top surface.

$$0 \le V \le 1$$

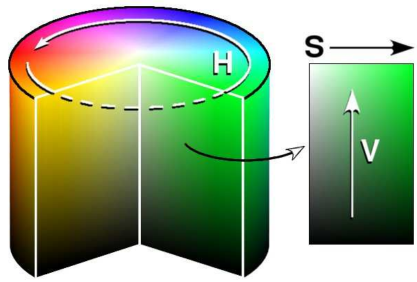
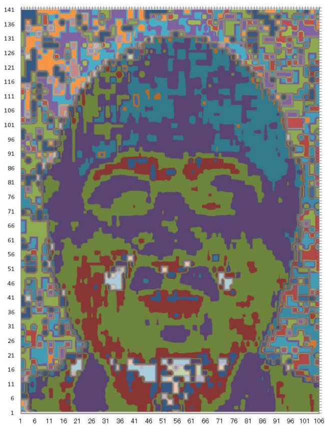
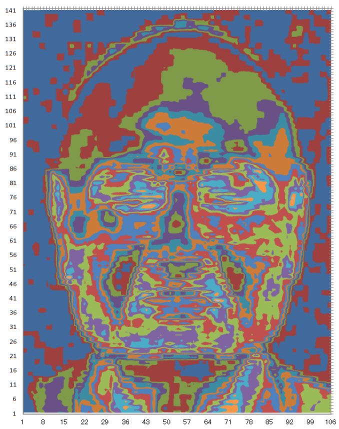


Fig.5 *HSV* color system[14]

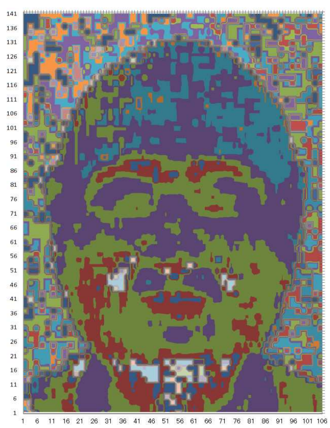
V=1 corresponds to "white", and V=0 correspond to "black". If $0 < V < 1$, *V* corresponds to gray. The shades of gray occur along the central axis.



(a) *H*-contour image



(b) *S*-contour image



(c) *V*-contour image

Fig.6 *HSV* image of Ganjin Wajo

As is in the previous section, here we also recover the color of “Ganjin Wajo” image using *HSV* color model. The statue has become dusty over a long period time and looks white (high *V* value and low *S* value) on his head. His cheeks and eyebrows are dark gray (low *S* value and low *V* value). Transforming the image into the *HSV* color space, and the contour line figures are shown in Figures 6(a), (b) and (c).

Looking at Figure 6 (a), the white region between the top of the head and the forehead and the dark region between the eyebrow and the cheek are preserving the information of “*H* value”. They have the almost same *H* values as other parts of the face. “*H* value” on his face takes a value of

$$0° \le H \le 60° \quad 300° \le H < 360° \tag{6}$$

That is magenta, red or yellow. See Figure 6(b). The region between the top of the head and the forehead, the eyebrow, and the cheek have a low value in *S*. The region looks white and the others look black.

Figure 7 shows the histogram of the *H* value. The distribution of *H* value is localized in the range 30°(Orange)±30°. Therefore, we estimate that the statue was colored in the *H* value of around 30° when it was built. Hence, we assume that

1. The whole face was colored in *H* value of 30° (Orange)
2. Due to dust, the color gamut has been expanded
3. Due to dust, the *S* value decreased

Based on the above assumptions we adopted the recovery method below.

1. The *H* value in the equation (6) is compressed into $15° \le H \le 45°$
2. The saturation of Hue-compressed pixels is raised.

We made a transformation:

for $0° < H < 60°$ set $H \leftarrow 0.5H + 15°$
for $300° < H < 360°$ set $H \leftarrow 0.5H - 135°$

This transforms *H* value between 15° and 45°. We give the pixels higher “Saturation *S* value” with the following method.

If $S < 0.8$ set $S \leftarrow 0.5S + 0.4$.

We set *S* value range from 0.4 to 0.8 by this transformation, though it originally covers from 0 to 0.8.

As is seen from the recovered image 8(b), the white region between the top of the head and the forehead was almost denoised. Further, the dark cheek and eyebrow were bright. However, the gray color still remained on the top of the head, which should be improved. As a result, we carried out the method listed below. The gray region on the top of the head consists of the two parts, one part is $S = 0$ and achromatic and the other is the part where *H* values changes randomly outside of the equation (6) (see Figure 6 (a)). The region is not thick vertically. When the pixel is at (i, j) we use the *S* and *H* values that are the mean of tree values before the line given in the equation (7) stated below.

$$\begin{cases} S_i^j = \dfrac{S_{i-1}^{j-1} + S_i^{j-1} + S_{i+1}^{j-1}}{3} \\ H_i^j = \dfrac{H_{i-1}^{j-1} + H_i^{j-1} + H_{i+1}^{j-1}}{3} \end{cases} \tag{7}$$

Applying this to Figure 8(b), we obtained Figure 8(c). It is seemed that the white region between the top of the head and the forehead and the darkened cheek and eyebrow were improved considerably. However, some black pixels still remained around the cheek and the jaw. Moreover, some

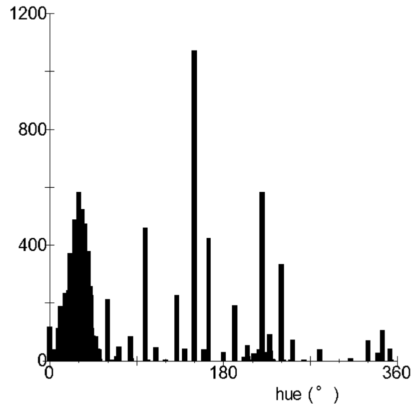
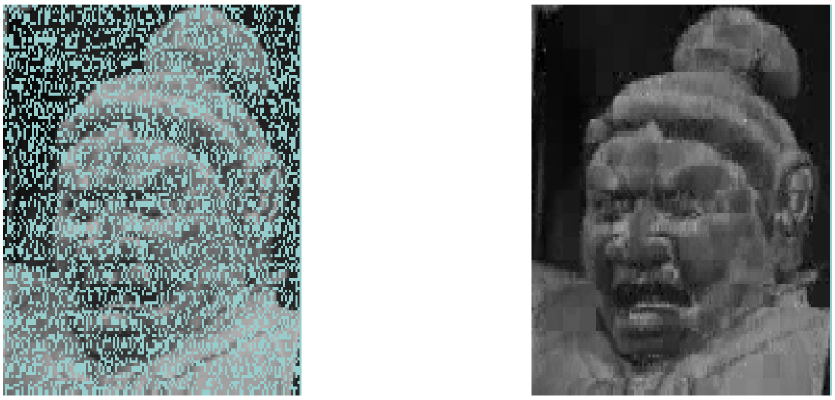


Fig.7 Histogram of *H*-values

(a) Original Image (b) Image after basic reprocessing (c) Image with additional local retouches

Fig.8 Color recovery by reprocessing of *H*-value compression and *S*-value elevation



(a) Noised (b) Denoised

Fig.9 Restoration of artificially noised image by sparse coding

pixels on the cheek and the jaw had different *H* values from the others. This problem should be resolved in the future study.

IV. Color Inpainting by Sparse Coding

Recently the sparse coding of signals has been actively studied. For the application to image analysis an image is divided into small patches and each patch is inpainted by a sparse combination of patch images from a redundant dictionary (example patch images with the same patch size). This is realized by the k-SVD algorithm which was proposed in Aharon et al.[1]. Several articles reported successful denoising results for grayscale images by the sparse coding. Mairal et al [7] reported an extension to the color image denoising and inpainting. In this article we tried to apply the sparse coding inpainting method to the grayscale image of “Zocho-ten”. As seen from Figures 9(a) and (b), from the artificially noised image, a clean image can be recovered. Here we used a redundant dictionary of 8×8patch images from 500 peoples face’s. However, this does not mean without a doubt that the damaged statue can be restored exactly as it was in the past. One idea is to use patches selected from photo databases of Japanese old statues as a dictionary. More detailed consideration will be given in the future research.

V. Conclusion

In this article, we mainly focused on color inpainting of the photo of the face of “Ganjin Wajo”, which was chosen due to the simplicity of the color structure of the image. We aimed to show to what extent the mathematical inpainting methods are useful to color restoration. As a first step four digital image-inpainting techniques were applied mainly to denoise and remove discoloration by mold or dust and it was shown that they have potentiality for such purpose. Though we recognize that our final goal to recover fully the color of “Shitenno” is very far, it was shown that the proposed methods are useful, at least, as pre-processes. For more complex situations, improved techniques based on this research will be pursued in our future work.

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