Partial Meet Contraction Based On Relevance Criterion*

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Abstract—**An agent usually holds a very large number of beliefs. Hopefully, efficient belief changes should be performed only in the part of its relevant states at a time. Parikh showed that AGM belief change operations do not always respect his relevance criterion. Kourousias and Makinson showed that they will do so if the given belief set K is in canonical form K'. However, even when K is closed under classical consequence, K' will be a belief base and usually is not closed under classical consequence. In this paper, we first show that there are two alternative approaches to guarantee the relevance criterion. They are constructed by replacing the family $K\perp x$ with the family $K\perp' x$ and $K\perp^* x$ respectively. The latter is general enough to generate all the relevance-respecting belief change operations definable on K itself.**

Keywords: Belief change, relevance criterion, AGM model.

1 Introduction

Belief revision is a topic of much interest in theoretical computer science and logic, and it forms a central problem in research into artificial intelligence (infer to [6,12]). Its notable methodology is AGM model, formulated by Alchourron, Gardenfors and Makinson in 1985 [1]. In the logic of belief revision, a belief state (or database) is often represented by a set of formulae. Agent's belief change is a canonical example in research of belief revision.

An agent in the world usually holds a very large number of beliefs and receives new information from the exterior world. Thus it may be inconsistent when new information is added into its beliefs. In order to deal with the contradictions, some original beliefs must be given up accordingly. What was given up is the motivation about why we introduce belief revision. Many works about belief revision have proposed many useful methods to deal with it. The AGM model provides a milestone work in belief revision. But it is possible to give up (or change) almost all original beliefs in AGM model. Recently some researches formulated that relevant beliefs should be affected only when beliefs are changed. For example, Parikh proposed that a belief change operator which respects his relevance criterion protects any irrelevant formulae in 1999 [10]. In that case, belief revision only changes locally in original belief. We will discuss the local change based on splitting of a set of formulae, carved up a theory into disjoint pieces about different subject matters(letters), formulated by Parikh in 1999 [10].

In order to deal with the local change, Parikh defined splitting of any set of formulates and the irrelevant formula in belief change over a language with a finite set of elementary letters in 1999 [10]. Kourousias and Makinson showed that every set K of formulae has a unique finest splitting and extended the definition of irrelevant formulae to an infinite language in 2007 [5]. Meanwhile, Parikh formulated a criterion for relevance in belief change and proved that the update operator in AGM does not satisfy it generally [10]. Many researches have been conducted to attack this issue syntactically by exploring some postulates to ensure the relevance criterion. These works are carried out by Parikh [10], Chopra, Georgatos and Parikh [3], Chopra and Parikh [4], Peppas, Chopra and Foo [11]. Kourousias and Makinson showed that the partial meet operations over canonical form K' of belief set K satisfies the relevance criterion [5].

In this paper, we will not begin with normalizing the belief set. But we will rebuild $K \perp x$, set of maximal subset A of belief set K such that $A \nvdash x$ in [1]. We choose two subsets, denoted by $K \perp' x$ and $K \perp^* x$, of $K \perp x$ such that $K \perp x \neq \emptyset$ iff $K \perp' x \neq \emptyset$ and $K \perp x \neq \emptyset$ iff $K \perp^* x \neq \emptyset$. Then we construct the partial meet contraction operator $\div'$ and $\div^*$ by replacing $K \perp x$ with $K \perp' x$ and $K \perp^* x$ respectively. The operators $\div'$ and $\div^*$ satisfy AGM postulates and relevance criterion formulated by Parikh. The converse holds too for $\div^*$. So that a representation theorem is obtained according to AGM.

The rest of the paper is organized as follows: In section 2, we recall many preliminaries for the paper. In section 3, two partial meet contraction $\div'$ and $\div^*$ protecting

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relevance criterion are constructed and the representation theorem for the latter is given. In section 4, we compare our approach with those of a number of related works.

2 Preliminaries

In this section we review the preliminaries for the paper. We always assume a propositional logic with set of infinite or finite letters including the zero-ary truth $\top$ among the primitive connectives. We use lower case letters $a, b, ..., x, y, z, \alpha, \beta, ...$ to range over formulae of classical propositional logic. Sets of formulae are denoted by upper case letters $A, B, ..., X, Y, ...$, reserving L for the set of all formulae, E for the set of all elementary letters (alias propositional variables) of the language. For any formula α, we write $E(\alpha)$ to mean the set of the elementary letters occurring in α; similarly for sets of formulae. Let $F \subseteq E$, $L(F)$ stands for the sub-language generated by F, i.e. the set of all formulae x with $E(x) \subseteq F$. Classical consequence is written as $\vdash$ when treated as a relation over $2^L \times L$, classical consequence operation is written as Cn when treated as an operation on 2^L into itself. The relation of classical equivalence is written $\dashv\vdash$. We say that set K of formulae is belief set if it is closed, i.e., $K = Cn(K)$. To lighten notation, $v(A) = 1$ is short for: $v(\alpha) = 1$ for all $\alpha \in A$, while $v(A) = 0$ abbreviates: $v(\alpha) = 0$ for some $\alpha \in A$.

Let K be any set of formulae.

For any belief set K, $K \perp x$ is the set of all maximal subsets A of K such that $A \nvdash x$. In other words, $A \in K \perp x$ iff

(1) $A \subseteq K$,

(2) $A \nvdash x$,

(3) any $\varphi \in K \setminus A$, $A, \varphi \vdash x$.

And γ is any function such that for every formula x, $\gamma(K \perp x)$ is a nonempty subset of $K \perp x$, if the latter is nonempty, and $\gamma(K \perp x) = \{K\}$ otherwise. Such a function is called a selection function for K. We say that γ is transitively relational over K iff there is a transitive relation $\leq$ over 2^K such that for all $x \notin Cn(\emptyset)$, $\leq$ marks off $\gamma(K \perp x)$ in the sense the following identity, which we call the marking off identity, holds:

$$\gamma(K \perp x) = \{B \in K \perp x : B' \leq B \text{ for all } B' \in K \perp x\}.$$

The operation $\div$ defined by putting $K \div x = \bigcap \gamma(K \perp x)$ for all x is called the partial meet contraction over K determined by γ.

Note that the concept of partial meet contraction includes, as special cases, those of maxichoice contraction and full meet contraction. The former is partial meet contraction with $\gamma(K \perp x)$ a singleton; the latter is partial meet contraction with $\gamma(K \perp x)$ the entire set $(K \perp x)$.

The following two lemmas in [1] are very important in belief revision.

Lemma 2.1([1]). Let $\div$ be a function defined for belief set K and a formula x. $\div$ is a partial meet contraction operation over K iff $\div$ satisfies AGM postulates $(\div 1) - (\div 6)$ for contraction over K.

Lemma 2.2([1]). Let K be any belief set, and $\div$ a partial meet contraction function over K, determined by a selection function γ. Then $\div$ is a transitive relation over K if and only if $\div$ satisfies AGM postulates $(\div 1) - (\div 8)$.

Lemma 2.3([1]). Let K be any belief set. Then $K \subseteq Cn((K \sim x) \cup \{x\})$.

Many matters relate each other in the world, but only partial relations between matters are essential. So that Parikh proposed the concept of splitting for belief set(set of formulate). It allows us carve up a belief set into disjoint pieces about different subject matters.

Definition 2.4(Splitting [5]) Let $\mathbf{E} = \{E_i\}_{i \in I}$ be any partition of the set E of all elementary letters of the language. we say that $\mathbf{E}$ is a splitting of set K of formulae iff $\bigcup\{Cn(K) \cap L(E_i)\}_{i \in I} \vdash K$, equivalently, iff there is a family set $\{B_i\}_{i \in I}$ of formulae with each $E(B_i) \subseteq E_i$ such that $\bigcup\{B_i\}_{i \in I} \dashv\vdash K$.

Generally, people often hope to split a belief set as fine as possible such that essential relation matters is in same piece possibly.

Definition 2.5(Fineness of a Partition[5]). Following customary terminology, we say that a partition $\mathbf{E} = \{E_i\}_{i \in I}$ of all elementary letters set E of the language is at least as fine as another partition $\mathbf{F} = \{F_j\}_{j \in J}$ of E, and we write $\mathbf{E} \leqslant \mathbf{F}$, iff every cell of $\mathbf{F}$ is the union of cells of $\mathbf{E}$. Equivalently, $R_{\mathbf{E}} \subseteq R_{\mathbf{F}}$, where $R_{\mathbf{E}}$ (resp. $R_{\mathbf{F}}$) is the equivalence relation over E associated with $\mathbf{E}$ (resp. $\mathbf{F}$).

Parikh showed that there is an unique finest splitting of K for finite language [10] and Kourousias and Makinson proved the result for any language [5]. They are the base of irrelevant formulate and relevance criterion over a language with set of infinite letters [10].

Theorem 2.6([5]). Every set K of formulae has a unique finest splitting.

The theorem says that there was a unique way to think of K as being composed of disjoint information about certain subject matters. The following parallel interpolation theorem, proposed by Kourousias and Makinson in 2007, was used to proved the theorem 2.6. We will use it in proof of our results.

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Theorem 2.7(Parallel interpolation theorem[5]). Let $A = \bigcup\{A_i\}_{i\in I}$ where the letter sets $E(A_i)$ are pairwise disjoint, and suppose $\bigcup\{A_i\}_{i\in I} \vdash x$. Then there are formulae b_i such that each $E(b_i) \subseteq E(A_i) \cap E(x)$, $A_i \vdash b_i$, and $\bigcup\{b_i\}_{i\in I} \vdash x$.

Parikh defined the irrelevant formulae in a belief change when the set of elementary letters was finite [10]. Kourousias and Makinson extended the definition for a language with set of infinite letters [5].

Definition 2.8 (Irrelevant formulae in a belief change [5]). Let K be any consistent set of formulae, with x a formula that is a candidate for contracting from K or integrating into K by a process of revision. Let $\mathbf{E} = \bigcup\{E_i\}_{i\in I}$ be the unique finest splitting of K. We say that a formula $y \in K$ is irrelevant to the contraction or revision of K by x (briefly: $y \in K$ is irrelevant to x modulo K) iff $E(y)$ is disjoint from $\bigcup\{E_j\}_{j\in J}$, where $\bigcup\{E_j\}_{j\in J}$ is the subfamily of cells in $\mathbf{E}$ that share some elementary letter with $E(x)$. We denote the set of the irrelevant formulae to x modulo K by $I_{K,x}$ and simply as I_x in contexts where the identity of K is clear. Formally,

$I_x = \{y \in K | E(y) \cap \bigcup_{j\in J}\{E_j\} = \emptyset$ where $E_j \cap E(x) \neq \emptyset$ for all $j \in J\}$

The *relevance criterion* may be put as follows: whenever an element y of K is irrelevant to x modulo K, then it remains an element of the result of contracting or revising K by x. In other words, $I_x \subseteq K \div x$ or $I_x \subseteq K * x$.

We recall some basic notions about essential formulae in [9]. A formula y is an essential formula of x iff $x \dashv\vdash y$ and for every formula z with $z \dashv\vdash x$ satisfied $E(y) \subseteq E(z)$. It is clear that the essential formula of a formula is not unique. But the essential formulae of a formula have the same elementary letters by the least letter-set theorem [9].

3 Partial Meet Contraction Based on Splitting

In this section, we will not begin with normalizing the belief set, but we will reduce $K \perp x$, set of maximal subset A of belief set K such that $A \nvdash x$ in [1]. We choose two subsets, denoted by $K \perp' x$ and $K \perp^* x$, of $K \perp x$ such that $K \perp x \neq \emptyset$ iff $K \perp' x \neq \emptyset$ and $K \perp x \neq \emptyset$ iff $K \perp^* x \neq \emptyset$. Then we construct the partial meet contraction operator $\div'$ and $\div^*$ by replacing $K \perp x$ with $K \perp' x$ and $K \perp^* x$ respectively. The operators $\div'$ and $\div^*$ satisfy AGM postulates and relevance criterion. The converse holds too for $\div^*$. So that a representation theorem is obtained according to AGM.

3.1 Contraction operator based on the finest splitting

Let $\mathbf{E} = \{E_i\}_{i\in I}$ be the finest splitting of a belief set K. Let $K_i = K \cap L(E_i)$ for all $i \in I$ and $K' = \bigcup\{K_i\}_{i\in I}$. As [1], we define $K \perp' x$ to be set of maximal subset A of K such that $A \nvdash x$ and $\bigcup_{i\in I}(A \cap L(E_i))$ is maximal subset of $\bigcup_{i\in I}(K \cap L(E_i))$ such that $\bigcup_{i\in I}(A \cap L(E_i)) \nvdash x$. i.e.

$K \perp' x = \{A \in K\perp x | \bigcup\{A \cap L(E_i)\}_{i\in I}$ is a maximal subset of K' such that $\bigcup\{A \cap L(E_i)\}_{i\in I} \nvdash x\}$

Let γ' to be any function such that for every formula x, $\gamma'(K \perp' x)$ is a nonempty subset of $(K \perp' x)$ when the latter is nonempty, and $\gamma'(K \perp' x) = \{K\}$ in the limiting case that $(K \perp' x)$ is empty. Such a function is called a selection function based on splitting for K.

Then the operation $\div'$ defined by putting $K \div' x = \bigcap \gamma'(K \perp' x)$ for all x is called the partial meet contraction based on splitting over K determined by γ'.

Immediately, we have that $K \perp' x \subseteq K \perp x$ by the above definitions. Our first lemma shows that these two sets are equivalent for non-emptiness. As in [1], we build partial meet contraction by section function γ over $K \perp' x$.

Lemma 3.1 $K \perp x \neq \emptyset$ iff $K \perp' x \neq \emptyset$.

Proof. It is enough to show that $K \perp x \neq \emptyset$ implies $K \perp' x \neq \emptyset$ since the inverse implication is obvious. Suppose $K \perp x \neq \emptyset$. It is clear that $\nvdash x$. There is a maximal subset A' of $K' = \bigcup\{K \cap L(E_i)\}_{i\in I}$ such that $A' \nvdash x$. By $K' \subseteq K$ and compactness, there is a maximal subset A of K such that $A' \subseteq A$ and $A \nvdash x$, i.e., $A \in K \perp x$. By the definition of $K \perp' x$, it is sufficient to show that $\bigcup\{A \cap L(E_i)\}_{i\in I}$ is a maximal subset of K' such that $\bigcup\{A \cap L(E_i)\}_{i\in I} \nvdash x$. Otherwise, there must exist $\alpha \in K'$ such that $\alpha \notin \bigcup\{A \cap L(E_i)\}_{i\in I}$ and $\bigcup\{A \cap L(E_i)\}_{i\in I} \bigcup\{\alpha\} \nvdash x$. Consequently, $A' \bigcup\{\alpha\} \nvdash x$ by $A' \subseteq \bigcup\{A \cap L(E_i)\}_{i\in I}$. It conflicts with the maximality of A'.

It is evident that $K \div' x = \bigcap \gamma'(K \perp' x)$ is a special AGM partial meet contraction since $K \perp' x \subseteq K \perp x$. So that it satisfies all properties of AGM operator.

The following lemma shows that any partial meet contraction $\div'$ based on splitting satisfies the relevance criterion. Hence we show that full meet contraction respect to relevance criterion, i.e. $I_x \subseteq K \sim' x$, since $K \sim' x \subseteq K \div' x$.

Lemma 3.2. Let $\mathbf{E} = \{E_i\}_{i\in I}$ be the finest splitting of belief set K and $K \sim' x = \bigcap(K \perp' x)$ full meet contraction based on splitting. Then $I_x \subseteq K \sim' x$.

Proof. For any $y \in I_x$, i.e., $y \in K$ is a irrelevant formula to x modulo K. Then $\bigcup\{K_i\}_{i\in I\setminus J} = \bigcup\{K \cap L(E_i)\}_{i\in I\setminus J} \vdash y$ since $\bigcup\{K_i\}_{i\in I} \vdash K$, parallel interpolation theorem and $(\bigcup\{K_j\}_{j\in J}) \cap E(y) = \emptyset$, where $\bigcup\{E_j\}_{j\in J}$ is the subfamily of cells in $\mathbf{E}$ that share some elementary letter with $E(x)$. Hence we show that, for all $i \in I\backslash J$, if α is any formula of K_i then $\alpha \in K \sim' x$.

Suppose for some $i \in I\backslash J$, $\alpha \in K_i$ but $\alpha \notin K \sim' x$, i.e, there exists $A \in K \perp' x$ such that $\alpha \notin A$. We have $\alpha \notin \bigcup\{A \cap L(E_i)\}_{i\in I} \nvdash x$. Hence we show that $\bigcup\{A \cap L(E_i)\}_{i\in I} \cup \{\alpha\} \nvdash x$ by $\alpha \in K_i \subseteq I_x$. It is a contradiction to $A \in K \perp' x$.

By lemma 2.1 and lemma 3.2, the following theorem is clear.

Theorem 3.3. Let $\mathbf{E} = \{E_i\}_{i\in I}$ be the finest splitting of belief set K. If $\div'$ is a partial meet contraction operation over K determined by γ' then $\div'$ satisfies AGM postulates $(\div 1) - (\div 6)$ for contraction and also satisfies the relevance criterion over K.

Corollary 3.4. Suppose the same conditions as for Theorem 3.3, If $*'$ is a partial meet revision operation over K determined by γ' then $*'$ satisfies AGM postulates $(*1) - (*6)$ for revision and also satisfies the relevance criterion over K.

Proof. By the definition of revision from contraction using the Levi identity, $K' \div' \neg x \subseteq K' * x$. Since $E(x) = E(\neg x)$, the preceding theorem tells us that $\alpha \in K' \div' \neg x$ for every $\alpha \in I_x$, and we are done.

Note. The following example shows that neither converse of the above two theorems holds.

Example: Let $K = Cn(\{p, q, r\})$, $x = p \wedge q$ and $K \div x = Cn(p \leftrightarrow q, r)$. It is clear that $Cn(r) \cap L(r) = I_x \subseteq K \div x$ and $K \div x$ satisfies AGM postulates $(\div 1) - (\div 8)$. But there is no γ' such that $\bigcap \gamma'(K \perp' x) = Cn(p \leftrightarrow q, r)$.

In next subsection, we will weaken the semantics of partial meet contraction based on splitting to gain a representation theorem.

3.2 Contraction operator based on relevance

In this subsection, we weaken the semantics of partial meet contraction based on splitting. Our method does not require on every part of the finest splitting of belief set. Now, it only depends on two parts, the relevance and irrelevance, of elementary letters of the language. We will define $K \perp^* x$ such that $K \perp' x \subseteq K \perp^* x \subseteq K \perp x$.

Given belief set K and $x \in K$. Let $\mathbf{E} = \{E_i\}_{i\in I}$ be the finest splitting of K. For any $A \subseteq K$ such that $A = Cn(A)$, we define $E_x = \bigcup\{E_i | i \in I \& E_i \cap E(\overline{x}) \neq \emptyset\}$, $A^r_x = A \cap L(E_x)$ and $A^i_x = A \cap L(E\backslash E_x)$ where $\overline{x}$ is some essential formula of x. It is clear that $K^i_x = I_{\overline{x}}$.

Similarly, we define $K \perp^* x$ to be set of maximal subset A of K such that $A \nvdash x$ and $A^r_x \cup A^i_x$ is a maximal subset of $K^r_x \cup K^i_x$ such that $A^r_x \cup A^i_x \nvdash x$. i.e.

$K \perp^* x = \{A \in K \perp x | A^r_x \cup A^i_x$ is a maximal subset of $K^r_x \cup K^i_x$ such that $A^r_x \cup A^i_x \nvdash x\}$

Let γ^* be a function such that for every formula x, $\gamma^*(K \perp^* x)$ is a nonempty subset of $(K \perp^* x)$ when the latter is nonempty, and $\gamma^*(K \perp^* x) = \{K\}$ in the limiting case that $(K \perp^* x)$ is empty. Such a function is called a selection function based on relevance for K.

Then the operation $\div^*$ defined by putting $K \div^* x = \bigcap \gamma^*(K \perp^* x)$ for all x is called the partial meet contraction based on relevance over K determined by γ^*.

By the way identical to lemma 3.1, we can prove the following lemma which shows that the two sets are equivalent for non-emptiness. A partial meet contraction over K was built as [1] by section function γ^* on $K \perp^* x$. Its proof is similar to that of lemma 3.1.

Lemma 3.5 $K \perp x \neq \emptyset$ iff $K \perp^* x \neq \emptyset$.

Lemma 3.6 $A \in K \perp x$ and $I_{\overline{x}} \subseteq A$ iff $A \in K \perp^* x$ where $\overline{x}$ is some essential formula of x.

Proof. Right to left. For any $A \in K \perp^* x$, by the definition of $K \perp^* x$, we have $A \in K \perp x$ and $A^r_x \cup A^i_x$ is a maximal subset of $K^r_x \cup K^i_x$ such that $A^r_x \cup A^i_x \nvdash x$. Then $I_{\overline{x}} \subseteq A^i_x \subseteq A$ since any $\alpha \in I_{\overline{x}} = K^i_x$ satisfies $A^r_x \cup A^i_x \cup \{\alpha\} \nvdash \overline{x}$.

For the converse. Suppose $A \in K \perp x$ and $I_{\overline{x}} \subseteq A$. It is suffices to show that $A^r_x \cup A^i_x$ is a maximal subset of $K^r_x \cup K^i_x$ such that $A^r_x \cup A^i_x \nvdash x$. Then we only need to show that A^r_x is maximal subset of K^r_x such that $A^r_x \nvdash x$ since $K^i_x = I_{\overline{x}} \subseteq A$, i.e., $K^i_x = I_{\overline{x}} = A^i_x$.

Let $\alpha \in K^r_x$ with $\alpha \notin A^r_x$. It is clear that $\alpha \notin A$ if and only if $\alpha \notin A^r_x$ whenever $\alpha \in K^r_x$. So $\alpha \notin A$ by $\alpha \notin A^r_x$. By $A \in K \perp x$ and $\alpha \in K^r_x \subseteq K$ then $A \cup \{\alpha\} \vdash x$, i.e., $A \cup \{\alpha\} \vdash \overline{x}$. Hence $A \vdash \alpha \to \overline{x}$. Using the interpolation theorem, there exists $\varphi \in L(E \cap E(\alpha \to \overline{x})) \subseteq L(E_x)$ such that $A \vdash \varphi \vdash \alpha \to \overline{x}$. So that $\varphi \in A^r_x$ and $\varphi \cup \alpha \vdash \overline{x}$. Furthermore, $A^r_x \cup \alpha \vdash \overline{x}$, i.e., $A^r_x \cup \alpha \vdash x$. Therefore A^r_x is a maximal subset of K^r_x such that $A^r_x \nvdash x$.

The lemma shows that every element in the set $K \perp^* x$ is only an element in $K \perp \overline{x}$ and it contains all irrelevant formulae to $\overline{x}$ where $\overline{x}$ is an essential letter of x. The converse holds too.

Given any $A \in K \perp' x$, we have $A \in K \perp x$ and $A \in K \perp \overline{x}$ where $\overline{x}$ is some essential formula of x. By Lemma 3.2, we have $I_{\overline{x}} \subseteq A$. Then $A \in K \perp^* x$ by Lemma 3.7. Hence $K \perp' x \subseteq K \perp^* x$. But the converse does not generally holds due to the example in the last subsection. We have weakened the semantics of the theory in the previous subsection and will gain a representation theorem in this subsection.

The example still shows that $K \perp^* x \subseteq K \perp x$ and $K \perp x \nsubseteq K \perp^* x$ usually. In other words, $K \perp^{'} x \subsetneq K \perp^* x \subsetneq K \perp x$ generally.

By the lemma 3.2 and lemma 2.2, the following theorem is clear.

Theorem 3.7. Let $\div^*$ be a function defined for belief set K and a formula x. The partial meet contraction operation $\div^*$ determined by γ^* is a transitive relation over K iff $\div^*$ satisfies AGM postulates $(\div 1) - (\div 8)$ for contraction and also satisfies the relevance criterion over K.

The theorem shows the semantics constructs an representation theorem for the relevance criterion and AGM postulates for contraction.

Corollary 3.8. Let $\star$ be a function defined for belief set K and a formula x. $\star$ is the partial meet revision operation $\star$ determined by $K \star x = Cn(K \div^* \neg x) \cap \{x\}$ iff $\star$ satisfies AGM postulates $(*1) - (*8)$ for revision and also satisfies the relevance criterion over K.

The result shows the semantics constructs an representation theorem for relevance criterion and AGM 8 postulates for revision.

4 Comparison with Related Works

In the sequent, we compare our methods with these of Kourousias and Makinson [5], and Hansson and Wassermann's local change depending on local implication [8] as well.

4.1 Partial meet operation over canonical form

In [5], Kourousias and Makinson first put the given belief set K into a canonical form $K^{'}$. Then they constructed partial meet contraction (or revision) over the canonical form $K^{'}$. However, even K is closed under classical consequence, K' will be a belief base and it is not closed under classical consequence usually. In the paper, we didn't first normalize belief set but rebuilt $K \perp x$ by replacing it with $K \perp^{'} x$ and $K \perp^* x$ respectively. Then we rebuilt partial meet contraction by select function over $K \perp^{'} x$ and $K \perp^* x$. The first case is equal to the partial meet contraction over canonical form $K^{'}$ on every piece belief $K_i = K \cap L(E_i)$ where $\{E_i\}_{i \in I}$ is the finest splitting of belief set K. $K \div^{'} x$ is a special kind of $K \div^* x$ since $K \div^{'} x \subseteq K \div^* x$. The following lemma shows that the partial meet contraction over normalized belief set satisfies relevance criterion.

Lemma 4.1([5]). Let K be any consistent belief set, $\mathbf{E} = \{E_i\}_{i \in I}$ the finest splitting of K, with $\{B_i\}_{i \in I}$ a family such that $K^{'} = \bigcup\{B_i\}_{i \in I} \dashv\vdash K$ and $E(B_i) \subseteq E_i$ for each $i \in I$, x a formula, and $K^{'} - x$ a partial meet contraction of x from $K^{'}$. Then $\alpha \in K^{'} - x$ if $\alpha \in K^{'}$ is irrelevant to x (modulo $K^{'}$).

Theorem 4.2. For any partial meet contraction operator $\dot{-}$ over $K^{'} = \bigcup\{K \cap L(E_i)\}_{i \in I}$ where K is a belief set, there exists a partial meet operator $\div^{'}$ over K based on the finest splitting of K such that $(K^{'} \dot{-} x) \cap L(E_i) = (K \div^{'} x) \cap L(E_i)$ for each $i \in I$.

Proof. Suppose $K^{'} \dot{-} x = \bigcap \gamma(K^{'} \perp x)$. Let $\mathbf{B} = \{A \in K \perp^{'} x | \bigcap\{A \cap L(E_i)\}_{i \in I} \in \gamma(K^{'} \perp x)\}$ and $K \div^{'} x = \bigcap \mathbf{B}$. For any $i \in I$ and any $\varphi \in (K^{'} \dot{-} x) \cap L(E_i)$, $\varphi \in A^{'}$ for all $A^{'} \in \gamma(K^{'} \perp x)$. It follows that $\varphi \in A \in \mathbf{B}$. So $\varphi \in K \div^{'} x$ and then $\varphi \in K \div^{'} x \cap L(E_i)$.

For the converse, note that for any $i \in I$ and any $\varphi \in (K \div^{'} x) \cap L(E_i)$, $\varphi \in A$ for all $A \in \mathbf{B}$. By definition of $\mathbf{B}$ and $\varphi \in L(E_i)$, we have $\varphi \in \gamma(K^{'} \perp x)$. Hence $\varphi \in (K^{'} \dot{-} x) \cap L(E_i)$.

The following theorem′s proof is similar to that of the above theorem.

Theorem 4.3. For any partial meet contraction operator $\div^{'}$ based on the finest splitting of belief set K, there exists a partial meet operator $\dot{-}$ over $K^{'} = \bigcup\{K \cap L(E_i)\}_{i \in I}$ such that $K^{'} \dot{-} x \cap L(E_i) = K \div^{'} x \cap L(E_i)$ for each $i \in I$.

The two theorems show that our operator is equal to the operator formulated by Kourousias and Makinson over local belief K_i for all $i \in I$.

4.2 Local change

Note that our local contraction or revision depends on splitting of K. Hansson and Wassermann, alternatively, considered local change depending on the following local implication [8]. Recall that, given a belief set K and a formula α, a subset X of K is in $K \perp\!\!\!\perp_C \alpha$ if X is a minimal subset of K such that $X \vdash \alpha$.

Definition 4.4([8]). Let C be the inference operation. The function c is the compartmentalization function based on C if and only if, for all $A, B \subseteq L$: $c(A, B) = \bigcup_{(\alpha \in A)} c(\alpha, B)$, where

- $c(\alpha, B) = \emptyset$ in the limit case $\alpha \in Cn(\emptyset)$ or $\neg\alpha \in Cn(\emptyset)$;
- $c(\alpha, B) = \bigcup((B \perp\!\!\!\perp_C \alpha) \cup (B \perp\!\!\!\perp_C \neg\alpha) \backslash B \perp\!\!\!\perp_C \perp)$ otherwise.

Let C be the classical consequence operation, $\mathbf{E} = \{E_i\}_{i \in I}$ the finest splitting of belief set K, $K_i = K \cap L(E_i)$ for all $i \in I$ and $K^{'} = \bigcup\{K_i\}_{i \in I}$. Then $c(x, K^{'}) \bigcap\{K_j\}_{j \in J} = \emptyset$ for all $x \in K$, where $\bigcup\{E_j\}_{j \in J}$

is the subfamily of cells in $\mathbf{E}$ that share some elementary letter with $E(\overline{x})$. But $c(x, K) \bigcap \{K_j\}_{j \in J} \neq \emptyset$ for all $x \in K$.

We secondly recall compartmentalization conception based on inference operation C. In spite of B is a belief set, the function $C(c(A, B))$ exhibits some expression. But it demonstrates clearly that B is a belief base. We recall the following definitions and theorems which depend on differ inference operation(include local inference operation).

Definition 4.5([8]). Let C be the inference operation on L and let c be the compartmentalization function derived from the classical consequence operation C. Then for any set A, the A-localization of C is the inference operation C_A such that for all sets B of formulae: $C_A(B) = C(c(A, B))$.

Theorem 4.6([8]).

If $\div$ is a partial meet contraction operation with respect to an operator C, then it is a kernel contraction operator with respect to the operator C.

It does not hold in general that if $\div$ is a kernel contraction operator with respect to an operator C, then it is a partial meet contraction operator with respect to an operator C.

Since our partial meet contraction based on splitting which satisfies the relevance criterion is a special kind of partial contraction. it is also a kernel contraction operator respect to the classical consequence operation by theorem 16 in [7]. The converse does not holds generally.

5 Conclusions and Future Work

In the paper, we select the class of subsets $K \perp^{'} x$ (resp. $K \perp^{*} x$) of $K \perp x$ such that $K \perp x \neq \emptyset$ iff $K \perp^{'} x \neq \emptyset$ (resp. $K \perp x \neq \emptyset$ iff $K \perp^{*} x \neq \emptyset$). Then we construct the partial meet contraction operator $\div^{'}$(resp. $\div^{*}$) by replacing $K \perp x$ with $K \perp^{'} x$(resp. $K \perp^{*} x$). The operators $\div^{'}$ and $\div^{*}$ satisfy the AGM postulates and the relevance criterion formulated by Parikh. The converse holds too for $\div^{*}$. So that a representation theorem, according to AGM, is obtained. We will discuss some representation theorems for maxichoice contraction and full meet contraction in future.

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