

Design of Total Sliding-Mode Control for Chua's Chaotic Circuit

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Abstract—This study mainly focuses on the development of a total sliding-mode control (TSMC) strategy for a Chua's chaotic circuit. The TSMC scheme, which is insensitive to uncertainties including parameter variations and external disturbance in the whole control process, comprises the baseline model design and the curbing controller design. In the baseline model design a computed torque controller is designed to cancel the nonlinearity of the nominal plant. In the curbing controller design an additional controller is designed using a new sliding surface to ensure the sliding motion through the entire state trajectory. Therefore, in the TSMC system the controlled system has a total sliding motion without a reaching phase. The effectiveness of the proposed TSMC scheme is verified by numerical simulations, and the advantages of good transient response and robustness to uncertainties are indicated in comparison with a conventional sliding-mode control (CSMC) system.

Index Terms—Total sliding-mode control, Baseline model design, Curbing controller design, Chua's circuit, Chaotic effect.

I. INTRODUCTION

Nowadays, electronic systems have been more complex than ever before, and new chaotic phenomena have been detected in the engineering and natural systems. These situations are generally produced from the analyses of nonlinear dynamical systems by ignoring regular, orderly, and long term predictable responses to simple inputs. As a characteristic of nonlinear system, chaos is a bounded unstable dynamic behavior that exhibits sensitive dependence on initial conditions and includes infinite unstable periodic motion. Thus, the chaotic is usually regarded as a nuisance and is designed out if possible in the past years [1]. Control of chaotic systems has been investigated intensively in various fields during recent years [2]–[5]. In general, the chaotic system can be performed by many forms like Lorenz [2], Rössler [3], Chua's circuit [4], [5], and so on. In this study, a chaotic system formed by Chua's circuit [4], [5], which is an extremely simple autonomous electrical circuit with a two double-scroll attractor, is investigated.

Variable structure control (VSC) with sliding mode, or sliding-mode control (SMC), is one of the effective nonlinear robust control approaches since it provides system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [6]–[12]. The first step of SMC design is to select a sliding surface that models the desired

closed-loop performance in state variable space. Then design the control such that the system state trajectories are forced toward the sliding surface and stay on it. The system state trajectory in the period of time before reaching the sliding surface is called the reaching phase. Once the system trajectory reaches the sliding surface, it stays on it and slides along it to the origin. The system trajectory sliding along the sliding surface to the origin is the sliding mode. The insensitivity of the controlled system to uncertainties exists in the sliding mode, but not during the reaching phase. Thus the system dynamic in the reaching phase is still influenced by uncertainties. To design the reaching phase, Gao and Hung [9] partially shaped the reaching law to specify the system dynamics in the reaching phase. However, the system dynamics are still subjected to uncertainties. Therefore, this study adopts the idea of total sliding-mode control (TSMC) [10] to get a sliding motion through the entire state trajectory. In other words, no reaching phase exists in the control process. Thus the controlled system through the whole control process is not influenced by uncertainties. Compared with the previous VSC controlled system, this study has two distinguishing features. First, the sliding surface is with an additional integral term. It is emphasized that the proposed use of an integral term is significantly different than integral terms used by other researchers. In the past researches [6], [7], integral terms were used when a boundary layer was introduced around the sliding surface, in an effort to eliminate steady-state error that results from a continuous approximation of switching control. In this study, no boundary layer is being considered. The special integral term is designed to eliminate the reaching phase, not to reduce steady-state error due to continuous control. Second, the equivalent controlled dynamics in the sliding mode is a second-order dynamics. This feature makes it easy to assign the system performance based on overshoot, rise time and settling time specifications.

This study is organized into five sections. Following the introduction, the Chua's chaotic circuit and its dynamic model are briefly described in Section II. Moreover, the detailed design procedure of the TSMC system are explained in Section III. In Section IV, comparative simulations with a conventional sliding-mode control (CSMC) for a Chua's chaotic circuit are performed to demonstrate the effectiveness and superiority of the proposed TSMC scheme. Finally, some conclusions are drawn in Section V.

II. SYSTEM DESCRIPTIONS

In this study, a Chua's circuit [4], [5] with a double-scroll chaotic phenomenon as shown in Fig. 1 is studied. In Fig. 1, L is an inductor, and its corresponding

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current is represented as i_L ; C_1 and C_2 are capacitors and their across voltages are repressed as v_{c1} and v_{c2} ; R is a resistor; u_i is a voltage source cascade with the inductor L ; N_D represents a nonlinear Chua's diode with a three-segment piecewise-linear characteristic of the capacitor voltage (v_{c1}) and the diode current (i_N) defined by

$$i_N = f(v_{c1}) \tag{1}$$

$$\hat{=} m_0 v_{c1} + \frac{1}{2}(m_1 - m_0)[|v_{c1} + b_p| - |v_{c1} - b_p|]$$

where $|\cdot|$ is the operator of absolute value; b_p is a predetermined boundary; m_0 and m_1 are the slopes in the inside and outside regions of the boundary as shown in Fig. 2. According to Kirchhoff's voltage and current laws, the state equations of the Chua's circuit can be represented as

$$C_1 \frac{dv_{c1}}{dt} = \frac{1}{R}(v_{c2} - v_{c1}) - f(v_{c1}) \tag{2a}$$

$$C_2 \frac{dv_{c2}}{dt} = \frac{1}{R}(v_{c1} - v_{c2}) + i_L \tag{2b}$$

$$L \frac{di_L}{dt} = u_i - v_{c2} \tag{2c}$$

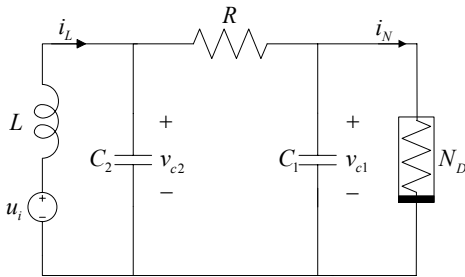


Fig. 1. Chua's chaotic circuit.

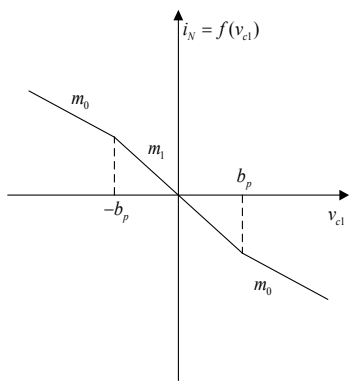


Fig. 2. Relation of capacitor voltage and diode current in non-linear resistor.

By introducing new variables $x_1 = v_{c1}$, $x_2 = v_{c2}$, and $x_3 = i_L$, then the system dynamics in (2) can be simplified as

$$\dot{x}_1 = a(x_2 - x_1) - bf(x_1) \tag{3a}$$

$$\dot{x}_2 = p(x_1 - x_2) + qx_3 \tag{3b}$$

$$\dot{x}_3 = r(u_i - x_2) \tag{3c}$$

where $a = 1/(RC_1)$, $b = 1/C_1$, $p = 1/(RC_2)$, $q = 1/C_2$, and $r = 1/L$. By considering the external disturbance and separating the normal parameters from the system dynamic, (3) can be expressed as

$$\dot{x}_1 = (a_n + \Delta a)[x_2 - x_1] - (b_n + \Delta b)f(x_1) \tag{4a}$$

$$\hat{=} a_n(x_2 - x_1) - b_n f(x_1) + d_{x1}$$

$$\dot{x}_2 = (p_n + \Delta p)(x_1 - x_2) + (q_n + \Delta q)x_3 \tag{4b}$$

$$\hat{=} p_n(x_1 - x_2) + q_n x_3 + d_{x2}$$

$$\dot{x}_3 = (r_n + \Delta r)(u_i - x_2) + d_u \tag{4c}$$

$$\hat{=} u - r_n x_2 + d_{x3}$$

where a_n , b_n , p_n , q_n and r_n are the normal values of $a, b, p, q,$ and r , respectively, and $\Delta a, \Delta b, \Delta p, \Delta q,$ and Δr are their corresponding parameter variations; d_u is an unpredictable external disturbance; $d_{x1} = \Delta a(x_2 - x_1) - \Delta b f(x_1)$, $d_{x2} = \Delta p(x_1 - x_2) + \Delta q x_3$ and $d_{x3} = \Delta r(u_i - x_2) + d_u$ denote respective lumped uncertainties in (4); $u = r_n u_i$ is a new control input. The control problem is to design a suitable control law to force the system states (x_1, x_2, x_3) to track specific reference commands (x_{1r}, x_{2r}, x_{3r}) for eliminating chaotic phenomena in the Chua's circuit.

III. TOTAL SLIDING-MODE CONTROL

In order to control the Chua's circuit effectively, a total sliding-mode control (TSMC) scheme as shown in Fig. 3 is introduced in this section. Define respective tracking errors as $e_{x1} = x_1 - x_{1r}$, $e_{x2} = x_2 - x_{2r}$, and $e_{x3} = x_3 - x_{3r}$, in which x_{1r} is the reference command voltage for the capacitor C_1 (v_{c1ref}); $x_{2r}(v_{c2ref})$ and $x_{3r}(i_{Lref})$ are the corresponding equilibrium points of x_2 and x_3 when $x_1 = x_{1r}$. By only considering the nominal system dynamics (i.e., $d_{x1} = d_{x2} = d_{x3} = 0$), x_{2r} and x_{3r} can be represented via (4a) and (4b) as

$$x_{2r} \hat{=} \frac{\dot{x}_{1r} + b_n f(x_{1r})}{a_n} + x_{1r} \tag{5a}$$

$$x_{3r} \hat{=} \frac{\dot{x}_{2r}}{q_n} + \frac{p_n [\dot{x}_{1r} + b_n f(x_{1r})]}{a_n q_n} \tag{5b}$$

From (4), one can organize the system dynamics in the following vector form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_n & a_n & 0 \\ p_n & -p_n & q_n \\ 0 & -r_n & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} d_{x1} - b_n f(x_1) \\ d_{x2} \\ d_{x3} \end{bmatrix} \tag{6a}$$

or

$$\dot{x} = Ax + hu + w \tag{6b}$$

where $x = [x_1 \ x_2 \ x_3]^T$, $A = \begin{bmatrix} -a_n & a_n & 0 \\ p_n & -p_n & q_n \\ 0 & -r_n & 0 \end{bmatrix}$, and

$h = [0 \ 0 \ 1]^T$, in which w is called the lumped uncertainty

vector and is defined as

$$\mathbf{w} = [d_{x1} - b_n f(x_1) \quad d_{x2} \quad d_{x3}]^T \quad (7)$$

The TSMC presentation for the Chua's circuit is divided into two main parts. The first part addresses performance design. The object is to specify the desired performance in terms of the nominal model, and it is referred to as baseline

model design. Following the baseline model design, the second part is the curbing controller design to totally eliminate the unpredictable perturbation effect from the parameter variations and external load disturbance so that the baseline model designs performance can be exactly assured.

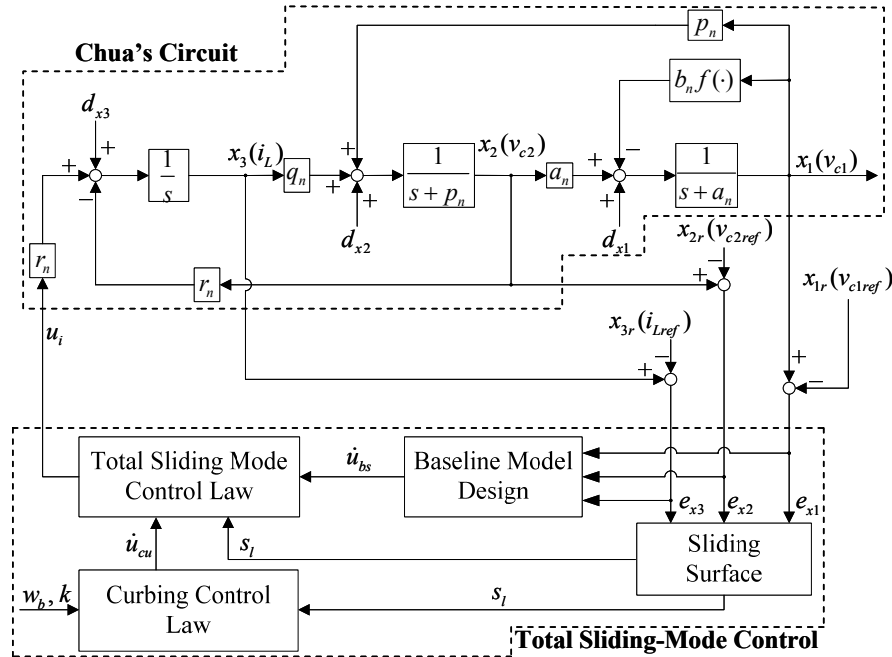


Fig. 3. Block diagram of TSMC system.

A. Baseline Model Design

By differentiating (6b) with respect to time and considering the absence of the lumped uncertainty vector (i.e., $\mathbf{w} = \mathbf{0}$), one can rewrite the system dynamic via a second-order vector form and devise in the control effort \dot{u}_{bs} in the baseline model design as follows:

$$\ddot{\mathbf{x}} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{h}u \quad (8)$$

$$\dot{u} = \dot{u}_{bs} \triangleq \mathbf{h}^+ [-\mathbf{A}\dot{\mathbf{x}} + \ddot{\mathbf{x}}_r - \mathbf{K}_v \dot{e} - \mathbf{K}_p e] \quad (9)$$

where $\mathbf{e} = \mathbf{x} - \mathbf{x}_r \triangleq [e_{x1} \quad e_{x2} \quad e_{x3}]^T$ is a tracking error vector, in which $\mathbf{x}_r = [x_{1r} \quad x_{2r} \quad x_{3r}]^T$ is a reference command vector; $\mathbf{K}_v \in R^{3 \times 3}$ and $\mathbf{K}_p \in R^{3 \times 3}$ are positive gain matrices; $\mathbf{h}^+ \in R^{1 \times 3}$ is the left penrose pseudo inverse of \mathbf{h} , i.e., $\mathbf{h}^+ = (\mathbf{h}^T \mathbf{h})^{-1} \mathbf{h}^T$. Substitute (9) into (8), one can obtain

$$\ddot{\mathbf{e}} + \mathbf{K}_v \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} = \mathbf{0} \quad (10)$$

Properly choosing the control gains in \mathbf{K}_v and \mathbf{K}_p , the desired system dynamics such as rise time, overshoot, and settling time can be easily designed by this second-order vector. However, if the uncertainties occur, i.e., the parameters of the system are deviated from the nominal value or an external load disturbance is added into the system, the baseline model design can not guarantee the performance specified by (10). Moreover, the stability of the controlled system may be destroyed. To ensure the system performance designed by (10) despite the existence of the uncertain system dynamics, an auxiliary control

design is necessary.

B. Curbing Control Design

The baseline model dynamic shown in (10) can be rewritten in the state variable form as

$$\dot{\mathbf{e}}_{cu} = \mathbf{A}_{cu} \mathbf{e}_{cu} \quad (11)$$

where $\mathbf{e}_{cu} = [\mathbf{e} \quad \dot{\mathbf{e}}]^T \in R^{6 \times 1}$ and $\mathbf{A}_{cu} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ -\mathbf{K}_p & -\mathbf{K}_v \end{bmatrix} \in R^{6 \times 6}$,

in which $\mathbf{I}_{3 \times 3}$ is a 3×3 identity matrix. Now, consider a sliding surface as follows [10]:

$$s_i(t) = c_{ub}(\mathbf{e}_{cu}) - c_{ub}(\mathbf{e}_{cu0}) - \int_0^t \frac{\partial c_{ub}}{\partial \mathbf{e}_{cu}^T} \mathbf{A}_{cu} \mathbf{e}_{cu} dt \quad (12)$$

where $c_{ub}(\mathbf{e}_{cu})$ is a scalar variable designed as $\frac{\partial c_{ub}}{\partial \mathbf{e}_{cu}^T} = [\mathbf{0}_{1 \times 3} \quad \mathbf{h}^+] \in R^{1 \times 6}$, and \mathbf{e}_{cu0} is the initial state of \mathbf{e}_{cu} . It is obvious that $s_i(0) = 0$ and

$$\dot{s}_i(t) = \frac{\partial c_{ub}}{\partial \mathbf{e}_{cu}^T} \dot{\mathbf{e}}_{cu} - \frac{\partial c_{ub}}{\partial \mathbf{e}_{cu}^T} \mathbf{A}_{cu} \mathbf{e}_{cu} = 0 \quad (13)$$

Thus, $s_i(t) = 0$ for all $t \geq 0$. Note that, since the function $s_i(t) = 0$ when $t = 0$, there is no reaching phase as in the traditional sliding-mode control. If the Chua's circuit subject to unknown parameter variations and external load disturbance is considered, the lumped uncertainly vector \mathbf{w} should be added in the system dynamics. Therefore, the system dynamic can be expressed as

$$\mathbf{h}^+ (\ddot{\mathbf{x}} - \mathbf{A}\dot{\mathbf{x}}) = \dot{u} + \mathbf{h}^+ \mathbf{w} \quad (14)$$

It is apparent that the control shown in (9) can not ensure that (14) satisfies the baseline model design and $s_i(t) = 0$ for $t > 0$. Thus, it is necessary to design an additional control such that the closed-loop dynamics of the controlled system is the same as the performance in the baseline model design. This is achieved by a control of the following form:

$$\dot{u} = \dot{u}_{bs} + \dot{u}_{cu} \quad (15)$$

where \dot{u}_{bs} is the same as given by (9), and \dot{u}_{cu} is given as

$$\dot{u}_{cu} = -w_b \text{sgn}[s_i(t)] - ks_i(t) \quad (16)$$

where $\text{sgn}[\cdot]$ is a sign function; w_b and k are positive constants. The objectives of this controller \dot{u}_{cu} are twofold. The first is to keep the controlled system dynamics on the surface $s_i(t) = 0$ for all time. Thus, \dot{u}_{cu} is called a curbing controller. Accordingly, the second objective is to guarantee that the closed-loop perturbed system has with the same performance shown in (10) as the baseline model design.

Substitute (9), (15) and (16) into (6b), the state variable form in (11) can be rewritten as follows:

$$\dot{e}_{cu} = \mathbf{A}_{cu} e_{cu} + \mathbf{h}_m [\dot{u}_{cu} + \mathbf{h}^+ \dot{w}] \quad (17)$$

where $\mathbf{h}_m = [\mathbf{0}_{3 \times 1} \quad \mathbf{h}^T] \in R^{6 \times 1}$. Now, $s_i(t) = 0$ when $t = 0$. To maintain the state on the surface $s_i(t) = 0$ for all time, on only needs to show that

$$s_i(t) \dot{s}_i(t) < 0 \quad \text{if } s_i(t) \neq 0 \quad (18)$$

By differentiating $s_i(t)$ in (12) with respect to time and using the error dynamic in (17), it yields

$$\begin{aligned} \dot{s}_i(t) &= \frac{\partial C_{ub}}{\partial e_{cu}^T} \dot{e}_{cu} - \frac{\partial C_{ub}}{\partial e_{cu}^T} \mathbf{A}_{cu} e_{cu} \\ &= \frac{\partial C_{ub}}{\partial e_{cu}^T} \{ \mathbf{A}_{cu} e_{cu} + \mathbf{h}_m [\dot{u}_{cu} + \mathbf{h}^+ \dot{w}] - \mathbf{A}_{cu} e_{cu} \} \\ &= \dot{u}_{cu} + \mathbf{h}^+ \dot{w} \end{aligned} \quad (19)$$

Multiplying $s_i(t)$ by (19) and inserting (7) into (19), one can obtain

$$\begin{aligned} s_i(t) \dot{s}_i(t) &= s_i(t) \dot{u}_{cu} + s_i(t) \mathbf{h}^+ \dot{w} \\ &\leq s_i(t) \dot{u}_{cu} + |s_i(t)| \left| \dot{d}_{x3} \right| \\ &= -ks_i^2(t) - |s_i(t)| (w_b - \left| \dot{d}_{x3} \right|) \end{aligned} \quad (20)$$

If the condition of $w_b > \left| \dot{d}_{x3} \right|$ holds, (20) can be rewritten as

$$s_i(t) \dot{s}_i(t) \leq -ks_i^2(t) < 0 \quad (21)$$

Thus, the sliding mode can be assured throughout the whole control period. Note that, the value of w_b can be roughly determined to be a small positive constant for reducing the chattering phenomena in (16) because the term of $-ks_i(t)$ via a large value of k is helpful for ensuring $s_i(t) \dot{s}_i(t) < 0$ in (20). The effectiveness of the proposed TSMC scheme is verified by the following numerical simulations.

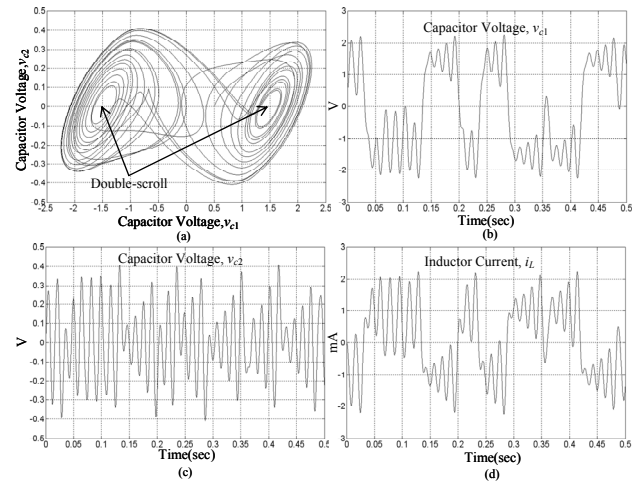


Fig. 4. Numerical simulations of open-loop examination for Chua's circuit: (a) Phase plane trajectory, $v_{c1} - v_{c2}$; (b) Capacitor voltage, v_{c1} ; (c) Capacitor voltage, v_{c2} ; (d) Inductor current, i_L .

IV. NUMERICAL SIMULATIONS

In order to exhibit the merits of the total sliding-mode control (TSMC) system, a conventional sliding-mode control (CSMC) scheme introduced from [12] is also examined in this study. In this CSMC strategy, a conventional sliding-surface vector is defined as

$$s_c(t) = e + \int_0^t \mathbf{G} e dt \quad (22)$$

where $\mathbf{G} \in R^{3 \times 3}$ is a positive-definite gain matrix. According to the standard derivation procedure [6], [7], [12], the CSMC law can be represented as

$$u_{CSMC} = \mathbf{h}^+ \{ \dot{x}_r - \mathbf{A}x - \mathbf{G}e - w_c \text{sgn}[s_c(t)] \} \quad (23)$$

where w_c is the upper bound of the lumped uncertainty vector, i.e., $\|w\| < w_c$, in which $\|\cdot\|$ is the Euclidean norm.

All numerical simulations are carried out using Windows packaged Matlab 6.5 edition software. The circuit specifications of a Chua's circuit are summarized as follows:

Sampling time	$T = 25 \mu s$;
Control time	$T_r = 50 \mu s$;
Inductor	$L = 0.707 H$;
Capacitor	$C_1 = 0.55 \mu F$;
	$C_2 = 4.95 \mu F$;
Resistor	$R = 1.428 k\Omega$;
Chua's diode	$m_0 = -0.5 \text{ mA/V}$;
	$m_1 = -0.8 \text{ mA/V}$;
	$b_p = 1V$

Moreover, the initial condition for the capacitor voltage is set at $v_{c1}(0) = 0.5V$. In addition, the control parameters of CSMC and TSMC systems are given as

$$\mathbf{K}_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 240 & 30000 \end{bmatrix}, \quad \mathbf{K}_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10^6 & 5 \times 10^7 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 20 & 2000 \end{bmatrix}, \quad w_b = 0.01, \quad k = 100, \quad (24)$$

$$w_c = 8, \quad x_{1r} = 2$$

All the parameters in (24) are chosen to achieve superior transient control performance in numerical simulations by considering the possible occurrence of operational conditions and the limitation of control efforts.

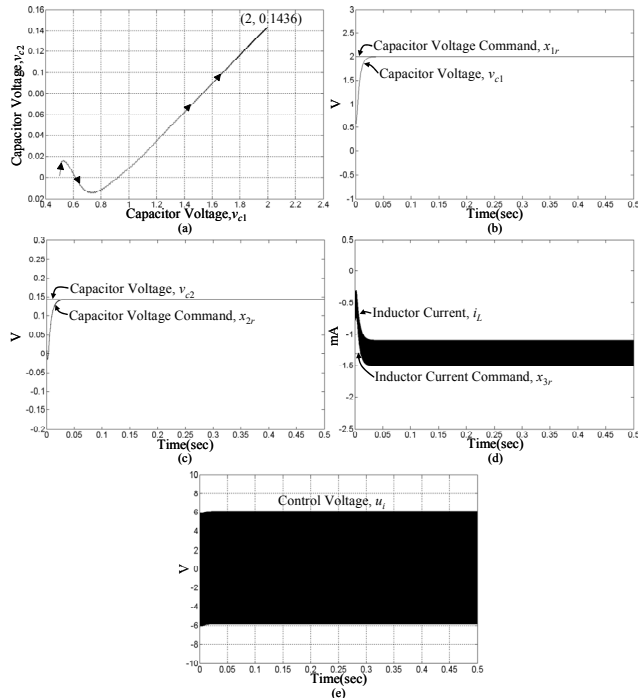


Fig. 5. Numerical simulations of CSMC scheme for Chua's circuit at case 1: (a) Phase plane trajectory, $v_{c1} - v_{c2}$; (b) Tracking response of capacitor voltage v_{c1} ; (c) Tracking response of capacitor voltage v_{c2} ; (d) Tracking response of inductor current i_L ; (e) Control voltage, u_i .

In order to investigate the robust characteristics of the proposed TSMC system, two examined cases are considered: Case 1 is the normal condition (i.e., $d_{x1} = d_{x2} = d_{x3} = 0$); Case 2 is the disturbance condition (i.e., $d_{x1} = 0.2 \cos(201\pi t) + 0.5 \cos(53\pi t)$, $d_{x2} = 0.2 \cos(201\pi t) + 0.5 \cos(53\pi t)$, and $d_{x3} = 2 \cos(201\pi t) + 5 \cos(53\pi t)$ occurred at beginning). Numerical simulations of the open-loop examination for the Chua's circuit are depicted in Fig. 4. From Fig. 4(a), one can see that the phase plane trajectory $v_{c1} - v_{c2}$ describe a double-scroll Chua's attractor. The corresponding responses of the capacitor voltages (v_{c1} and v_{c2}) and inductor current (i_L) are depicted in Fig. 4(b)–(d), respectively. Comparative simulations of the CSMC scheme at cases 1 and 2 are depicted in Figs. 5 and 6, respectively. At case 1, one can see that the phase trajectory $v_{c1} - v_{c2}$ as shown in Fig. 5(a) can be controlled to its command (2, 0.1436), and the responses of the capacitor

voltage (v_{c1} and v_{c2}) as shown in Fig. 5(b) and (c) can arrive at their command voltages (x_{1r} and x_{2r}), respectively. However, the chattering control voltage in Fig. 5(e) caused by the sign function in (23) results in the shaking response of the inductor current in Fig. 5(d). At case 2, the phase trajectory $v_{c1} - v_{c2}$ as shown in Fig. 6(a) also can be controlled to its command (2, 0.1436), but the trajectory in comparison with the one in Fig. 5(a) is obviously sensitive to external disturbances. Besides, the responses of the capacitor voltages and inductor current as shown in 6(b)–(d) become slow, and their corresponding tracking responses degenerate due to the occurrence of external disturbances.

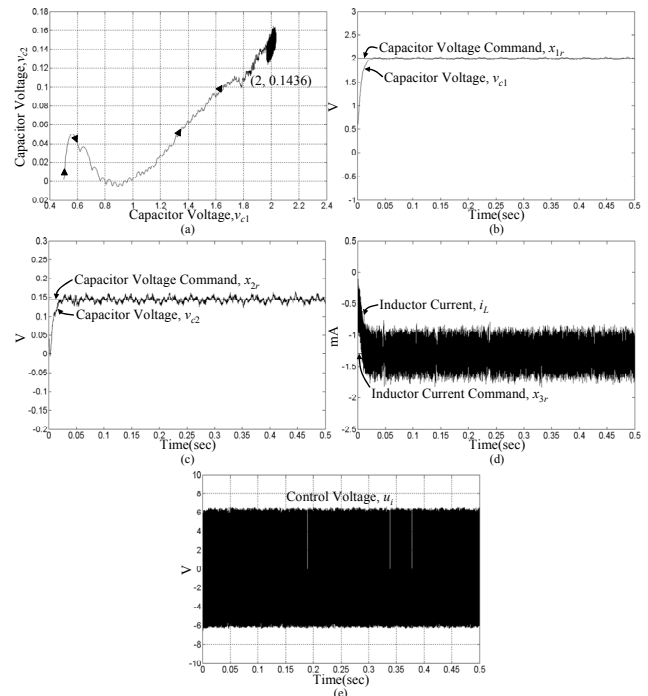


Fig. 6. Numerical simulations of CSMC scheme for Chua's circuit at case 2: (a) Phase plane trajectory, $v_{c1} - v_{c2}$; (b) Tracking response of capacitor voltage v_{c1} ; (c) Tracking response of capacitor voltage v_{c2} ; (d) Tracking response of inductor current i_L ; (e) Control voltage, u_i .

Although a large upper bound of the lumped uncertainty vector can solve the problem of slow or degenerate tracking responses, more serious chattering control voltage is inevitable. This situation should be prevented as far as possible in practical applications. In addition, numerical simulations of the TSMC scheme at cases 1 and 2 are given in Figs. 7 and 8, respectively. The simulated results show not only that perfect tracking responses as shown in Figs. 7(b)–(d) and 8(b)–(d) can be achieved, but also that related control voltages as shown in Figs. 7(e) and 8(e) can be obtained without chattering. From Figs. 7(a) and 8(a), one can see that the phase trajectories are less sensitive to external disturbances according to the property of total sliding-mode control. By comparing Figs. 7–8 and Figs. 5–6, the proposed TSMC scheme indeed yields superior control performance than the conventional CSMC system.

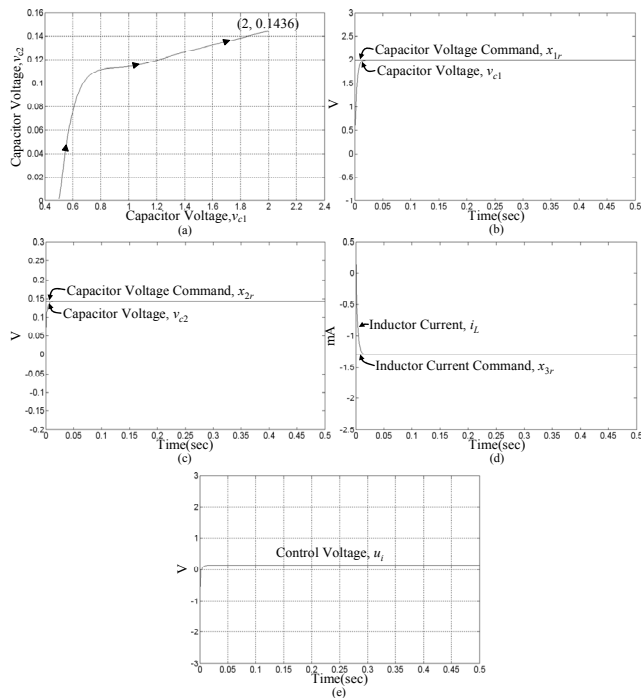


Fig. 7. Numerical simulations of TSMC strategy for Chua's circuit at case 1: (a) Phase plane trajectory, $v_{c1} - v_{c2}$; (b) Tracking response of capacitor voltage v_{c1} ; (c) Tracking response of capacitor voltage v_{c2} ; (d) Tracking response of inductor current i_L ; (e) Control voltage, u_i .

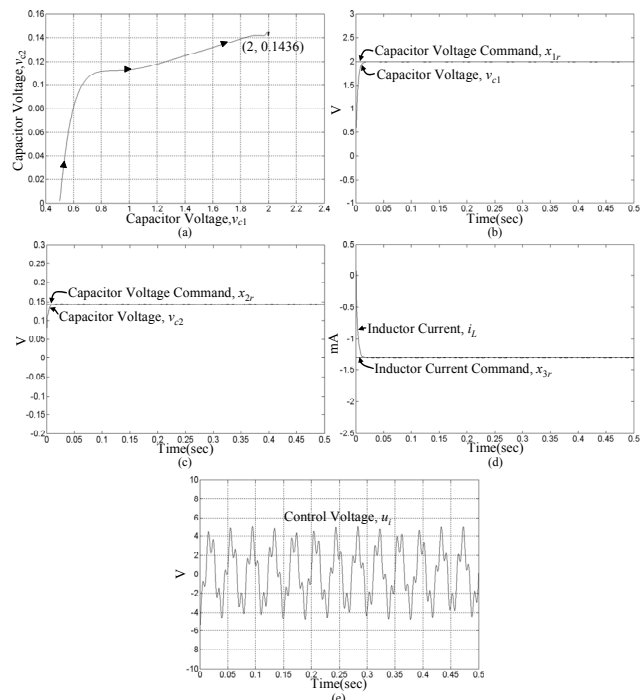


Fig. 8. Numerical simulations of TSMC strategy for Chua's circuit at case 2: (a) Phase plane trajectory, $v_{c1} - v_{c2}$; (b) Tracking response of capacitor voltage v_{c1} ; (c) Tracking response of capacitor voltage v_{c2} ; (d) Tracking response of inductor current i_L ; (e) Control voltage, u_i .

V. CONCLUSIONS

This study has successfully developed a total sliding-mode control (TSMC) system to handle a Chua's chaotic circuit. Numerical simulations are presented to illustrate the effectiveness of the proposed TSMC scheme, and its superiority is indicated in comparison with a conventional sliding-mode control (CSMC) system. According to the simulated results, it is obvious that the chattering control efforts in the CSMC can be greatly alleviated, and the TSMC has a total sliding motion without a reaching phase so that the control performances are less sensitive to system uncertainties. Consequently, the proposed TSMC scheme is more suitable for the state control of the Chua's chaotic circuit than the CSMC system.

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