Unknown Input High Gain Observer for Parametric Fault Detection and Isolation of Dynamical Systems

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Abstract: An unknown input high gain observer based parametric fault detection and isolation scheme is presented. First, a reduced order unknown input high gain observer is derived. Then, using these observers, a fault detection and isolation technique is devised to detect and isolate the parametric fault of a system whose parameters are uncertain to some extent. The proposed FDI algorithm consists of two steps. In the first step, the detection of fault and the isolation of faulty region are achieved and in the next step, the faulty parameter is isolated from the faulty region. Effectiveness of the observer as well as the FDI technique is shown with the help of a numerical example.

Keywords: Unknown input high gain observer; parametric fault; fault detection and isolation; parameter estimation.

I. INTRODUCTION

With the rising demands of high reliability and safety of advanced processes like avionics, nuclear power stations, automobiles etc have led to increasing requirements of developing new methods for supervision and monitoring as a part of overall process control scheme. Different fault detection and isolation (FDI) schemes have been developed for avoiding failure of the plants. Model based fault detection techniques (like Kalman filter or observer based) have received increasing attention following the pioneering work of Beard [1].

The FDI concept using observers or Kalman filters is based on the assumption that the mathematical model of the system is perfectly known. In reality, however this assumption does not hold, because the

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parameters of a process are in general uncertain or time varying. Again the characteristics of disturbances or noise are not completely known; hence they cannot be perfectly modeled. There is always a mismatch between the actual process model and its mathematical model (even if there is no fault in the process), which sometimes produces false alarms corrupting the performance of the FDI process. To avoid false alarms, the FDI method should be made robust i.e., insensitive to modeling uncertainties. But the algorithm should not be too robust to ignore the fault i.e., a significantly large variation of the parameter values.

Over the years, various kinds of robust fault detection and isolation techniques have been developed to diagnose different types of faults like sensors, actuators or components [2, 5, 7, 8, 12, 13]. Frank [6], in a survey paper, described different types of observer based robust fault diagnosis techniques. Patton and Chen [11] discussed various robustness issues related observer based fault diagnosis techniques. Linear matrix inequality (LMI) based robust fault detection techniques for uncertain systems have been developed in [14]. The identification based FDI techniques have been used by many researchers [9] for different types of fault diagnosis. Daley and Wang [3] used a high gain observer, which was developed by Petersen and Hollot [10], as a tool for sensor fault detection.

In the present work, an unknown input high gain observer (UIHGO) based parametric (i.e., component) fault detection and isolation technique is presented. First, an unknown input high gain observer is developed for an uncertain system. Such type of unknown input observers has wide applications in control systems where the uncertainties (modeling or parametric or both) are unavoidable. Next, these observers are used in developing a fault diagnosis technique for a parametrically uncertain system on the assumptions that sensors and actuators are fault free. Since the high gain observer [3, 10] is robust against parameter uncertainties to some extent, the FDI technique is also robust against the uncertainties. The FDI process works in two steps. In step-1, the detection of fault and isolation of faulty zone is accomplished. In the next step, faulty parameter is isolated by parameter isolation method. In the present work, a part of the system parameters (i.e., the parameters of the faulty subsystem) is estimated and only when a fault occurs in the system. In this respect the complexity of fault isolation is drastically reduced in comparison with standard parameter identification technique [9] where all the parameters of a system are estimated at every instant and compared with actual values. A numerical example is presented to show the effectiveness of the observer as well as the FDI technique.

The basic methodology of designing the unknown input high gain observer for an uncertain system is discussed in section II. The fault detection and isolation algorithm is explained in section III. A numerical example is presented in section IV. The concluding remarks are included in section V.

II. UNKNOWN INPUT HIGH GAIN OBSERVER

In this section, an unknown input high gain observer is developed. The sufficient conditions for existence of such observers are provided.

Consider a linear time-invariant uncertain system with unknown inputs

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta \boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta \boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t)$$
(1)

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \,. \tag{2}$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ - the state vector, $\mathbf{u}(t) \in \mathbf{R}^m$ - the measurable input vector, $\mathbf{y}(t) \in \mathbf{R}^p$ - the output vector and $\mathbf{d}(t) \in \mathbf{R}^q$ - the unknown input vector. The matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and \mathbf{E} of suitable dimensions are known. The matrices $\Delta \mathbf{A}$ and $\Delta \mathbf{B}$ are the uncertainties of the system and input matrices. It is assumed that $(\mathbf{A} + \Delta \mathbf{A})$ is always asymptotically stable for all $\Delta \mathbf{A}$.

It is assumed that the system satisfies the rank condition: rank(CE) = rank(E).

Now, using a state transformation matrix T, the states are redefined as z(t) = Tx(t) such that

 $TE = \begin{bmatrix} \phi_{n-r \times q} \\ \overline{E}_2 \end{bmatrix}$ where \overline{E}_2 is $r \times q$ dimensional matrix

with $rank(\overline{E}_{2}) = rank(E)$ and ϕ is a null matrix.

The system and output equations can be recast as follows

$$\begin{cases} \dot{z}_{1} \\ \dot{z}_{2} \end{cases} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix} \begin{cases} z_{1} \\ z_{2} \end{cases} + \begin{bmatrix} \overline{B}_{1} \\ \overline{B}_{2} \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} \boldsymbol{\Phi}_{(n-r)\times q} \\ \overline{E}_{2} \end{bmatrix} \boldsymbol{d} + \begin{bmatrix} \Delta \overline{A}_{11} & \Delta \overline{A}_{12} \\ \Delta \overline{A}_{21} & \Delta \overline{A}_{22} \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix} + \begin{bmatrix} \Delta \overline{B}_{1} \\ \Delta \overline{B}_{2} \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} \overline{C}_{1} & \overline{C}_{2} \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix} \cdot$$
(3)

Now it is assumed that the measurement signals are such that the following rank condition is satisfied: rank(C) > rank(E). This is a necessary condition for

designing this observer as the extra measurement signals are used to design the reduced order observer after decoupling the unknown inputs.

This condition allows the rearrangement of the output equation in the following form with the help of a transformation $\begin{bmatrix} \overline{y}_1 \\ \overline{y}_2 \end{bmatrix} = Vy$, where V is a nonsingular

matrix, as

$$\begin{cases} \overline{\mathbf{y}}_1 \\ \overline{\mathbf{y}}_2 \end{cases} = \begin{bmatrix} \overline{\mathbf{C}}_{11} & \mathbf{\phi} \\ \overline{\mathbf{C}}_{21} & \overline{\mathbf{C}}_{22} \end{bmatrix} \begin{cases} \mathbf{z}_1 \\ \mathbf{z}_2 \end{cases}.$$
(5)

Now the equations (3) and (5) can be written in expanded form as follows

$$\dot{\boldsymbol{z}}_{1} = \boldsymbol{\overline{A}}_{11}\boldsymbol{z}_{1} + \boldsymbol{\overline{A}}_{12}\boldsymbol{z}_{2} + \boldsymbol{\overline{B}}_{1}\boldsymbol{u} + \Delta \boldsymbol{\overline{A}}_{11}\boldsymbol{z}_{1} + \Delta \boldsymbol{\overline{A}}_{12}\boldsymbol{z}_{2} + \Delta \boldsymbol{\overline{B}}_{1}\boldsymbol{u}$$
(6)

$$\dot{\boldsymbol{z}}_2 = \boldsymbol{A}_{21}\boldsymbol{z}_1 + \boldsymbol{A}_{22}\boldsymbol{z}_2 + \boldsymbol{B}_2\boldsymbol{u} + \boldsymbol{E}_2\boldsymbol{d} + \Delta \boldsymbol{A}_{21}\boldsymbol{z}_1 + \boldsymbol{A}_{22}\boldsymbol{z}_2 + \Delta \boldsymbol{B}_2\boldsymbol{u} \quad (7)$$

 $\overline{\mathbf{y}}_1 = \overline{\mathbf{C}}_{11} \mathbf{z}_1 \tag{8}$

$$\overline{\mathbf{y}}_2 = \overline{\mathbf{C}}_{21} \mathbf{z}_1 + \overline{\mathbf{C}}_{22} \mathbf{z}_2 \,. \tag{9}$$

Eliminating z_2 from the equation (6) by using the equation (9), one can get

$$\dot{\boldsymbol{z}}_{1} = \overline{\boldsymbol{A}}_{11}\boldsymbol{z}_{1} + \overline{\boldsymbol{A}}_{12}\overline{\boldsymbol{C}}_{22}^{-1}(\overline{\boldsymbol{y}}_{2} - \overline{\boldsymbol{C}}_{21}\boldsymbol{z}_{1}) + \overline{\boldsymbol{B}}_{1}\boldsymbol{u} + \Delta\overline{\boldsymbol{A}}_{11}\boldsymbol{z}_{1} + \Delta\overline{\boldsymbol{A}}_{12}\boldsymbol{z}_{2} + \Delta\overline{\boldsymbol{B}}_{1}\boldsymbol{u} \qquad (10)$$

It can be seen that \overline{C}_{22}^{-1} should be nonsingular, which will be always so as rank(CE) = rank(E).

Now, the equation (10) can be written in a simplified form as

$$\dot{\boldsymbol{z}}_1 = \boldsymbol{\bar{A}}_s \boldsymbol{z}_1 + \boldsymbol{\bar{B}}_s \boldsymbol{\bar{u}} + \boldsymbol{E}_u \boldsymbol{d}_u, \qquad (11)$$

where
$$\overline{A}_{s} = \overline{A}_{11} - \overline{A}_{12}\overline{C}_{22}^{-1}\overline{C}_{21}, \quad \overline{B}_{s} = \begin{bmatrix} \overline{B}_{1} & \overline{A}_{12}\overline{C}_{22}^{-1} \end{bmatrix},$$

 $\overline{u} = \begin{cases} u \\ \overline{y}_{2} \end{cases}$ and $E_{u}d_{u} = \Delta \overline{A}_{11}z_{1} + \Delta \overline{A}_{12}z_{2} + \Delta \overline{B}_{1}u$ with E_{u} -

known matrix and d_{u} - unknown signal.

For designing an observer, the system should satisfy the observability conditions: rank(O(A, C)) = n.

Now one can design an observer for the system (11) and (8) as

$$\dot{\hat{z}}_1 = \overline{A}_s \hat{z}_1 + \overline{B}_s \overline{u} + K(\overline{y}_1 - \hat{\overline{y}}_1)$$
(12)

$$\hat{\overline{y}}_1 = \overline{C}_{11} \hat{z}_1. \tag{13}$$

The gain matrix K is found out by solving the following algebraic Riccati equation [3,10]

$$\overline{A}_{s}\boldsymbol{P} + \boldsymbol{P}\overline{A}_{s}^{T} + \boldsymbol{Q} + \frac{q^{2}\boldsymbol{E}_{u}\boldsymbol{E}_{u}^{T}}{\sigma} - \boldsymbol{P}\overline{\boldsymbol{C}}_{11}^{T}\overline{\boldsymbol{C}}_{11}\boldsymbol{P} + \frac{\boldsymbol{P}\boldsymbol{E}_{u}\boldsymbol{E}_{u}^{T}\boldsymbol{P}}{q^{2}\sigma} = 0$$
(14)
with $\boldsymbol{K} = \boldsymbol{P}\overline{\boldsymbol{C}}^{T}$
(15)

with
$$\mathbf{K} = \mathbf{P}\mathbf{C}_{11}^{T}$$
, (15)

where Q is a pre-chosen positive definite matrix and the constants $q \& \sigma$ are specified numbers. It was shown in [10] that for any $\sigma > 0$, there exists q such that gain obtained from the above equations will lead to $\|\vec{C}_{11}(jwI - \vec{A}_s + K\vec{C}_{11})E_u\| < \sigma$ for $\forall w \in \mathbb{R}^1$ where w is the frequency. This condition implies that the effect of unknown signal d_u becomes very small in error dynamics for an appropriate value of σ .

The states \hat{z}_2 are estimated from equation (9) as

$$\hat{z}_2 = \bar{C}_{22}^{-1} (\bar{y}_2 - \bar{C}_{21} \hat{z}_1).$$
 (16)

Now using $\hat{z} = \{\hat{z}_1 \ \hat{z}_2\}^T$ the estimated state vector \hat{x} is found out as $\hat{x} = T^{-1} \hat{z}$.

III. FAULT DETECTION AND ISOLATION ALGORITHM

In this section, a parametric fault detection and isolation technique for an uncertain system is described. It consists of two steps. In the first step, a set of residuals is generated with the help of a bank of high gain observers to detect the fault and isolate the faulty zone. In the second step, faulty parameter is isolated from the faulty zone.

Consider a linear time invariant system as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta \boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta \boldsymbol{B})\boldsymbol{u}(t)$$
(17)

where the significance of the matrices and vectors are same as described in the previous section.

Suppose a parametric fault occurs in the plant. The detection and isolation of the fault are carried out in two steps as follows.

Step-1: Detection and partial isolation of fault

The faulty system is written as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta \boldsymbol{A}_f + \Delta \boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta \boldsymbol{B}_f + \Delta \boldsymbol{B})\boldsymbol{u}(t), \quad (18)$$

where ΔA_f and ΔB_f are the faulty parts of the matrices A and B respectively. The state equation (18) can now be rearranged as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta \boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta \boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad (19)$$

where E is a known matrix and $d(t) \in \mathbf{R}^{q}$ is the unknown input satisfying the relation

$$\boldsymbol{E}\boldsymbol{d}(t) = \Delta \boldsymbol{A}_{f}\boldsymbol{x}(t) + \Delta \boldsymbol{B}_{f}\boldsymbol{u}(t) \,. \tag{20}$$

Now the system is divided into N number of subsystems with each characterized by a few parameters. The choice of subsystems is arbitrary. In a physical system, the subsystems are chosen based on the physical proximity of different parameters. Assume that the fault has occurred in the *i*-th subsystem.

The system equations considering the fault in the *i*-th subsystem is written as

$$\dot{x}_{(i)}(t) = (A + \Delta A)x_{(i)}(t) + (B + \Delta B)u(t) + E_{(i)}d_{(i)}(t)$$
 (21)

where the subscript (i) indicates that the fault has occurred only in the *i*-th subsystem.

The output equation for this system is written as

$$\mathbf{y}_{(i)}(t) = \mathbf{C}_{(i)} \mathbf{x}_{(i)}(t),$$
 (22)

The equations (21) and (22) are similar to equations (1) and (2). Now, following the procedure discussed in

the previous section, an unknown input high gain observer is designed to estimate the states $\hat{x}_{(i)}(t)$.

Once the states $\hat{x}_{(i)}(t)$ are estimated, the residuals are calculated as

$$\mathbf{r}_{(i)}(t) = \mathbf{y}_{(i)}(t) - \hat{\mathbf{y}}_{(i)}(t) = \mathbf{y}_{(i)}(t) - \mathbf{C}_{(i)}\hat{\mathbf{x}}_{(i)}(t).$$
(23)

Now, an unknown input observer, if properly designed, can estimate the states irrespective of unknown inputs. So the residual $\mathbf{r}_{(i)}(t)$, calculated from the equation (23), converges within bounded value (i.e., threshold value) if the fault occurs in the *i*th subsystem or there is no fault in the system as the effect of possible faults in *i*-th is considered as unknown inputs. In this way, one can detect a fault and isolate the faulty subsystem using N number of UIHGOs. However (N-1) such observers will be sufficient to isolate a faulty subsystem when N > 2because once (N-1) subsystems are found fault free, the remaining subsystem is automatically identified as the faulty one. A decision table is drawn to isolate the faulty subsystem from observation of (N-1)residuals.

Step-2: Total isolation of fault

In this step, the faulty parameter in the faulty subsystem is isolated. The effect of the faulty subsystem is now simulated as an unknown input signal, say $F_u(t)$. The relationship between $F_u(t)$, the parameters of the faulty subsystem, say s, and the states x(t) are known and can be written as

$$F_{\mu}(t) = f(s, \boldsymbol{x}), \qquad (24)$$

where the function 'f' is linear for a linear system. The system equation for this case becomes

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta \mathbf{A})\mathbf{x}(t) + (\mathbf{B} + \Delta \mathbf{B})\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t), \qquad (25)$$

where $d(t) = F_u(t)$. With a measurement matrix C, an observer is then designed to estimate the states. Knowing the states, the unknown input signal is then estimated from the state equations neglecting the uncertainties using the nominal values of the parameters of the other non-faulty subsystems.

The estimated signal $\hat{F}_{u}(t)$ is now used to estimate the

parameters
$$s$$
 's from the relation (24) as

$$\hat{F}_{\mu}(t) = f(s, \hat{\boldsymbol{x}}(t)) \,. \tag{26}$$

Different parameter estimation techniques can be used to estimate s from equation (26). However, a very simple logical estimation approach is applied in the present work in order to isolate the faulty element.

Let us consider the k-th parameter s_k as the faulty one. From the above relation, s_k can be estimated using nominal values of rest of the parameters. Mathematically, $\hat{s}_k = g(s_1, s_2, \dots, s_{k-1}, s_{k+1}, \dots, \hat{x}(t), \hat{F}_u(t))$ (27) where g is a functional.

In steady state, the estimated values vary very less if the assumption is correct. The moving averages technique is used to smoothen the fluctuation of the estimated values. If the assumption is wrong, the estimated values will vary significantly large. Now, as the single fault case is being considered, there will be only one case when the estimated parameter will vary less. The particular parameter for which it happens is the faulty one. In this way, the faulty parameter is isolated. With this the isolation process is completed. In this way, any parametric fault can be detected and isolated following the above two steps.

IV. NUMERICAL EXAMPLE

Consider a mechanical system as shown in figure 1. The state space model of the system can be written as follows:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta \boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta \boldsymbol{B})\boldsymbol{u}(t) ,$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{(K_1 + K_2)}{M_1} & \frac{K_2}{M_1} & -\frac{(C_1 + C_2)}{M_1} & \frac{C_2}{M_1} \\ \frac{K_2}{M_3} & -\frac{(K_2 + K_3)}{M_3} & \frac{C_2}{M_3} & -\frac{(C_2 + C_3)}{M_3} \end{bmatrix},$$

 $\boldsymbol{B} = \begin{bmatrix} 0 \ 0 \ 0 \ (1/M_3) \end{bmatrix}^T, \qquad \boldsymbol{x} = \begin{bmatrix} X_1 & X_2 & \dot{X}_1 & \dot{X}_2 \end{bmatrix}^T, \\ \boldsymbol{u}(t) = F(t), \text{ where } X_i \text{ and } \dot{X}_i \text{ are the displacement and velocity of the mass element } \boldsymbol{M}_i \text{ respectively, } \\ \boldsymbol{K}_j \text{ - the stiffness element and } \boldsymbol{C}_j \text{ - the damping coefficient } (i=1,2 \text{ and } j=1, 2, 3). \text{ The matrices } \Delta \boldsymbol{A} \text{ and } \Delta \boldsymbol{B} \text{ are uncertainties.} \end{cases}$

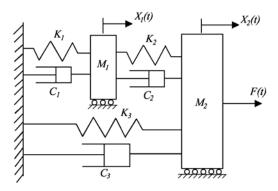


Figure 1: Mechanical system having two masses and three sets of spring-damper

The numerical values of the parameters are $M_1 = 870 \text{ kg}$, $M_3 = 1550 \text{ kg}$, $K_1 = 280000 \text{ N/m}$, $K_2 = 370000 \text{ N/m}$, $K_3 = 340000 \text{ N/m}$,

$$C_1 = 3500 \text{ Ns/m}$$
, $C_2 = 3000 \text{ Ns/m}$ and $C_2 = 5675 \text{ Ns/m}$.

A fault is introduced at t = 50 sec in the spring of stiffness K_2 . The new value of K_2 is set to 185000 N/m. Now using FDI algorithm, discussed in section 3, the fault (here K_2) is detected and isolated as follows.

Step-1: Detection and partial isolation of the fault

First, the system is divided into three subsystems as follows: $SS1: K_1$, $C_1 \& M_1$; $SS2: K_2 \& C_2$ and $SS3: K_3$, $C_3 \& M_3$.

The uncertainties are as follows: $\Delta A = M\Sigma N_1$ and $\Delta B = M\Sigma N_2$ with $M = I_n$, $N_1 = 0.05 \times A_u$, $N_2 = 0.05 \times B_u$ and $\Sigma = \Sigma_0 \sin(w_1 t)$ where $\Sigma_0 = 0.25 \times I$ and $w_1 = 0.05$ rad/s. The matrices A_u and B_u are same as A and B excepting the elements containing constant terms are replaced with zeros. The sinusoidal variation in system parameters is applied. The following input signal is applied in numerical simulation: $u = u_0 s \ln(wt)$ with $u_0 = 100$ N and w = 1 rad/s.

As the system is divided into three subsystems, so two UIHGOs are sufficient as a part of step-1. The observers are designed for SS1 and SS3. The unknown input matrices E's and unknown input signals d's for those observers are given as

$$E_{(1)} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^{T}, \ d_{(1)} = \Delta A_{(1)} \mathbf{x}_{(1)} + \Delta B_{(1)} \mathbf{u}_{(1)}$$
$$E_{(3)} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^{T}, \ d_{(3)} = \Delta A_{(3)} \mathbf{x}_{(3)} + \Delta B_{(3)} \mathbf{u}_{(3)}.$$
The output matrices are

 $\boldsymbol{C}_{(1)} = \begin{bmatrix} 0 \ 1 \ 0 \ 0; 0 \ 0 \ 1 \ 0 \end{bmatrix}^T \text{ and } \boldsymbol{C}_{(3)} = \begin{bmatrix} 0 \ 1 \ 0 \ 0; 0 \ 0 \ 1 \ 1 \end{bmatrix}^T.$

Now applying the algorithm discussed in section-2, two E_u 's appear as $E_{u(1)} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ and $E_{u(3)} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. The tuning parameters σ and qare considered as $\sigma_{(1)} = 0.05$, $\sigma_{(3)} = 0.05$, $q_{(1)} = 15$ and $q_{(3)} = 15$. The value of Q is chosen as $Q = 5I_{n_1}$ for both the observers.

Two high gain observers are then designed for the above systems and the gain matrices are calculated from the equations (14) and (15). The values of the observer gains for the above observers are $\mathbf{K}_{(1)} = \begin{bmatrix} 0.7253 & 4.7760 & 14.8934 \end{bmatrix}^T$ and $\mathbf{K}_{(3)} = \begin{bmatrix} 0.4368 & 2.9387 & -13.1569 \end{bmatrix}^T$ respectively. The residuals are plotted in figure 2 and figure 3.

In ideal (fault free and without presence of parameter uncertainties) situation the residuals should be zero. However in the present case these will not be perfectly zero due to presence of parameter uncertainties. Hence two small threshold values $\varepsilon_{(1)} = \{1.5 \times 10^{-5} \ 1 \times 10^{-20}\}^T$ and $\varepsilon_{(3)} = \{3 \times 10^{-9} \ 3 \times 10^{-20}\}^T$ units are chosen. These are calculated when there is no fault in the system.

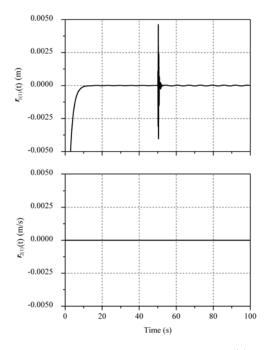


Figure 2: Components of the residual $\mathbf{r}_{(1)}(t)$

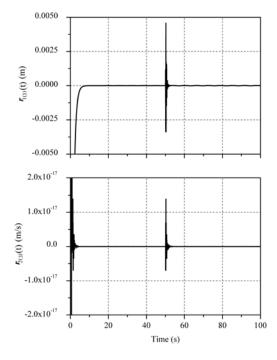


Figure 3: Components of the residual $r_{(3)}(t)$

As both the residuals cross the threshold values, the existence of fault is confirmed. In order to isolate the faulty subsystem a decision table is constructed as shown in table 2.

Table 1: Decision table for isolation of faulty
subsystem

Observation	Is $\mathbf{r}_{(i)} > \boldsymbol{\varepsilon}_{(i)}$ (use '1') or $\mathbf{r}_{(i)} \leq \boldsymbol{\varepsilon}_{(i)}$ (use '0') ?		Decision
	$r_{(i)} \leq r_{(i)}$ (us $r_{(1)}$	r ₍₃₎	Decision
Case	1	1	Fault: SS2.

From table-1, it is seen that the fault is in subsystem 2. Now the next step (i.e., step-2) is carried out to isolate faulty parameter.

Step-2: Total isolation of the fault

Here the faulty subsystem (SS2) is replaced by an unknown force $F_{u}(t)$ as

$$F_{u}(t) = K_{2}(X_{2} - X_{1}) + C_{2}(\dot{X}_{2} - \dot{X}_{1}) + C_{2}(\dot{X$$

The system is remodeled as follows $\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta \mathbf{A})\mathbf{x}(t) + (\mathbf{B} + \Delta \mathbf{B})\mathbf{u}(t) + \mathbf{E}\mathbf{d}(t)$ where $\mathbf{E} = \begin{bmatrix} 0 & 0 & 1/M_1 & -1/M_3 \end{bmatrix}^T$ and $\mathbf{d}(t) = F_u(t)$. It can be noticed that the parameter uncertainties for this system are only in 3rd and 4th rows of \mathbf{A} and \mathbf{B} matrices for this problem, which indicate $\Delta \mathbf{A}$ and $\Delta \mathbf{B}$ have non-zero elements in 3rd and 4th rows only. For this advantage here the system equation can be rewritten combining the unknown inputs and uncertainties as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}_{c}\boldsymbol{d}_{c}(t)$$

where $E_{c} = [0 \ 0 \ 1 \ 0; 0 \ 0 \ 0 \ 1]^{T}$ and

 $d_{c} = \left[\Delta A_{3,1-4} \mathbf{x} + \Delta B_{3,1} \mathbf{u} + F_{u} / M_{1} \quad \Delta A_{4,1-4} \mathbf{x} + \Delta B_{4,1} \mathbf{u} - F_{u} / M_{3} \right]^{T}$ This special situation may not occur in all system. A full order unknown input observer [4] is designed with output matrix $C = \begin{bmatrix} 0 & 1 & 2 & 0; 0 & 0 & -1 & 1; 1 & 0 & 0 \end{bmatrix}^{T}$ to estimate the states $\hat{\mathbf{x}}(t)$. Using the estimated states $\hat{\mathbf{x}}(t)$ and the nominal values of the parameters of subsystem-1, the unknown input $F_{u}(t)$ is estimated from the following relationship

$$\hat{F}_{u}(t) = M_1 \hat{x}_3 + K_1 \hat{x}_1 + C_1 \hat{x}_3$$

where \hat{x}_3 is calculated taking the derivative of \hat{x}_3 with respect to time.

Finally the faulty parameters are estimated using the following relation

$$\hat{F}_{u}(t) = K_{2}(\hat{x}_{2} - \hat{x}_{1}) + C_{2}(\hat{x}_{4} - \hat{x}_{3})$$

First, the fault is assumed in K_2 and the stiffness K_2 is estimated using the nominal value of C_2 =3000 Ns/m. The moving averages are taken to smoothen the estimated values and the estimated values are plotted in figure 4. The plot shows that estimated values vary very less from it mean value. Now C_2 is assumed to be faulty and C_2 is estimated using the nominal value of K_2 =370000 N/m. The estimated values after taking moving averages are plotted in figure 5. The plot shows that the estimated values of C_2 vary widely, which is because of wrong assumption. This confirms that fault is in K_2 and thereby the fault isolation process is completed.

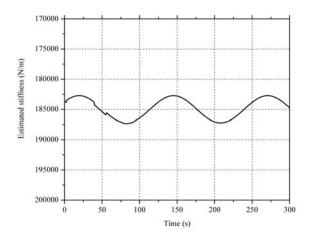


Figure 4: Estimated stiffness

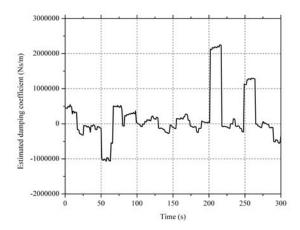


Figure 5: Estimated damping coefficient

Thus it is seen that the FDI scheme works well for the occurrence of fault in subsystem 2. It can be shown easily that the method works with equal ease for any parameter fault in other subsystem.

V. CONCLUSIONS

An UIHGO based parameter FDI scheme is presented. First an UIHGO for uncertain systems is derived. These types of observers have wide applications in robust control and fault diagnosis. Then, using these observers, a FDI technique is devised. The main advantage of the FDI algorithm is that it is capable of estimating faults even if the parameters are coupled in the system matrix. It also reduces the complexity of estimating all the parameters at every time instant unlike existing identification based techniques. The same FDI technique can also be used to detect a fault of a noisy system provided other types of unknown input estimators capable of handling noise should be used.

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