Pressure Limits of Thick-Walled Cylinders

A. B. Ayob, M. N. Tamin, and M. Kabashi Elbasheer

***Abstract*— The effect of autofrettage on thick-walled cylinders, operating under high internal pressure, has become a significant area of development, both in research and practice. In optimal design of thick-walled cylinders, there are two main objectives to be achieved: increasing its strength-weight ratio and extending its fatigue life. This can be achieved by generating a residual stress field in the cylinder wall prior to use. Both analytical and numerical techniques have been used for the investigation of the effects of residual stresses on the load-carrying capacity. The scope of the current paper includes application of ABAQUS finite element code to the direct problem of finding thick-walled cylinder autofrettage solutions. The results reveal three scenarios in the design of thick-walled cylinders. For maximum load carrying capacity, non-autofrettage is suitable when, in service, the whole wall thickness will be yielded. Full autofrettage is suitable when, during subsequent operation, yielding is limited at the inner surface. Optimum autofrettage of the cylinder is suitable if a minimum equivalent stress is to be achieved. The analytical solutions were compared to numerical results and a very good correlation in form and magnitude was obtained.**

***Index Terms*— Autofrettage, elastic-plastic junction line, finite element analysis, loads capacity, residual stress.**

I. Introduction

Due to the ever-increasing industrial demand for axisymmetric pressurized cylindrical components which have had wide applications in chemical, nuclear, fluid transmitting plants, power plants and military equipment, the attention of designers has been concentrated on this particular branch of engineering. The increasing scarcity and higher cost of materials have led researchers not to confine themselves to the customary elastic regime but moved their attention to the elastic–plastic approach which offers more efficient use of materials [4]. Autofrettage is a common process of producing residual stresses in the wall of a thick-walled cylinder prior to use. An appropriate pressure, large enough to cause yielding within the wall, is applied to the inner surface of the cylinder and then removed. Upon release of this pressure, compressive residual stresses are developed to a certain radial depth at the bore. These residual stresses serve to reduce the tensile stresses developed as a result of subsequent application of an operating pressure, thus increasing the load bearing capacity [1, 2]. Usually large scale yielding occurs in the autofrettaged thick-walled cylinder wall [3]. Theoretical and numerical methods used to determine limit loads involve some assumptions and parameters that affect the accuracy of the results. The purpose of this study is to investigate the effect of autofrettage parameters on the limit loads. The simplest and most general theoretical treatment of the partially-plastic cylinder has been the use of the Tresca yield criterion, with the assumption of an elastic perfectly-plastic material. Using the Tresca yield criteria together with an autofrettage level parameter, a precise solution for residual stress was developed.

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II. Operating Pressure Limitations

For a cylinder subjected to an internal pressure, P_i , the radial stress, σ_r , and circumferential (hoop) stress, σ_θ , distributions are given by Lame's formulation:

$$\sigma_r = \frac{P_i}{k^2-1}\left[1-\frac{r_o^2}{r^2}\right] \tag{1}$$

$$\sigma_\theta = \frac{P_i}{k^2-1}\left[1+\frac{r_o^2}{r^2}\right] \tag{2}$$

For a cylinder with end caps and free to change in length, the axial stress is given by [5]:

$$\sigma_z = \frac{P_i}{k^2-1} \tag{3}$$

where $k = \frac{r_o}{r_i}$ (4)

According to Tresca yield theory, yielding occurs when the equivalent stress is [5]:

$$\sigma_{Tr} = (\sigma_\theta - \sigma_r) = \sigma_Y \tag{5}$$

Two important pressure limits, $P_{Y,i}$ and $P_{Y,o}$, are considered to be of importance in the study of pressurized cylinders. $P_{Y,i}$ corresponds to the internal pressure required at the onset of yielding of the inner surface of the cylinder, and $P_{Y,o}$ is the internal pressure required to cause the whole wall to yield completely. The magnitudes of $P_{Y,i}$ and $P_{Y,o}$, according to Tresca yield strength criterion, are [1, 6 and 7]:

For $P_{Y,i}$:

$$\frac{p_i}{k^2-1}\left(1+k^2\right)-\frac{p_i}{k^2-1}\left(1-k^2\right)=\sigma_Y$$

$$\frac{p_i}{k^2-1}\left(2k^2\right)=\sigma_Y$$

$$P_{Tr,Y,i}=\frac{(k^2-1)}{2k^2}\sigma_Y$$

$$\frac{P_{Tr,Y,i}}{\sigma_Y}=\frac{(k^2-1)}{2k^2} \tag{6}$$

Eqn. (6) is the non-dimensionalized inner surface pressure limit of thick-walled cylinder using Tresca yield criterion.

As for $P_{Y,o}$:

$$\frac{2\,p_i}{k^2-1}=\sigma_Y$$

$$P_{Tr,Y,o}=\frac{(k^2-1)}{2}\sigma_Y$$

$$\frac{P_{Tr,Y,o}}{\sigma_Y}=\frac{(k^2-1)}{2} \tag{7}$$

Eqn. (7) corresponds to the outer surface pressure limit. The relationships between $P_{Tr,Y,i}$ and $P_{Tr,Y,o}$ with r_o and r_i are graphically shown in Fig. 1.

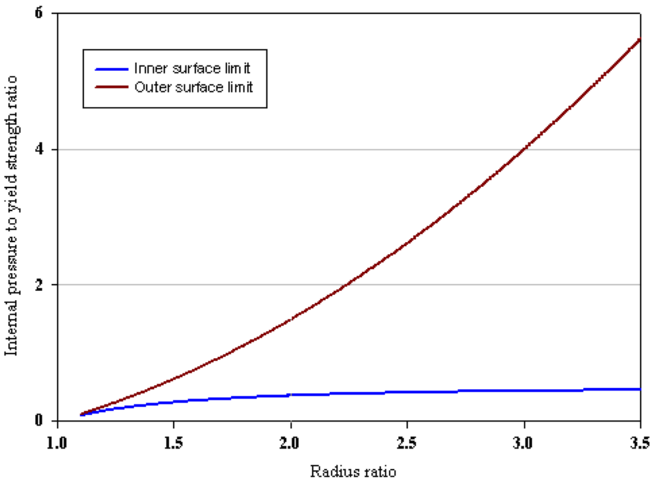

Fig. 1: Pressure limits of thick-walled cylinder

III. AUTOFRETTEGE PROCESS

The autofrettage pressure, P_a, is a sufficiently high internal pressure applied before a cylinder is put into use. If the internal pressure is removed after part of the cylinder thickness has become plastic, a residual stress is set up in the wall. Assuming that during unloading the material follows Hooke's Law; the residual stresses can be obtained from equations [8]:

For the plastic region, $r_i \le r \le r_a$

$$\sigma_{r,p,R}=\frac{\sigma_Y}{2}\left\{\left[2\ln\left(\frac{r}{r_a}\right)-1+\frac{m^2}{k^2}\right]-\left[2\ln(m+1)-\frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\left(1-\frac{r_o^2}{r^2}\right)\right\} \tag{8.a}$$

$$\sigma_{\theta,p,R}=\frac{\sigma_Y}{2}\left\{\left[2+2\ln\left(\frac{r}{r_a}\right)-1+\frac{m^2}{k^2}\right]-\left[2\ln(m+1)-\frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\left(1+\frac{r_o^2}{r^2}\right)\right\} \tag{8.b}$$

$$\sigma_{z,p,R}=\frac{\sigma_Y}{2}\left\{\left[1+2\ln\left(\frac{r}{r_a}\right)-1+\frac{m^2}{k^2}\right]-\left[2\ln(m+1)-\frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\right\} \tag{8.c}$$

For the elastic region, $r_a \le r \le r_o$

$$\sigma_{r,e,R}=\frac{\sigma_Y}{2}\left[1-\frac{r_o^2}{r^2}\right]\left\{\frac{m^2}{k^2}-\left(1-\frac{m^2}{k^2}+2\ln(m)\right)\left(\frac{1}{k^2-1}\right)\right\} \tag{9.a}$$

$$\sigma_{\theta,e,R}=\frac{\sigma_Y}{2}\left[1+\frac{r_o^2}{r^2}\right]\left\{\frac{m^2}{k^2}-\left(1-\frac{m_2}{k^2}+2\ln(m)\right)\left(\frac{1}{k^2-1}\right)\right\} \tag{9.b}$$

$$\sigma_{z,e,R}=\frac{\sigma_Y}{2}\left\{\frac{m^2}{k^2}-\left(1-\frac{m^2}{k^2}+2\ln(m)\right)\left(\frac{1}{k^2-1}\right)\right\} \tag{9.c}$$

where $$m=\frac{r_a}{r_i} \tag{10}$$

and r_a is the autofrettage radius. By substituting $r = r_a$, the residual stresses at junction radius, r_a, is obtained.

$$\sigma_{r,R}=\frac{\sigma_Y}{2}\left[1-\frac{k^2}{m^2}\right]\left\{\frac{m^2}{k^2}-\left(1-\frac{m^2}{k^2}+2\ln(m)\right)\left(\frac{1}{k^2-1}\right)\right\} \tag{11.a}$$

$$\sigma_{\theta,R}=\frac{\sigma_Y}{2}\left[1+\frac{k^2}{m^2}\right]\left\{\frac{m^2}{k^2}-\left(1-\frac{m^2}{k^2}+2\ln(m)\right)\left(\frac{1}{k^2-1}\right)\right\} \tag{11.b}$$

$$\sigma_{z,R}=\frac{\sigma_Y}{2}\left\{\frac{m^2}{k^2}-\left[1-\frac{m^2}{k^2}+2\ln(m)\right]\left(\frac{1}{k^2-1}\right)\right\} \tag{11.c}$$

The plot of the above residual stress distributions are shown in Fig. 2. On application of the operating pressure the total stress of the partially autofrettaged cylinder is the summation of the residual stress and the stress due to the operating pressure, i.e.:

$$\sigma_{r,T}=\sigma_r+\sigma_{r,R} \tag{12.a}$$

$$\sigma_{\theta,T}=\sigma_\theta+\sigma_{\theta,R} \tag{12.b}$$

$$\sigma_{z,T}=\sigma_z+\sigma_{z,R} \tag{12.c}$$

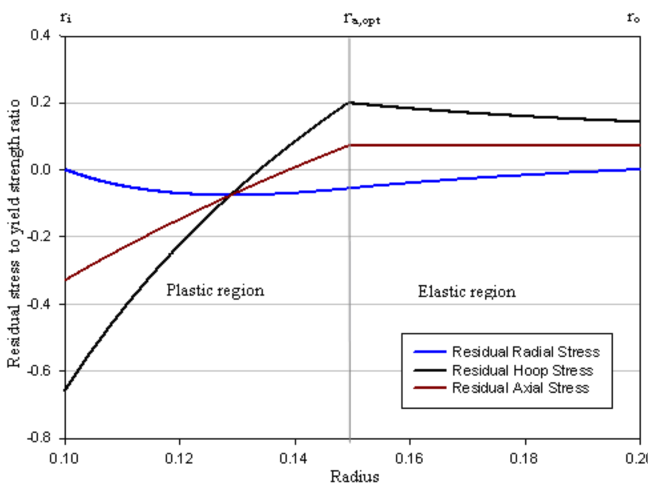

Fig. 2: Residual stress distributions, after autofrettage

The comparison of total stress distributions between autofrettaged and non-autofrettaged cylinders is shown in Fig. 3. In operation, autofrettage causes the high hoop stress at the inner surface to be reduced, and moves the location of the peak hoop stress from the inner surface to a location $r = r_a$. The equivalent stresses at $r = r_a$ is a maximum value.

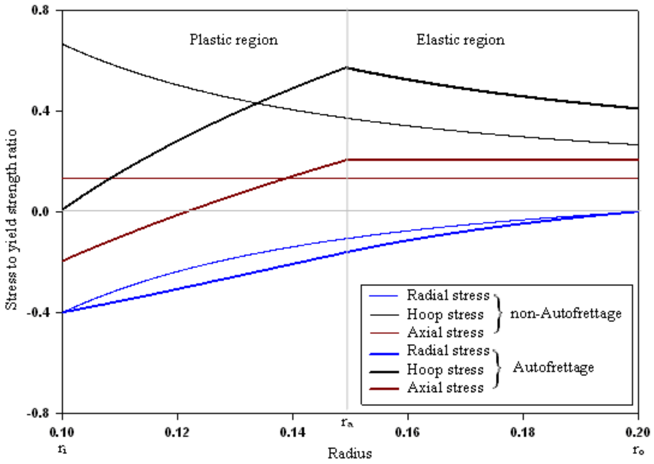

Fig. 3: Total stress distributions of autofrettaged and non-autofrettaged cylinders

The autofrettage process leads to a decrease in the maximum Tresca equivalent stress during the working stage. This means that the cylinder can now be subjected to an increase in the pressure capacity. A key problem in the analysis of autofrettage is to determine the optimum autofrettage pressure and the corresponding radius of the elastic–plastic boundary where the maximum equivalent stress in the cylinder becomes a minimum.

IV. Optimum Autofrettage

During autofrettage, the cylinder is yielded to the elastic-plastic junction line, called the autofrettage radius r_a. Using Tresca yield theory the equivalent stress at $r = r_a$ is obtained, when the cylinder is subjected to internal operating pressure, after being treated by autofrettage:

$$\sigma_{Tr} = \sigma_Y \frac{k^2}{m^2}\left[\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\frac{1}{k^2-1}\right] + \left[\frac{2P_{opr}}{k^2-1}\right]\left(\frac{k^2}{m^2}\right) \quad (13)$$

Differentiating and equating the differential to zero, i.e.

$$\frac{d\sigma_{Tr}}{dm} = 0$$

Hence, $m = \exp(\frac{P_{opr}}{\sigma_Y})$ is obtained.

Letting, $n = \frac{P_{opr}}{\sigma_Y}$

Therefore, $m_{Tr} = \exp(n)$ (14)

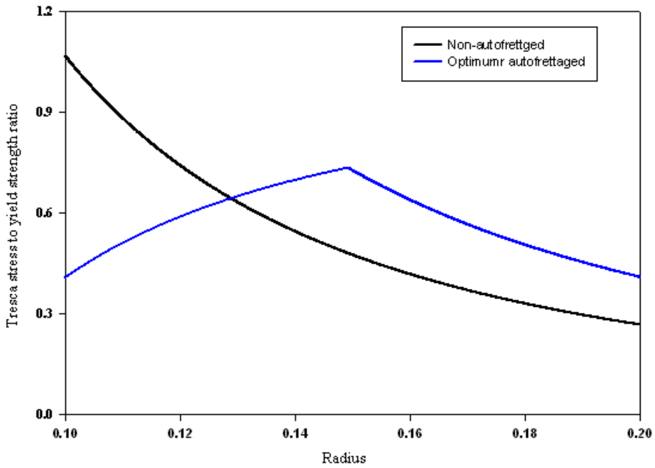

Fig. 4: Total Tresca equivalent stress

The internal pressure to cause yielding to a depth of r is:

$$P = \frac{\sigma_Y}{2}\left[1 - \frac{r^2}{r_o^2} + 2\ln\left(\frac{r}{r_i}\right)\right] \quad (15)$$

From Eqn. (14) the optimum autofrettage radius is obtained: $r_{a,opt} = r_i e^n$

Therefore the optimum autofrettage pressure is:

$$P_{a,opt,Tr} = \frac{\sigma_Y}{2}\left[1 - \frac{e^{2n}}{k^2} + 2n\right] \quad (16)$$

Using an arbitrary value of autofrettage pressure the total Tresca equivalent stress, using Eqns. 1-3 and Eqns. 8-12, can be found for all values of r. The equivalent stress is a maximum at all values of $r=r_a$, but with $P=P_{opt,a}$, the maximum equivalent stress has a minimum value but is still lower than that in non-autofrettaged condition, as shown in Fig. 4. The analytical results can be validated by numerical analysis, as shown in Fig. 5.

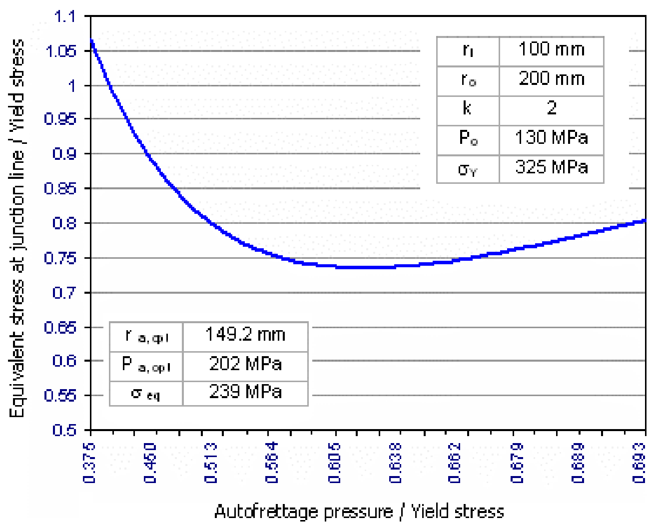

Fig. 5: Optimum autofrettage pressure and radius, from FEM

V. Allowable Internal Pressure Of Autofrettaged Cylinder

Eqns. (6) and (7) are used to obtain the maximum internal pressure to cause different stages of yielding in a cylinder which is not treated with autofrettage. For a cylinder treated with autofrettage, and using Tresca yield criterion, the internal pressure to cause the inner surface to yield again, can be obtained. Substituting Eqns. (1), (2) and (8) into Eqn. (12), when $r = r_i$, the internal pressure to cause yielding at the inner surface is:

$$P_{Y,i} = \frac{\sigma_Y}{2}\left[(2\ln(m) + 1 - \frac{m^2}{k^2}\right] \tag{17}$$

When $r = r_o$, by substituting Eqns. (1), (2) and (9) into Eqn. (12), the internal pressure to cause the whole wall thickness to yield is,

$$P_{Y,o} = \frac{\sigma_Y}{2}\left[(2\ln(m) + k^2 - m^2\right] \tag{18}$$

The values of pressure in Eqns. (17) and (18) can be graphically represented in Figs. 6 and 7, respectively. These pressures are influenced by different optimum autofrettage levels which were obtained when an operating pressure was initially known. From Fig. 6, the internal pressure to cause yielding at the inner surface of a cylinder which is treated with optimum autofrettage pressure is greater than that for a non-treated cylinder. On the other hand, the internal pressure to cause full yielding in a cylinder which has been treated with optimum autofrettage is lower than that which is non-treated with autofrettage (Fig. 7).

VI. Full Autofrettage

A special case is when the cylinder is fully autofrettaged, i.e. $r_a = r_o$. Therefore m = k and the equivalent stress at any radius can be obtained:

$$\sigma_{Tr} = \sigma_Y\left[1 - \frac{2\ln(k)}{k^2 - 1}\left(\frac{r_o^2}{r^2}\right)\right] + \frac{2P_{opr}}{k^2 - 1}\left(\frac{r_o^2}{r^2}\right) = \sigma_Y \tag{19}$$

Therefore the internal pressure to cause the internal surface and whole thickness to yield is, by substituting $r = r_i$ and $r = r_o$ into Eqns. (17) and (18) respectively. Table 1 and Fig. 8 show the influence of autofrettage level pressure on the allowable internal pressure of the cylinder, calculated according to Tresca yield criterion. Comparisons of the internal pressure are made between a cylinder which is not treated with autofrettage, treated with optimum autofrettage and full autofrettage.

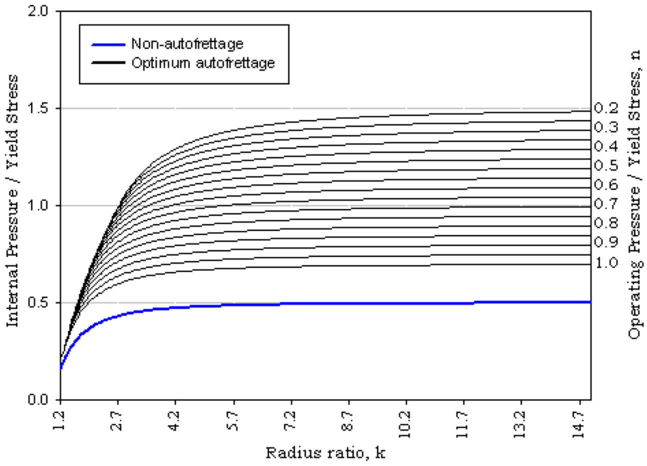

Fig. 6: Internal pressure to cause the inner surface to yield, with different levels of optimum autofrettage.

VII. Pressure Limitation of Optimum Autofrettage Cylinder

Rearranging Eqn. (16), the relation between the optimum autofrettage pressure and the operating pressure is:

$$\frac{P_{a,opt}}{\sigma_Y} = \left[n + \frac{k^2 - e^{2n}}{2k^2}\right] \tag{20}$$

Fig. 9 shows that, increasing the operating pressure results in an increase in the optimum autofrettage pressure. For high radius ratio (k>5), increasing the cylinder thickness does not affect the magnitude of the optimum autofrettage pressure significantly. For low radius ratio (k<3), decreasing the cylinder thickness leads to a dramatic decrease in optimum autofrettage pressure.

Table 1: Allowable internal pressure of cylinder treated with different levels of autofrettage.

	Internal pressure to cause the internal surface to yield		Internal pressure to cause the whole cylinder thickness to yield	
		k=2 n=0.25		k=2 n=0.25
No treatment	$\frac{(k^2-1)}{2k^2}\sigma_y$	0.375 σ_Y	$\frac{(k^2-1)}{2}\sigma_y$	1.500 σ_Y
Partial treatment	$\frac{\sigma_y}{2}\left[2\ln(m)+1-\frac{m^2}{k^2}\right]$	0.533 σ_Y	$\frac{\sigma_y}{2}\left[2\ln(m)+k^2-m^2\right]$	1.428 σ_Y
Full treatment	$\sigma_y \ln(k)$	0.693 σ_Y	$\sigma_y \ln(k)$	0.693 σ_Y

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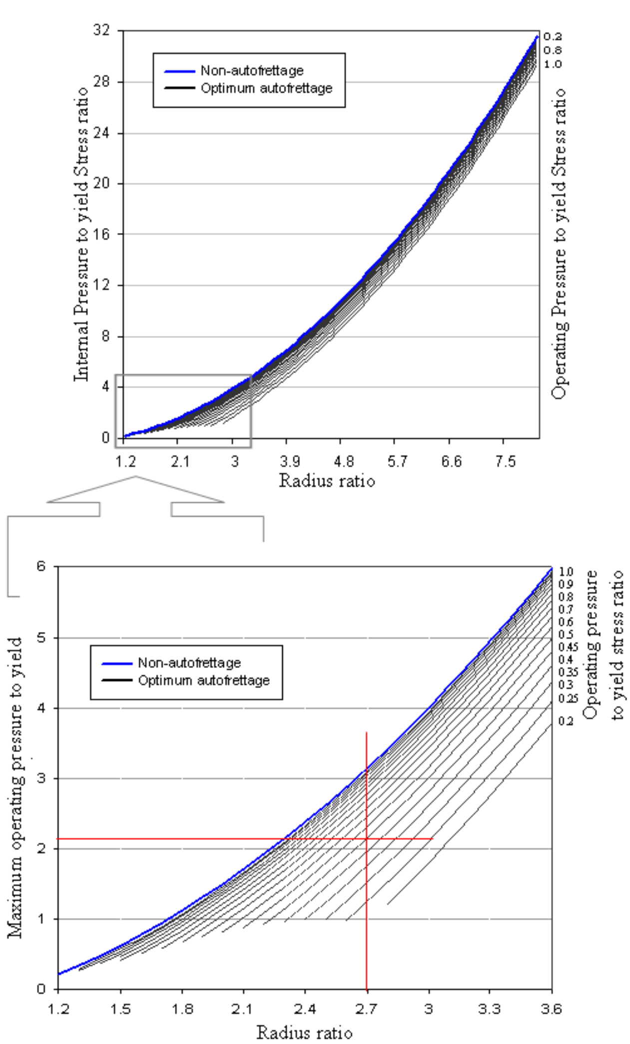

Fig. 7: Internal pressure to cause the whole thickness to yield, with different optimum autofrettage level.

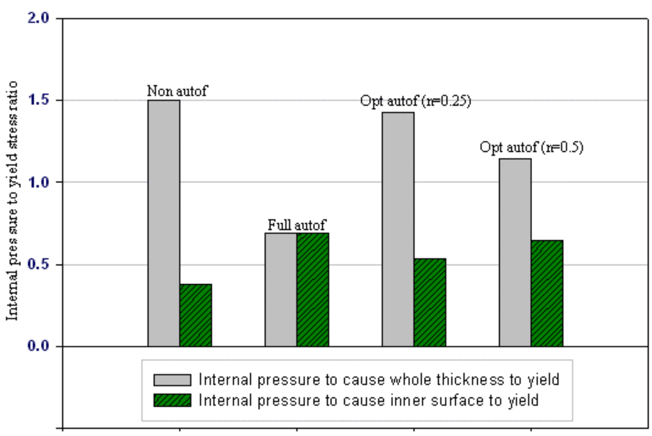

Fig. 8: Allowable internal pressure of none, full and optimum autofrettaged cylinder, using Tresca yield criteria

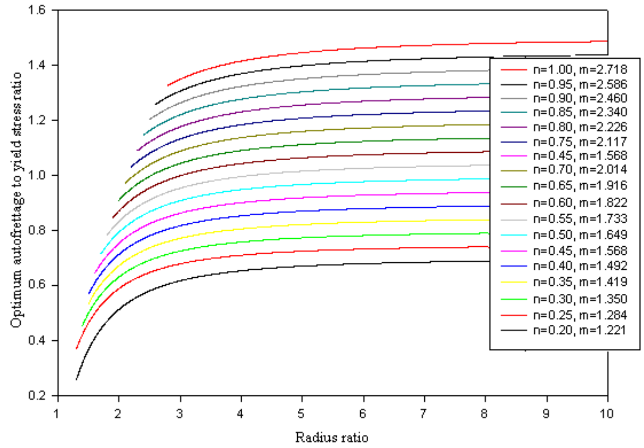

Fig. 9: Optimum autofrettage for different values of operating pressure and radius ratio.

VIII. Finite Element Analysis

The autofrettage process may be simulated by finite element methods, making use of elastic-plastic analysis. Using the ABAQUS code, an FE model of a cylinder with an inside radius 100 mm and outside radius of 200 mm was generated, as shown in Fig. 10(a). Symmetry conditions were fully utilized to reduce computing time. The FE model contained 30 elements and 62 nodes, as shown in Fig. 10(b). The material used is steel which has the following properties:

$E = 203$ GPa

$\sigma_Y = 325$ MPa

$\nu = 0.33$

The material is assumed to be elastic-perfectly plastic, having Tresca plasticity response.

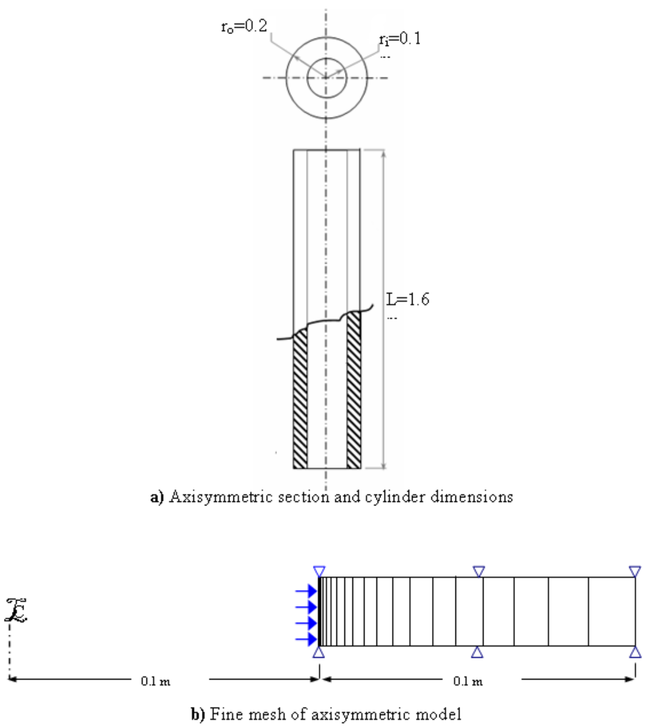

Fig. 10: Dimensions of plain thick-walled cylinder axisymmetric model

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An internal (autofrettage) pressure of 202 MPa was applied, and then removed. The residual stress distributions were then evaluated in the thick-walled cylinder. Using subsequent operating pressure of 160-220 MPa, FEM results show that the total equivalent stress becomes a minimum value of 237 MPa at an operating pressure of 200 MPa, as shown in Fig. 11.

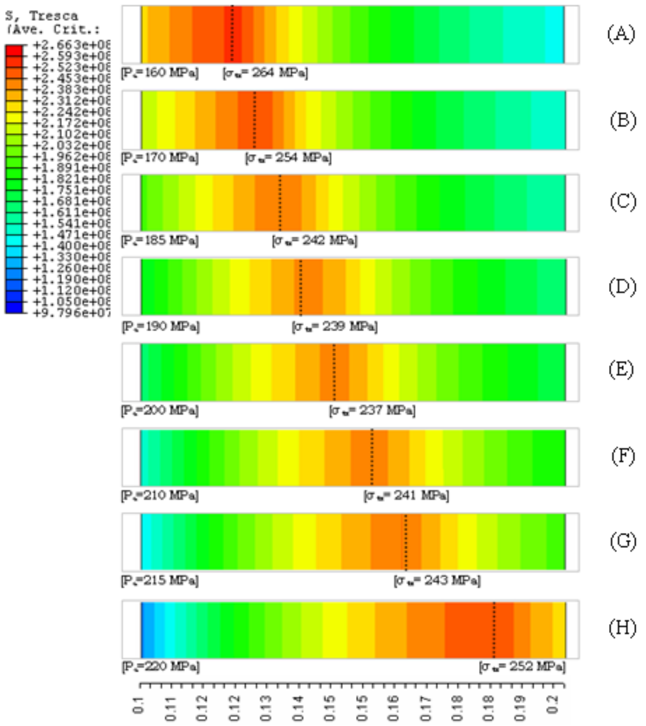

Fig. 11: Occurrence of a minimum of the maximum equivalent Tresca stress

IX. Conclusion

Two-dimensional FE simulations of the thick-walled cylinder were carried out to validate the analytical optimum autofrettage pressure and radius. The effects of autofrettage level parameters on the pressure capacity of cylinders were studied. From the results of this study, the following points may be concluded.

- After autofrettage, the largest residual stress is the hoop stress which is compressive and occurs at the inner surface of the thick-walled cylinder. This is beneficial in reducing the largest tensile hoop stress in subsequent repressurization with an internal operating pressure.
- The total equivalent stress values increases from the inner surface to the maximum value at the elastic-plastic autofrettage radius, and then decreases toward the outer surface.
- The autofrettage process increases the allowable internal pressure and elastic strength of a cylinder.
- The autofrettage process has a negligible effect on increasing the pressure capacity which can cause the whole cylinder wall to yield.
- The optimum autofrettage pressure is unique for a given operating pressure.
- There are three cases in the design of pressurized thick-walled cylinder:
  - Non-autofrettage is suitable if yielding is allowed throughout the cylinder wall thickness.
  - Full autofrettage is suitable if yielding is allowed at the inner surface only.
  - Optimum autofrettage case is suitable if the maximum equivalent stress is to be optimized.

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Nomenclature

P	pressure
r	radius
k	outer to inner radius ratio
m	autofrettage to inner radius ratio
n	operating pressure to yield stress ratio
σ	normal stress
ν	poisson's ratio
E	elastic modulus
i	inner
o	outer
a	autofrettage
r	radial
θ	hoop
z	axial
Y	yield
p	plastic
e	elastic
opt	optimum
opr	operating
Tr	Tresca
R	residual
T	total

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