Integrated Inventory Model With Fuzzy Order Quantity And Fuzzy Shortage Quantity

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Abstract—**This paper investigates an integrated inventory model with fuzzy order quantity and fuzzy shortage quantity. We express order quantity and shortage quantity as the normal triangular fuzzy numbers and then we will find the membership function of fuzzy cost and its centroid. We find that the estimated value of the total cost in the fuzzy sense is higher than in the crisp model.**

***Index Terms*— fuzzy inventory, fuzzy cost, buyer, vendor, membership function.**

I. INTRODUCTION

In traditional inventory management systems, the economic lot size (E.L.S) for a vendor and a buyer are managed independently, that is, the vendor and buyer find their own optimal order quantity. As a result, the E.L.S of buyer may not result in an optimal policy for the vendor and vice-versa. To overcome this problem, researchers have studied joint economic lot size (J.E.L.S) model where the joint total relevant cost (J.T.R.C) for the buyer as well as the vendor has been optimized. Goyal first introduced an integrated inventory policy for a single supplier and a single customer and derived the minimum joint variable cost for the supplier and the customer [1]. Banerjee introduced the J.E.L.S model for a single vendor and a single customer and obtained the minimum joint total relevant cost for both buyer and vendor at the same time with the assumption that the vendor makes the production set up every time the buyer places an order and supplies on a lot for lot basis [2]. Goyal modified Banerjee's [2] paper on the assumption that vendor may possibly produce a lot size that may supply an integer number of orders to the buyer [3]. Lu relaxed the assumption of Goyal [3] and developed a model with the assumption that the vendor can ship a subbatch to the supplier even before the entire batch is completed [4]. Goyal provided an alternative shipment policy where all the subbatches are not necessarily of same size [5].

Recently, fuzzy concepts have been introduced in the economic order quantity (E.O.Q) models. Zadeh showed the intention of accommodating uncertainty in the non stochastic sense rather than the presence of random variables [6]. Sommer applied fuzzy dynamic programming to an inventory and production-scheduling problem in which the management wishes to fulfill a contract for providing a product and then withdraw from the market [7]. Park examined the E.O.Q model in the fuzzy set theoretic perspective associating the fuzziness with the cost data [8]. Yao and Lee used extension principle to solve E.O.Q model with shortage. They fuzzified the order quantity into triangular fuzzy number, trapezoidal fuzzy number and got the optimal solution in the fuzzy sense [9]-[10]. Later, Chang *et al.* fuzzified the shortage quantity into triangular fuzzy number and the order quantity was a positive real variable and then deduced the membership function of the fuzzy total cost and its centroid [11]. Wu & Yao fuzzified both order quantity and shortage quantity into triangular fuzzy numbers and got the centroid of fuzzy total cost [12]. For the first time, Mahata *et al.* investigated the J.E.L.S model for both buyer and vendor in fuzzy sense. In this paper they have extended Banerjee's [2] J.E.L.S model with the assumption that the order quantity for the buyer/vendor is fuzzy variable[13].

In this article, we use from Mahata *et al.* [13] and Wu & Yao [12] models and investigate an integrated inventory model with fuzzy order quantity and fuzzy shortage quantity that these are a normal triangular fuzzy number. First in section II, we introduce the assumptions and notations of the model and then in section III, we model a fuzzy total cost for the buyer and vendor at the same time, then obtain a membership function of the fuzzy total cost and its centroid. In section IV, we solve an example and then we summarize the conclusions in section V.

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II. ASSUMPTIOINS AND NOTATIONS

Following assumptions and notations are considered:

A. assumptions

1) The demand rate and production rate are deterministic.
2) Manufacturing set-up cost, ordering cost, unit inventory holding cost for the vendor and the buyer, are known.
3) Single vendor and single buyer are considered.
4) There is a single product.
5) Shortage is allowed for buyer and fully backordered.
6) The vendor makes the production set up every time the buyer places an order and supplies on a lot for lot basis.
7) Order quantity and shortage quantity are normal triangular fuzzy numbers.

B. notations

D: Annual constant demand
P: Vendor's annual constant rate of production
C_V: The unit production cost
C_P: The unit purchase cost paid by the purchaser
A: The purchaser's ordering cost per order
S: The vendor's setup cost per setup
r: The annual inventory carrying cost per dollar invested in stocks
π : The shortage cost per unit quantity per year
q: The order quantity
b :The shortage quantity

III. THE MEMBERSHIP FUNCTION AND THE CENTROID OF FUZZY TOTAL COST

First we consider a crisp sense. Thus, Joint total relevant cost by considering shortage, is as follow

$$F(q,b)=\frac{D}{q}\cdot(s+A)+\frac{q}{2}\cdot r\cdot(\frac{D}{p}c_v+c_p)+\frac{(rc_p+\pi)\cdot b^2}{2q}-rc_p\cdot b \qquad (1)$$

Therefore the optimal solution in crisp case is

$$b^*=\frac{rc_p}{rc_p+\pi}\cdot q^* \qquad (2)$$

$$q^*=\sqrt{\frac{2D\cdot(s+A)\cdot(rc_p+\pi)}{r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)-(rc_p)^2}} \qquad (3)$$

$$F(q_*,b_*)=\sqrt{\frac{2D\cdot(s+A)\cdot\left[r\cdot(\frac{D}{p}c_v+c_p)\cdot(rc_p+\pi)-(rc_p)^2\right]}{(rc_p+\pi)}} \qquad (4)$$

Equation (1) and its derivatives have been obtained under the assumption that all the lead time (i.e., the period from the ordering time to the arrival time) in each cycle are the same. In the reality, such as the traffic condition may vary as well as other situations may affect the lead time among each cycle. Hence in (1) we cannot assume the lead time are all the same in each cycle. This will affect to the certainty of order quantity q and shortage quantity b too. Therefore we shall fuzzify both q and b at the same time, i.e., using a triangular fuzzy number $\tilde{q}=(q_1,q_0,q_2)$ and $\tilde{b}=\frac{rc_p}{rc_p+\pi}\cdot\tilde{q}$.

So

$$\mu_{\tilde{q}}(q)=\begin{cases}\frac{q-q_1}{q_0-q_1}, & q_1\le q\le q_0\\ \frac{q_2-q}{q_2-q_0}, & q_0\le q\le q_2\\ 0, & otherwise\end{cases} \qquad (5)$$

where $0\prec q_1\prec q_0\prec q_2$; q_1,q_0,q_2 are unknown.

For deffuzification , we use the centroid method. Therefore, the centroid of $\tilde{q}$ is

$$c(\tilde{q})=\frac{\int_{-\infty}^{\infty}x\mu_{\tilde{q}}(x)dx}{\int_{-\infty}^{\infty}\mu_{\tilde{q}}(x)dx}=\frac{q_1+q_0+q_2}{3} \qquad (6)$$

where $c(\tilde{q})$ denotes the estimated value of the order quantity in the fuzzy sense.

From $b=\frac{rc_p}{(rc_p+\pi)}.q$ and the Extension Principle, we have

$$\mu_{\tilde{b}}(b)=\begin{cases}\frac{b\cdot(rc_p+\pi)-rc_p\cdot q_1}{rc_p\cdot(q_0-q_1)} & \frac{rc_p}{rc_p+\pi}q_1\le b\le\frac{rc_p}{rc_p+\pi}q_0\\ \frac{rc_p\cdot q_2-b\cdot(rc_p+\pi)}{rc_p\cdot(q_2-q_0)} & \frac{rc_p}{rc_p+\pi}q_0\le b\le\frac{rc_p}{rc_p+\pi}q_2\\ 0 & otherwise\end{cases} \qquad (7)$$

The centroid of $\tilde{b}$, by (7) is

$$C(\tilde{b})=\frac{1}{3}\cdot(\frac{rc_p}{rc_p+\pi})\cdot(q_1+q_0+q_2)=(\frac{rc_p}{rc_p+\pi})\cdot C(\tilde{q})) \qquad (8)$$

The process of fuzzifying both q and b at the same time and finding out the fuzzy total cost $F(q,b)$ and obtaining the membership function by using Extension Principle, is very tedious and difficult. Instead, we shall use the *property 1*, and then apply the Extension Principle to obtain the membership function.

Property 1. The minimum total cost F(q,b) with respect to *q, b* is the same as the minimum total cost $G(q) = F(q, \frac{rc_p}{rc_p + \pi}.q)$ with respect to *q* and $b = \frac{rc_p}{rc_p + \pi}.q$.

$$\min_{0 \prec b \prec q} F(q,b) = F(q_*, b_*) = G(q_*) = \min_{0 \prec q} G(q_*)$$

With respect to *property 1*, we replace $b = \frac{rc_p}{rc_p + \pi}.q$ in (1).

Therefore, we will have

$$G(q) = \frac{D}{q} \cdot (s + A) + \frac{q}{2} \cdot r \cdot (\frac{D}{p} c_v + c_p) - \frac{(rc_p)^2 \cdot q}{2(rc_p + \pi)} \qquad (9)$$

Let $G(q) = z$, then the roots of $G(q) = z$ are

$$d_1(z) = \frac{(rc_p + \pi)}{\left[r \cdot (\frac{D}{p} c_v + c_p) \cdot (rc_p + \pi) - (rc_p)^2 \right]} \cdot \left[z - \sqrt{z^2 - G(q_*)^2} \right] \qquad (10)$$

and

$$d_2(z) = \frac{(rc_p + \pi)}{\left[r \cdot (\frac{D}{p} c_v + c_p) \cdot (rc_p + \pi) - (rc_p)^2 \right]} \cdot \left[z + \sqrt{z^2 - G(q_*)^2} \right] \qquad (11)$$

From $G(q) = z$ and the Extension Principle, we have the membership function of the fuzzy total cost $G(\tilde{q})$ as follow

$$\mu_{G(\tilde{q})}(z) = \begin{cases} \max[\mu_{\tilde{q}}(d_1(z)), \mu_{\tilde{q}}(d_2(z))] & \text{if } z \geq G(q_*) \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

In order to solve (12), we use the Table I and equations (13)-(25).

In table I, we consider the different position of $d_1(z), d_2(z)$ with respect to q_1, q_0, and q_2 and obtain $\mu_{G(\tilde{q})}(z)$.

Also we have

$$if \quad q_j \geq q_k \quad and \quad q_j.q_k \geq q_*^2 \Rightarrow G(q_j) \geq G(q_k) \qquad (13)$$

$$if \quad q_j \geq q_k \quad and \quad q_j.q_k \leq q_*^2 \Rightarrow G(q_j) \leq G(q_k) \qquad (14)$$

and

$$q_j \succ q_* \Leftrightarrow G(q_j) \prec \left[\frac{r.(\frac{D}{p} c_v + c_p).(rc_p + \pi) - (rc_p)^2}{(rc_p + \pi)} \right] .q_j \qquad (15)$$

$$q_j \succ q_* \Leftrightarrow \left[\frac{r.(\frac{D}{p} c_v + c_p).(rc_p + \pi) - (rc_p)^2}{(rc_p + \pi)} \right] .q_j \succ G(q_*) \qquad (16)$$

$$G(q_*) \prec G(q_j) \qquad (17)$$

Table I
The position of $d_1(z)$, $d_2(z)$ and $\mu_{G(\tilde{q})}(z)$ for $z \geq G(q_*)$

case		q_1	q_0	q_2	$\mu_{G(\tilde{q})}(z)$
1	$d_1(z)$, $d_2(z)$				0
2	$d_1(z)$	$d_2(z)$			$\frac{d_2(z) - q_1}{q_0 - q_1}$
3	$d_1(z)$		$d_2(z)$		$\frac{q_2 - d_2(z)}{q_2 - q_0}$
4	$d_1(z)$			$d_2(z)$	0
5		$d_1(z)$, $d_2(z)$			$\frac{d_2(z) - q_1}{q_0 - q_1}$
6		$d_1(z)$	$d_2(z)$		$\max\left[\frac{d_1(z) - q_1}{q_0 - q_1}, \frac{q_2 - d_2(z)}{q_2 - q_0}\right]$
7		$d_1(z)$		$d_2(z)$	$\frac{d_1(z) - q_1}{q_0 - q_1}$
8			$d_1(z)$, $d_2(z)$		$\frac{q_2 - d_1(z)}{q_2 - q_0}$
9			$d_1(z)$	$d_2(z)$	$\frac{q_2 - d_1(z)}{q_2 - q_0}$
10				$d_1(z)$, $d_2(z)$	0

Under the condition $z \geq G(q_*)$ and after some calculations, we get the following results.

$$when(q_j \leq d_1(z)) \wedge (q_j \succ q_*) \Rightarrow there \ is \ no \ solution \qquad (18)$$

$$when(q_j \leq d_1(z)) \wedge (q_j \prec q_*) \Rightarrow G(q_*) \leq z \leq G(q_j) \qquad (19)$$

$$when(d_1(z) \leq q_j) \wedge (q_j \prec q_*) \Rightarrow z \geq G(q_j) \qquad (20)$$

$$when(d_1(z) \leq q_j) \wedge (q_j \succ q_*) \Rightarrow z \geq G(q_*) \qquad (21)$$

$$when(q_j \leq d_2(z)) \wedge (q_j \prec q_*) \Rightarrow z \geq G(q_*) \qquad (22)$$

$$when(q_j \leq d_2(z)) \wedge (q_j \succ q_*) \Rightarrow z \geq G(q_j) \qquad (23)$$

$$when(d_2(z) \leq q_j) \wedge (q_j \prec q_*) \Rightarrow \mu_{F(\tilde{Q})}(z) = 0, so \ we \ don't \ consider \ it \qquad (24)$$

$$when(d_2(z) \leq q_j) \wedge (q_j \succ q_*) \Rightarrow G(q_*) \leq z \leq G(q_j) \qquad (25)$$

Now, in order to find $\mu_{G(\tilde{q})}(z)$ easier, we shall divide the region $0 \prec q_1 \prec q_0 \prec q_2$ into the following four cases:

(1) $q_* \prec q_1 \prec q_0 \prec q_2$
(2) $0 \prec q_1 \prec q_0 \prec q_2 \prec q_*$
(3) $0 \prec q_1 \prec q_* \prec q_0 \prec q_2$
(4) $0 \prec q_1 \prec q_0 \prec q_* \prec q_2$

Then, by table I and equations (12)-(25), we find the membership function $\mu_{G(\tilde{q})}(z)$ of fuzzy total cost $G(\tilde{q})$ in each case and after that, we obtain centroid of membership function with undermentioned equation.

$$E(q_1,q_0,q_2) = R/P \quad (26)$$

Where

$$P = \int_{-\infty}^{\infty} \mu_{G(\tilde{q})}(z)dz$$

and

$$R = \int_{-\infty}^{\infty} z\mu_{G(\tilde{q})}(z)dz$$

As a result, the centroid of fuzzy total cost is given by

$$\begin{aligned} E(q_1,q_0,q_2) &= E_1(q_1,q_0,q_2)I(T_1) + E_2(q_1,q_0,q_2)I(T_2) \\ &+ E_{31}(q_1,q_0,q_2)I(T_3)I(T_{31}) \\ &+ \sum_{j=1}^{7} E_{32j}(q_1,q_0,q_2)I(T_3)I(T_{32})I(T_{32j}) \\ &+ \sum_{j=1}^{3} E_{33j}(q_1,q_0,q_2)I(T_3)I(T_{33})I(T_{33j}) \\ &+ \sum_{j=1}^{7} E_{41j}(q_1,q_0,q_2)I(T_4)I(T_{41})I(T_{41j}) \\ &+ \sum_{j=1}^{3} E_{42j}(q_1,q_0,q_2)I(T_4)I(T_{42})I(T_{42j}) \\ &+ E_{43}(q_1,q_0,q_2)I(T_4)I(T_{43}) \end{aligned} \quad (27)$$

Here $E(q_1,q_0,q_2)$ denotes the estimated value of the total cost in the fuzzy sense when (q_1,q_0,q_2) is given and the order quantity can be found from (6) and the shortage quantity can be found from (8).

And

$$T_{31} = \{(q_1,q_0,q_2) | q_0 q_1 \succ q_*^2\}$$

$$T_{32} = \{(q_1,q_0,q_2) | q_1 q_0 \prec q_*^2 \quad and \quad q_2 q_1 \succ q_*^2\}$$

$$T_{321} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \le 0\}$$

$$T_{322} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad s_1 \prec s_2 \le G(q_0) \prec G(q_1)\}$$

$$T_{323} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad s_1 \le G(q_0) \prec s_2 \le G(q_1)\}$$

$$T_{324} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad s_1 \le G(q_0) \prec G(q_1) \le s_2\}$$

$$T_{325} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad G(q_0) \le s_1 \prec s_2 \le G(q_1)\}$$

$$T_{326} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad G(q_0) \le s_1 \prec G(q_1) \le s_2\}$$

$$T_{327} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) \succ 0 \, and \quad G(q_0) \prec G(q_1) \le s_1 \prec s_2\}$$

$$T_{33} = \{(q_1,q_0,q_2) | q_1 q_2 \prec q_*^2\}$$

$$T_{331} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) = 0 \, and \quad s_3 \le G(q_0) \prec G(q_2)\}$$

$$T_{332} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) = 0 \, and \quad G(q_0) \prec s_3 \le G(q_2)\}$$

$$T_{333} = \{(q_1,q_0,q_2) | \Delta^*(q_1,q_0,q_2) = 0 \, and \quad G(q_0) \prec G(q_2) \le s_3\}$$

$$T_{41} = \{(q_1,q_0,q_2) | q_2 q_1 \succ q_*^2\}$$

$$T_{42} = \{(q_1,q_0,q_2) | q_1 q_2 \prec q_*^2 \quad and \quad q_2 q_0 \succ q_*^2\}$$

$$T_{43} = \{(q_1,q_0,q_2) | q_0 q_2 \prec q_*^2\}$$

$$T_{41j} = T_{32j}, \; j = 1,...,7$$

$$T_{42j} = T_{33j}, \; j = 1,2,3$$

Where, s_1, s_2, s_3 are the roots of following equation

$$\frac{d_1(z) - q_1}{q_0 - q_1} = \frac{q_2 - d_2(z)}{q_2 - q_0}$$

and

$$I(A) = \begin{cases} 1 & if \quad (q_1,q_0,q_2) \in A \\ 0 & if \quad (q_1,q_0,q_2) \notin A \end{cases}$$

IV. EXAMPLE

We use from numbers of mahata' article[13] for solving an example.

D= 1000 ,P= 3200 ,A= 100 ,S= 400 ,C_P= 25 ,C_V= 20 , r = 0.2,Π= 10

Then we can have the crisp optimal solution: the optimal order quantity $q_* = 467$, the optimal shortage quantity $b_* = 155.7$ and the minimal total cost $F(q_*,b_*) = 2140.872$

We consider the following ratios

$$r_1(\tilde{q}) = \frac{E(q_1,q_0,q_2) - G(c(\tilde{q}))}{G(c(\tilde{q}))} * 100$$

$$r_2(\tilde{q}) = \frac{E(q_1,q_0,q_2) - G(q_*)}{G(q_*)} * 100$$

and we will calculate these ratios for different quantity of q_1, q_0, q_2 for the four following cases and summarize them in tables II-IV.

$467 \le q_1 \prec q_0 \prec q_2$;

$0 \prec q_1 \le 467 \prec q_0 \prec q_2$;

$0 \prec q_1 \prec q_0 \le 467 \prec q_2$;

$0 \prec q_1 \prec q_0 \prec q_2 \le 467$

Table II

For the case $467 \le q_1 \prec q_0 \prec q_2$

q_1	q_0	q_2	$G(c(\tilde{q}))$	$E(q_1,q_0,q_2)$	$r_1(\tilde{q})$	$r_2(\tilde{q})$
469	471	473	2140.95	2140.959	0.000	0.004
469	471	477	2141.01	2141.053	0.002	0.008
469	475	479	2141.132	2141.19	0.003	0.015
469	475	483	2141.234	2141.35	0.005	0.022
471	475	479	2141.181	2141.218	0.002	0.016
471	475	483	2141.291	2141.379	0.004	0.024
471	479	481	2141.352	2141.415	0.003	0.025
471	479	487	2141.561	2141.706	0.007	0.039
473	477	479	2141.291	2141.312	0.001	0.021
473	477	483	2141.418	2141.477	0.003	0.028
473	479	487	2141.638	2141.751	0.005	0.041
473	481	483	2141.561	2141.623	0.003	0.035
475	479	483	2141.561	2141.597	0.002	0.034
475	479	485	2141.638	2141.696	0.003	0.038
475	483	487	2141.895	2141.978	0.004	0.052
475	483	491	2142.086	2142.228	0.007	0.063
477	481	489	2141.989	2142.073	0.004	0.056
477	483	485	2141.895	2141.934	0.002	0.050
477	485	489	2142.188	2142.269	0.004	0.065
477	485	493	2142.403	2142.543	0.007	0.078

Table III

For the case $0 \prec q_1 \le 467 \prec q_0 \prec q_2$

q_1	q_0	q_2	$G(c(\tilde{q}))$	$E(q_1,q_0,q_2)$	$r_1(\tilde{q})$	$r_2(\tilde{q})$
465	469	470	2140.877	2140.890	0.001	0.001
465	469	475	2140.907	2140.968	0.003	0.004
465	473	481	2141.046	2141.196	0.007	0.015
465	475	485	2141.181	2141.413	0.011	0.025
461	471	479	2140.926	2141.092	0.008	0.010
461	473	474	2140.899	2140.984	0.004	0.005
461	473	476	2140.916	2141.028	0.005	0.007
461	473	483	2141.01	2141.272	0.012	0.019
457	471	479	2140.892	2141.091	0.009	0.010
457	473	481	2140.926	2141.184	0.012	0.015
457	475	483	2140.978	2141.298	0.015	0.020
457	477	484	2141.028	2141.387	0.017	0.024
453	477	484	2140.963	2141.381	0.019	0.024
453	483	486	2141.109	2141.669	0.026	0.037
453	483	487	2141.132	2141.721	0.028	0.040
453	483	493	2141.291	2142.087	0.037	0.057
451	477	487	2140.978	2141.529	0.026	0.031
451	483	487	2141.087	2141.716	0.029	0.039
451	483	489	2141.132	2141.827	0.032	0.045
451	485	495	2141.352	2142.306	0.045	0.067

Table IV

For the case $0 \prec q_1 \prec q_0 \le 467 \prec q_2$

q_1	q_0	q_2	$G(c(\tilde{q}))$	$E(q_1,q_0,q_2)$	$r_1(\tilde{q})$	$r_2(\tilde{q})$
462	464	470	2140.886	2140.919	0.002	0.002
462	464	477	2140.874	2141.006	0.006	0.006
462	464	487	2140.950	2141.449	0.023	0.027
462	464	497	2141.132	2142.143	0.047	0.059
457	459	469	2141.013	2141.108	0.004	0.011
457	459	471	2140.980	2141.102	0.006	0.011
457	459	473	2140.951	2141.098	0.007	0.011
457	459	475	2140.927	2141.096	0.008	0.010
451	453	469	2141.308	2141.54	0.011	0.031
451	453	471	2141.248	2141.527	0.013	0.031
451	455	473	2141.140	2141.442	0.014	0.027
451	455	479	2141.013	2141.426	0.019	0.026
443	447	469	2141.863	2142.349	0.023	0.069
443	449	471	2141.681	2142.211	0.025	0.063
443	451	475	2141.443	2142.082	0.030	0.057
443	447	483	2141.308	2142.251	0.044	0.064
437	447	477	2141.770	2142.785	0.047	0.089
437	447	479	2141.681	2142.772	0.051	0.089
437	447	481	2141.597	2142.761	0.054	0.088
437	447	483	2141.518	2142.751	0.058	0.088

Table V

For the case $0 \prec q_1 \prec q_0 \prec q_2 \le 467$

q_1	q_0	q_2	$G(c(\tilde{q}))$	$E(q_1,q_0,q_2)$	$r_1(\tilde{q})$	$r_2(\tilde{q})$
462	464	466	2140.917	2140.927	0.000	0.003
459	461	465	2141.013	2141.036	0.001	0.008
456	460	462	2141.165	2141.19	0.001	0.015
454	460	462	2141.219	2141.265	0.002	0.018
453	457	463	2141.308	2141.373	0.003	0.023
452	456	458	2141.557	2141.582	0.001	0.033
451	457	463	2141.373	2141.468	0.004	0.028
450	458	464	2141.34	2141.471	0.006	0.028
450	454	460	2141.639	2141.705	0.003	0.039
450	454	458	2141.725	2141.768	0.002	0.042
449	453	461	2141.681	2141.778	0.005	0.042
449	451	459	2141.863	2141.937	0.003	0.050
447	455	463	2141.597	2141.767	0.008	0.042
447	453	461	2141.77	2141.901	0.006	0.048
445	455	463	2141.681	2141.901	0.010	0.048
445	455	459	2141.863	2142.008	0.007	0.053
444	452	460	2142.013	2142.186	0.008	0.061
444	450	458	2142.228	2142.362	0.006	0.070
443	453	461	2141.962	2142.184	0.010	0.061
441	451	459	2142.285	2142.51	0.010	0.077

V. CONCLUSIONS AND FUTURE RESEARCHES

A. For $\tilde{q} = (q_1, q_0, q_2)$, compare $E(q_1, q_0, q_2)$ with $G(c(\tilde{q}))$.

Let $q_2 - q_0 = \Delta_{20} (\succ 0), q_0 - q_1 = \Delta_{01} (\succ 0)$

After computing $r_1(\tilde{q})$ for different quantity of q_1, q_0, q_2 in tables II-V, we see that when Δ_{20}, Δ_{01} are small, $E(q_1, q_0, q_2)$ are close to $G(c(\tilde{q}))$ and when Δ_{20}, Δ_{01} are larger, $E(q_1, q_0, q_2)$ are away from $G(c(\tilde{q}))$.

B. Comparison of the estimate of the total cost in the fuzzy sense $E(q_1, q_0, q_2)$ with the crisp minimal total cost $G(q_*)$.

From tables II-V and with considering $r_2(\tilde{q})$, we see that the estimate of the total cost in the fuzzy sense is larger than the crisp minimal total cost $G(q_*)$ and when Δ_{20}, Δ_{01} become larger, $E(q_1, q_0, q_2)$ are away from $G(q_*)$.

Equation (1) is obtained by assuming the lead times are fixed, and then get the minimal total cost $G(q_*)$. But in the reality, usually the time from the ordering point to the delivering point are not fixed and will vary a little. Therefore, we should not use the crisp minimal total cost $G(q_*)$, in stead, we should consider the fuzzy case to suit the real situation.

C. Comparison of our article with Mahata et al.' article

We compare $E(q_1, q_0, q_2)$ of our article with $E(q_1, q_0, q_2)$ of Mahata' article and we see that the estimate of the total cost in the fuzzy sense in our article is smaller than Mahata' article.

With this comparison, we conclude that, in fuzzy inventory models, like crisp inventory models, the total cost in models with backorder is smaller than the models without shortage.

For the future research, we can solve this model with numerical methods and/or genetic algorithm and get the optimal quantity for the fuzzy total cost.

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