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# A Modified Statistical Design Model of Double Sampling $\overline{X}$ Control Chart

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*Abstract* - The double sampling (DS) control chart can efficaciously reduce sample sizes and increase performance of process monitoring. Hsu [*Int. j. prod. res.,* vol. 42, no. 5, pp. 1043-1047] mentioned that the statistical design model of DS $\overline{X}$ control chart could merely minimize sample sizes during in-control process monitoring but fail to decrease sample sizes during detection of process shifts. In this study, with minimization of sample sizes for both in-control process and out-of-control process, a multi-objective programming method and genetic algorithm are proposed for statistical designs of DS $\overline{X}$ control chart. In comparison with both statistical design models, it is quite obvious that our model can effectively lower sample sizes of two process situations.

*Keywords* - **double sampling $\overline{X}$ control chart; statistical design; multi-objective programming; genetic algorithm**

## I. INTRODUCTION

The Shewhart's control chart has been extensively used as a tool for process monitoring in current industries. For Shewhart's $\overline{X}$ control chart, the performance to detect process mean shift can be increased through increase of sample sizes without any change in probability of occurrence of false alarms. However, increase in sample sizes signifies raise of costs and inspection time. Daudin[5] applied the concept of double sampling plans to the Shewhart's $\overline{X}$ control chart and used the two-stage Shewhart's $\overline{X}$ control chart to monitor processes, so that it was called as Double Sampling $\overline{X}$ control chart (DS $\overline{X}$ control chart). With this alternative method used, the advantage of the Shewhart's control chart, i.e., simplification at setup and calculation, can be maintained in addition to improvements in capability of detecting process mean shift and reduction of sample sizes.

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Additionally, through modification of sampling methods of the Shewhart's $\overline{X}$ control chart, the methods such as $\overline{X}$ charts with variable sample sizes (VSS) and $\overline{X}$ charts with variable sampling intervals (VSI) are also provided. For both charts, changes in sample sizes and sampling intervals of the Shewhart's $\overline{X}$ control chart lead to VSS and VSI respectively, which own better performances to detect process mean shift[1][13]. However, with Costa[2] comparing performances of process mean shift detection for VSS, VSI and DS $\overline{X}$ control chart, the best performances occur at the DS $\overline{X}$ control chart. On account of this reason, the DS $\overline{X}$ control chart is an arresting subject in our study.

Before using the DS $\overline{X}$ control chart, we have to design five parameters for this control chart: widths of two sets of control limits, sample sizes of two stages and widths of a set of warning limits. During process monitoring, various designs will cause different statistic performances for the DS $\overline{X}$ control chart. After considering statistic viewpoints, Irianto and Shinozaki[7] selected the Single-Objective programming method to determine the optimal design of the DS $\overline{X}$ control chart. Additionally, while referring to methods by Irianto and Shinozaki[7] (namely I&S model in the following sections), He *et al.*[4][5][6] designed various DS $\overline{X}$ control charts. In the I&S model, the expected sample size under in-control process becomes the objective function of the model and the best design of the DS $\overline{X}$ control chart determined by subject to risk probabilities of two process states in control charts. However, according to designs of the DS $\overline{X}$ control chart by He *et al.*[5], Hsu[11] listed the expected sample size of out-of-control process and found failure in I&S methods that the expected sample size for detection of out-of-control process cannot be reduced and its sample size is even larger than that of the Shewhart's control chart. Thus, using I&S methods cannot find the optimal design of the DS $\overline{X}$ control chart.

On the basis of above-mentioned reasons, the concept to minimize the expected sample size of out-of-control process and the multi-objective programming method are used to modify the I&S statistical design model in this study. Finally, we will use statistic performance to illustrate the differences of our modified model and I&S methods.

## II. DS $\overline{X}$ CONTROL CHART

### *A. Principles of the DS $\overline{X}$ control chart*

The DS $\overline{X}$ control chart proposed by Daudin[9] integrated two Shewhart's $\overline{X}$ control charts with different widths of control limits for process monitoring and added warning limits in the first-stage control chart. The graphic view of the DS $\overline{X}$ control chart is shown in Fig. 1 that the process observations are transformed to a standard normal distribution. Therefore, the central lines of control charts in two stages are 0. $L_1$ and $L_2$ are the width of control limits in the first-stage control chart and the second-stage control chart respectively. $W$ is the width of warning limits in the first-stage control chart.

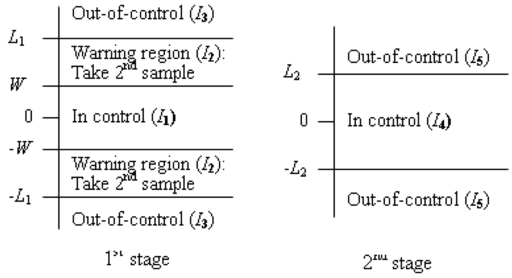


Fig. 1. Graphic view of DS $\overline{X}$ control chart

Under an assumption that process state is in control, each control region in Fig. 1 can be defined as $I_1 = [-W, W]$, $I_2 = [-L_1, -W) \cup (W, L_1]$, $I_3 = (-\infty, -L_1] \cup [L_1, +\infty)$ and $I_4 = [-L_2, L_2]$ and $I_5 = (-\infty, -L_2] \cup [L_2, +\infty)$.

Daudin[9] has explicitly illustrated the control procedure of the DS $\overline{X}$ control chart. First, take a small sample size, $n_1$, and calculate the sample mean $\overline{X_1}$. Then, calculate $z$ using a normalize approach, that is, $z = \sqrt{n_1}\left(\overline{X_1} - \mu\right)/\sigma$. If $z$ falls in $I_3$, it will be considered as an out-of-control process. If $z$ falls in $I_1$, it will be deemed an in-control process. For the case that $z$ falls in $I_2$, it is necessary to conduct a second-time sampling and monitor processes with the second-stage control chart. With the second-time sampling occurring, the sample size will be $n_2$ (usually $n_1 < n_2$) and the sample mean $\overline{X_2}$ for the second-time sampling needs to be calculated. Then, the total sample mean $\overline{Y}$ for both sampling stages can be calculated with $\overline{Y} = \left(n_1\overline{X_1} + n_2\overline{X_2}\right)/(n_1 + n_2)$. Afterward, normalize value of $\overline{Y}$ will be represented with $z_2$, $z_2 = \sqrt{n_1 + n_2}\left(\overline{Y} - \mu\right)/\sigma$. When $z_2$ falls in $I_4$, it will be considered as an in-control process. Otherwise, it will be regarded as an out-of-control process.

Irianto and Shinozaki[7] assumed a normal distribution for the observation of process and displayed calculations of Type I error probability and Type II error probability. Both He *et al.*[5] and Hsu[9] adopted same methods to calculate probabilities of Type I and Type II errors in DS $\overline{X}$ control chart and evaluated statistic performances of process monitoring. Supposing the real process is an in-control state but the sample mean falls in $I_3$ or $I_5$, it will be concluded that is called Type I error or false alarm and its probability can be calculated by following equation:

$$\begin{aligned}\alpha &= P(z \in I_3) + P(z \in I_2) \times P(z_2 \in I_5) \\ &= P(z \in I_3) + P(z \in I_2) \times [1 - P(z_2 \in I_4)] = 1 - [\Phi(L_1) - \Phi(-L_1)] \\ &+ \int_{z \in I_2} \left\{1 - \left[\Phi\left(cL_2 - z\sqrt{\frac{n_1}{n_2}}\right) - \Phi\left(-cL_2 - z\sqrt{\frac{n_1}{n_2}}\right)\right]\right\}\varphi(z)dz\end{aligned} \tag{1}$$

$\Phi$ is the cumulative distribution function of a standard normal distribution. $\varphi$ is the probability density function of a standard normal distribution. $c = \sqrt{(n_1 + n_2)/n_2}$. $P(\bullet)$ is the probability that $z$ falls in some region. By principles of the DS $\overline{X}$ control chart, it is not consequential to addition to sample size $n_2$ for each sampling. The probability of addition to sample size $n_2$ is $P(z \in I_2)$. Using this probability can estimate the expected sample size $E_0(N)$ of the DS $\overline{X}$ control chart under in-control processes.

$$\begin{aligned}E_0(N) &= n_1 + n_2 P(z \in I_2) \\ &= n_1 + n_2 \times [\Phi(-W) - \Phi(-L_1) + \Phi(L_1) - \Phi(W)]\end{aligned}$$
(2)

In case of process variation that the process mean is shifted to $\mu_1 = \mu_0 + \delta\sigma$ from initial mean $\mu_0$, where $\delta$ is the shift size of process mean (by equation $\delta = (\mu_1 - \mu_0)/\sigma$), the process will be wrong determined as an in-control state that is called Type II error. The probability of this false determination is

$$\begin{aligned}\beta_\delta &= P(z \in I_1^*) + P(z \in I_2^*) \times P(z_2 \in I_4^*) = \Phi\left(L_1 + \delta\sqrt{n_1}\right) \\ &- \Phi\left(-L_1 + \delta\sqrt{n_1}\right) + \int_{z \in I_2^*} \left[\Phi\left(cL_2 + c\delta\sqrt{n_1 + n_2} - z\sqrt{\frac{n_1}{n_2}}\right)\right. \\ &\left. - \Phi\left(-cL_2 + c\delta\sqrt{n_1 + n_2} - z\sqrt{\frac{n_1}{n_2}}\right)\right]\varphi(z)dz\end{aligned} \tag{3}$$

where $I^*$ means the control region that process mean has shifted, $I_2^* = \left[-L_1 + \delta\sqrt{n_1}, -W + \delta\sqrt{n_1}\right) \cup \left(W + \delta\sqrt{n_1}, L_1 + \delta\sqrt{n_1}\right]$. The expected sample size $E_\delta(N)$ for a shift size $\delta$ is

$$\begin{aligned}E_\delta(N) &= n_1 + n_2 P(z \in I_2^*) = n_1 + n_2 \times \left[\Phi\left(-W + \delta\sqrt{n_1}\right)\right. \\ &\left. - \Phi\left(-L_1 + \delta\sqrt{n_1}\right) + \Phi\left(L_1 + \delta\sqrt{n_1}\right) - \Phi\left(W + \delta\sqrt{n_1}\right)\right]\end{aligned} \tag{4}$$

Average run length (ARL) is usually taken as an index to evaluate the statistic performances of process monitoring in control charts. It is defined as the average times to detect process mean shift. Thus, the statistic performances of process monitoring in control charts can be more completely shown through ARL than $\alpha$ and $\beta$. For in-control process, the average run length is written as $ARL_0$, $ARL_0 = 1/\alpha$. When process mean had shifted, and its shift size is $\delta$, the necessary average sampling times for detecting shift in a control chart can be expressed as $ARL_\delta = 1/(1-\beta_\delta)$.

*B. The I&S statistical design*

By statistic viewpoints, Irianto and Shinozaki[7] constructed a nonlinear programming model to solve the designs of the DS $\overline{X}$ control chart. The major consideration in the I&S model is to emphasize economic benefits of costs at sampling inspections, meanwhile, objective function of this model is to minimize $E_0(N)$. For better performances in process monitoring of control charts, $\alpha$ and $\beta$ have to be restricted to less than some specific values. The I&S model is described as follows,

Min $E_0(N) = n_1 + n_2 P(z \in I_2)$

Subject to

$\alpha \le \alpha^*$ (5)

$\beta_\delta \le \beta^*$ (6)

$1 < n_1 < n_2 \quad n_1, n_2 \in$ Integer (7)

$0 < W < L_1$ (8)

$0 < L_1 < L_{1u}$ (9)

$L_2 > 0$

In Eq. (5) and (6), $\alpha$ and $\beta_\delta$ must be less than the maximal specific values $\alpha^*$ and $\beta^*$ respectively. Furthermore, in accordance with statistic viewpoints, these two equations restrict performances of control charts. In two stages of Eq. (7), the sample sizes must be positive integer and $n_1 < n_2$, where $n_1$ is greater than 1, else, the first stage control chart will become a control chart of individual measurement. In Eq. (8) and (9), $L_1$ must be greater than $W$ and keeps positive. Besides, $L_1$ cannot exceed the assigned upper limit $L_{1u}$ or else $L_1$ will increase unlimitedly during processes for solutions and the sensitivity of detecting process mean shift in the first stage control chart will also decrease.

The optimal design of a DS control chart provided by He *et al.*[4][5][6] was based on this model. But in this model, the sample size $E_\delta(N)$ is not restricted during process mean shift, hence the necessary sample size in detecting process shifts cannot be reduced.

## III. MODIFIED MODEL AND SOLUTION

*A. Modified model*

Considering a concept of minimizing sample sizes for process shifts, we add an objective function such as Eq. (10) into the original I&S statistic design model to minimize $E_\delta(N)$. ARL showing better statistic performances in control charts than $\alpha$ and $\beta_\delta$, we select ARL as a norm to restrict capabilities of process monitoring of control charts. The modified model becomes

Min $E_0(N) = n_1 + n_2 P(z \in I_2)$

Min $E_\delta(N) = n_1 + n_2 P(z \in I_2^*)$ (10)

Subject to

$ARL_0 \ge ARL_0^*$ (11)

$ARL_\delta \le ARL_\delta^*$ (12)

$1 < n_1 < n_2 \quad n_1, n_2 \in$ Integer

$0 < W < L_1$

$0 < L_1 < L_{1u}$

$L_2 > 0$

In addition to the original objectives function in I&S model, an extra objective function to minimize $E_\delta(N)$, as shown in Eq. (10), is newly added into the modified model. Eq. (11) and (12) limit the expected sampling times of false alarm occurrence and detecting process mean shift. It should be noted that $ARL_0$ should be greater than the minimal tolerance value $ARL_0^*$ because the less occurrence frequency of false alarms will be better. In addition, $ARL_\delta$ should be less than the specific value $ARL_\delta^*$ since the faster detecting is better when process shift has occurred. These two equations have identically statistical meanings with Eq. (5) and (6) of the I&S model. Other constraints in this model are identical to those in the I&S model.

The modified model is a double-objective mathematical programming model, hence the techniques for solutions in conventional single-objective programming model will inadequate for this model. The weight method proposed by Zadeh[10] assigned a weight to each objective function and then combined them as a single objective function using the weight average to obtain solutions with this single objective method. Each weight value represents the importance degree of the objective function and the sum of weights of all objective functions will be 1. As regards our modified model, the weight method is adopted to integrate all objective functions that $U$ is the weight value of $E_0(N)$. The combined objective function $f$ is written as follows,

Min $f = U[n_1 + n_2 P(z \in I_2)] + (1-U)[n_1 + n_2 P(z \in I_2^*)]$ (13)

Because our modified model belongs to a nonlinear programming method and mixes continuous-discrete variables and discontinuous and nonconvex solution space. The genetic algorithms (GA) being adequate to solve this type of problems, He *et al.*[4][5][6] applied it to solve the I&S model.

*B. Using genetic algorithms for solutions*

Genetic Algorithm (GA) is a technology of global optimization. For non-linear programming model, mixed continuous-discrete variables or discontinuous and nonconvex solution space, the application of GA offers immediate optimal solution for a model.

GA executes global search with multiple solutions simultaneously. If the number of population is $m$, it means that there are $m$ sets of chromosome searching for optimal solutions at the same time. The solution procedure of GA consists of the following key steps: (1) The generation of initial solutions for decision variables randomly based on number of population $m$; (2) evaluation of the fitness for these solutions; (3) selection and crossover of chromosomes, where the rate that chromosomes of better fitness are chosen is higher; (4) part of the chromosomes experience mutation; and (5) next generation of solution are generated and repeat from step (2). The evolution goes on and on, and eventually a convergent solution will be obtained.

In this paper, GA is used to determine the solutions for the design of DS. The solutions of a set of decision variables, $n_1$, $n_2$, $L_1$, $W$ and $L_2$ are considered a genes, and the initial solutions for multiple sets of decision variables are generated based on the number of population $m$. The solution that satisfies all the limitations in Eq. (11) and (12) is called a feasible solution. The Eq. (13) is calculated using each of the feasible solutions. Smaller values of Eq. (13) mean better fitness for feasible solutions. By iterating the above GA solution procedure, the optimal DS design is achieved when the Eq. (13) of all feasible solutions converge to the same value.

Since Palisade[12] developed Evolver, the GA tool software attached to Microsoft Excel. In this study, Evolver 4.0 is selected for solutions of our modified model. The optimal configurations for number of chromosomes, crossover rate and mutation rate will be different according to various conditions. In view of this reason, to conduct repeated tests will find the best parameter values for solutions. With repeatedly testing conducted, the best parameters acquired for this study are number of chromosomes=100, crossover rate=0.7 and mutation rate with self adjustment.

## IV. COMPARISONS AND DISCUSSIONS

This section is aimed at comparisons in designs of the DS $\overline{X}$ control chart with I&S model and our modified model. The expected sample size of the DS $\overline{X}$ control chart will vary with different shift sizes. Hence, to select an adequate and correct shift size for solutions and guarantee a minimal expected sample size for all shift detection is necessary. By computer programming simulation, 100 thousand different sets of designs of the DS $\overline{X}$ control chart will be generated randomly and expected sample sizes with various shift sizes can be calculated. Matlab7 is selected as a programming tool for simulation and results are shown in Fig. 2.

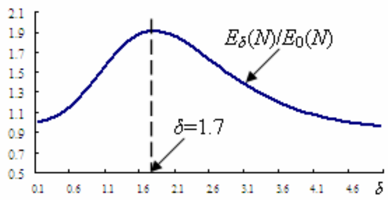


Fig. 2. A simulation of $E_\delta(N)$ for DS $\overline{X}$ charts

The curves in Fig. 2 represent average numbers of simulating 100 thousand sets of $E_\delta(N)/E_0(N)$. The larger the $E_\delta(N)$ is, the bigger the $E_\delta(N)/E_0(N)$ becomes. In Fig. 2, the maximal $E_\delta(N)$ occurs at $\delta$ =1.7 or so. Thus, to reduce $E_\delta(N)$, we select $\delta$=1.7 for solutions and compare solutions acquired with different $\delta$=1.0 and 3.0. The criterion for solutions is ARL from the standard Shewhart's control chart with a sample size 5 (ARL is 4.5, 1.27 and 1.00 respectively for $\delta$=1.0, 1.7 and 3.0). In addition, to contrast differences of expected sample size $E(N)$ for different models, we assume $U$=0.5 and $L_{1u}$ =4.5, and then use GA to solve designs of the DS $\overline{X}$ control chart for I&S model and our modified model respectively. The control chart designs and their ARL and $E(N)$ of several shift sizes are shown in Table I.

TABLE I.
ARL and $E(N)$ for DS $\overline{X}$ control chart designs of two models

| Solved for given $\delta$ | Model | DS $\overline{X}$ chart designs $n_1$ | $n_2$ | $L_1$ | $W$ | $L_2$ | | $\delta$ 0 | 0.5 | 1 | 1.5 | 1.7 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta$=1.0 | Modified | 2 | 6 | 3.268 | 1.980 | 2.759 | *ARL* | 370.09 | 30.67 | 4.50 | 1.84 | 1.51 | 1.25 | 1.01 |
| | | | | | | | $E(N)$ | 2.28 | 2.60 | 3.53 | 4.58 | 4.82 | 4.83 | 2.92 |
| | I&S | 2 | 6 | 3.328 | 1.706 | 2.849 | *ARL* | 370.73 | 26.63 | 3.68 | 1.57 | 1.33 | 1.15 | 1.01 |
| | | | | | | | $E(N)$ | 2.52 | 2.97 | 4.15 | 5.28 | 5.48 | 5.36 | 3.05 |
| $\delta$=1.7 | Modified | 3 | 4 | 3.293 | 2.095 | 2.880 | *ARL* | 370.02 | 28.75 | 3.89 | 1.51 | 1.27 | 1.09 | 1.00 |
| | | | | | | | $E(N)$ | 3.14 | 3.42 | 4.21 | 4.80 | 4.76 | 4.39 | 3.11 |
| | I&S | 3 | 6 | 3.199 | 2.134 | 2.849 | *ARL* | 370.49 | 25.62 | 3.53 | 1.49 | 1.27 | 1.10 | 1.00 |
| | | | | | | | $E(N)$ | 3.19 | 3.56 | 4.64 | 5.43 | 5.35 | 4.82 | 3.13 |
| $\delta$=3.0 | Modified | 2 | 6 | 3.170 | 1.912 | 2.907 | *ARL* | 370.14 | 32.89 | 4.48 | 1.77 | 1.47 | 1.22 | 1.00 |
| | | | | | | | $E(N)$ | 2.33 | 2.67 | 3.62 | 4.61 | 4.80 | 4.72 | 2.79 |
| | I&S | 2 | 6 | 3.283 | 1.342 | 2.991 | *ARL* | 370.39 | 24.57 | 3.16 | 1.36 | 1.19 | 1.08 | 1.00 |
| | | | | | | | $E(N)$ | 3.07 | 3.67 | 5.00 | 5.96 | 6.00 | 5.64 | 3.00 |
| Standard Shewhart's | | 5 | | 3.00 | | | *ARL* | 370.40 | 33.40 | 4.50 | 1.57 | 1.27 | 1.08 | 1.00 |

Comparing results of the two models in Table I, we can find that designs of the control chart with I&S model for small shifts are better than our modified model and standard Shewhart's control chart. Notwithstanding that ARL for other shifts with I&S model is close to that of the standard Shewhart's control chart, the sample size is still greater than that of the standard Shewhart's control chart. As regards designs of the control chart using our modified model, the sample size is lower than that of the standard Shewhart's control chart and ARL of common shift size is similar.

The following is about comparisons in designs of the DS $\overline{X}$ control chart with our modified model for $\delta$=1.0, 1.7 and 3.0. From solutions for designs of control charts at $\delta$=1.0 and 3.0, the detection capability at $1.5 \le \delta \le 2$ is the weakest and the sample size at $1.5 \le \delta \le 2$ is larger than other situations. However, the detection capabilities and the sample sizes for $\delta \ge 3$ and $\delta \le 1$ are superior to those of the standard Shewhart's control chart. In regard to

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designs of the DS $\overline{X}$ control chart for solving $\delta$=1.7, not only the sample size but also capability at shift detection is optimal.

The following step is analyses that $U$ and $L_{1u}$ affect upon control chart designs. Because each $L_1$ derived from our modified model is less than 3.2, the assumption, $L_{1u}$ =4.5, is reasonable. During processes for control chart designs, any $L_{1u}$, larger than 3.2, will not affect solutions. Data in Table II are designs of the DS $\overline{X}$ control chart at $\delta$=1.7 under various weight conditions. It can be clearly seen that either ARL or $E(N)$ displays little variation, hence the weight value $U$ will not substantially affect solutions of the DS $\overline{X}$ control chart. That is, $U$ can be an arbitrary value during processes for solved the control chart designs.

For some shift size in this example, the detection capabilities in designs of control charts derived from I&S model is worse than our modified model and standard Shewhart's control chart (especially for conditions 1.5≤$\delta$≤2). However, using our modified model to solve $\delta$=1.7 can effectively reduce the sample size for each shift size. Thus, according to the above comparisons, designs of the DS $\overline{X}$ control chart and statistic performances using our modified model are better than those using I&S model and Shewhart's control chart.

TABLE II.
A sensitivity analysis for modified $U$ in solved case of $\delta$=1.7

| Weights | DS $\overline{X}$ chart designs | | | | | | $\delta$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n_1$ | $n_2$ | $L_1$ | $W$ | $L_2$ | | 0 | 0.5 | 1 | 1.5 | 1.7 | 2 | 3 |
| $U$=0.1 | 3 | 4 | 3.246 | 2.083 | 2.921 | $ARL$ | 371.02 | 29.53 | 3.94 | 1.51 | 1.27 | 1.09 | 1.00 |
| | | | | | | $E(N)$ | 3.14 | 3.42 | 4.19 | 4.75 | 4.70 | 4.32 | 3.10 |
| $U$=0.3 | 3 | 4 | 3.177 | 2.070 | 3.000 | $ARL$ | 370.97 | 31.19 | 4.17 | 1.52 | 1.27 | 1.09 | 1.00 |
| | | | | | | $E(N)$ | 3.15 | 3.42 | 4.06 | 4.68 | 4.60 | 4.22 | 3.08 |
| $U$=0.5 | 3 | 4 | 3.292 | 2.088 | 2.884 | $ARL$ | 370.02 | 28.75 | 3.89 | 1.51 | 1.27 | 1.09 | 1.00 |
| | | | | | | $E(N)$ | 3.14 | 3.42 | 4.21 | 4.80 | 4.76 | 4.39 | 3.11 |
| $U$=0.7 | 3 | 4 | 3.287 | 2.095 | 2.884 | $ARL$ | 370.11 | 28.87 | 3.90 | 1.52 | 1.27 | 1.10 | 1.00 |
| | | | | | | $E(N)$ | 3.14 | 3.41 | 4.19 | 4.79 | 4.75 | 4.38 | 3.11 |
| $U$=0.9 | 3 | 4 | 3.245 | 2.050 | 2.936 | $ARL$ | 371.16 | 28.78 | 4.24 | 1.50 | 1.25 | 1.09 | 1.00 |
| | | | | | | $E(N)$ | 3.16 | 3.41 | 3.88 | 4.80 | 4.73 | 4.34 | 3.11 |

## V. CONCLUSION

The statistical design model of the DS $\overline{X}$ control chart by Irianto and Shinozaki[7] displayed excellent performance at process shift detection but failed to efficaciously curtail sample sizes. In this study, the model proposed by Irianto and Shinozaki[7] is modified and the sample sizes for out-of-control process is added into this model for solutions of new designs of the DS $\overline{X}$ control chart. Comparing results from both models, we find that designs of the DS $\overline{X}$ control chart with our modified model can lower sample sizes without changing original detection capability that has apparently improved drawbacks existing in methods provided by Irianto and Shinozaki[7]. Additionally, by simulation results, a large sample size will occur when the DS $\overline{X}$ control chart is detecting process mean shifts with 1.5 to 2 standard deviations. Meanwhile, by various solutions for different shift sizes, the design of the DS $\overline{X}$ control chart with the optimal statistic performances can be acquired according to solutions at shift size=1.7.

He *et al.*[4][5][6] adopted methods proposed by Irianto and Shinozaki[7] as well to solve several designs of the DS $\overline{X}$ control charts. As regards our modified model, how to apply it to evaluate performances of other DS-type control charts deserves further exploration. Weight method is used to solve our model. However, this weight method that minimizes the integrated objective function value cannot simultaneously minimize $E_0(N)$ and $E_\delta(N)$ so that the solution derived from this method might be regarded as an approximate solution only. Nevertheless, having been an uncomplicated method, it has been extensively applied to solutions of the multi-objective programming model. In further study, we suggest that using other optimal technology of multi-objective programming solves our model to obtain better designs of DS $\overline{X}$ control charts.

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