Research on Gear Plunge Shaving for Gears with Tooth Modifications

Jia-Hung Liu Shinn-Liang Chang

Abstract—**Plunge shaving is the most advanced gear finishing technique which only needs radial infeed. Its advantages include increased productivity, accuracy, long tool life, and a simple machine structure. For the plunge shaving method, the gear tooth modification only depends on the surface geometry of the plunge shaving cutter. The analytical description of the gear with tooth modifications is firstly constructed by B-spline surface fitting. Then, the grinding wheel profile is parameterized and optimized for minimizing the surface deviations of theoretical and ground (from re-sharpening machine) tooth surfaces of the plunge shaving cutter, in which the topographic error has been reduced. The cutting trace of plunge shaving cutter has also been analyzed so that the shaving efficiency can be improved.**

Index Terms—**Gear plunge shaving, Cone grinding wheel, Topographic error, Shaving cutter serration.**

I. Introduction

Gears are the most important components in transmission systems. Modifications of gear teeth can accommodate errors and deformations encountered in the manufacture, assembly, and operation of gear pairs. Litvin [1] provided a double crowned gear modified both in lead and in profile with improved transmission error. Wagaj and Kahraman [2] investigated the durability of helical gear affected by tooth modifications. Kahraman et al. [3] presented a gear wear model analyzing the influences of tooth modification.

Gear shaving is one of the most efficient and economical processes for gear finishing after hobbing or shaping. Among the four basic shaving methods, plunge shaving is the most advanced gear finishing technique which only needs radial infeed. Its advantages include increased productivity, accuracy, long tool life, and a simple machine structure [4]. The basic meshing condition of 3D crossed-axis helical gear pair was first derived by Litvin [5], and it has been widely adopted as the fundamental assumption for simulation of gear shaving.

In recent years, how to induce gear tooth modification by shaving has become an important subject of research. Chang et al. and Hung et al. [6]-[8] derived the mathematical model of traditional shaving machine and investigated the influences of parameters on tooth lead modifications. Litvin et al. [9] proposed a method for shaving gears with double crowning by CNC shaving machine. For shaving methods other than plunge shaving, the gear tooth modification is accomplished by tooth modifications of the shaving cutter and the coordinated motions between cutter and gear. For the plunge shaving method, however, it only depends on the surface geometry of the plunge shaving cutter. Focusing on the surface geometry of shaving cutter, what's really significant is precision of the region between SAP. (start of active profile) and EAP. (end of active profile) as shown in Fig. 1. In gear shaving, SAP. of cutter tooth shaves the gear tooth tip while EAP. shaves the gear tooth root. Traditionally, the cutter surface geometry results from a cutter re-sharpening machine by trial and error, which is very time-consuming.

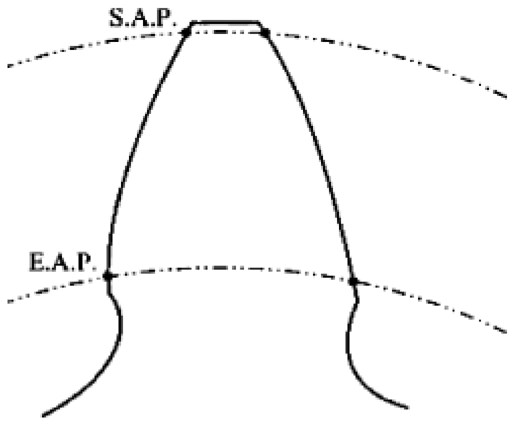

Fig. 1 SAP. and EAP. of a shaving cutter tooth.

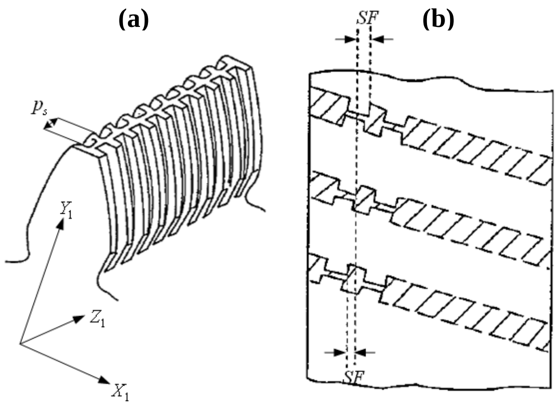

Fig. 2. Design parameters of plunge shaving cutter serration (a) serration pitch (b) serration displacement.

Moreover, the shaving cutter tooth surfaces have serrations extending from the top land to the root fillet of the teeth whose sharp edges exert a cutting action on the work

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gear due to the relatively lengthwise sliding motion. Serrations are an important feature for all shaving cutters, but for plunge cutters their importance is vital. The serrations related to such type of cutter must have a helical pattern, as shown in Fig. 2, and they have to be manufactured with a very high precision CNC slotting machine in order to obtain the best results. Thus, the surface roughness of the work gear after shaving is affected primarily by the arrangement of these serrations. In this paper, the analytical description of the gear with tooth modifications is first constructed by B-spline surface fitting. Then, the grinding wheel profile is parameterized and optimized for minimizing the surface deviations of theoretical and ground (from re-sharpening machine) tooth surfaces of the plunge shaving cutter. Efficiency is greatly improved by avoiding the traditional trial and error method. Then, based on the optimized cutter surface, a method for optimizing the serration displacement is proposed to improve the cutting efficiency.

II. Surface Interpolation of the Modified Gear Tooth Surface

To integrate the modified gear tooth surfaces into the analytical process, especially for those with both lead and profile modifications, B-spline surface interpolation is selected for its ease of manipulation. In practice, the sampling points of the modified surface can be obtained by CMM (Coordinate Measuring Machine) or from other sources. In this paper, for studying purposes, the most commonly used numerical model is adopted for generation of interpolating points: the tooth flank is modified in profile (root to tip) and lead (side to side) directions independently as shown in Fig. 3 [2]. The magnitude at the tip a_t and the gear roll angle at the start α_t define the boundaries of the tip relief. Between these two points, the profile follows a linear trajectory. Similarly, the magnitude a_r and the starting roll angle α_r define the starting point of a root modification. The amount of lead crowning is denoted by h.

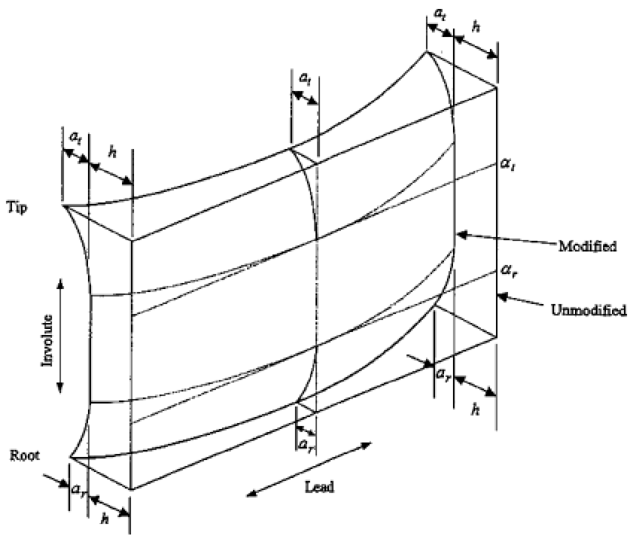

Fig. 3. Model of gear tooth modifications [2].

A B-spline representation enables the simulation of surface irregularities and the control of small tooth geometric modifications, such as rounding and reliving. Given a grid of sampling points $D_{k,\ell}$ ($0 \le k \le m$ and $0 \le \ell \le n$) and orders p and q (degrees $p-1$ and $q-1$), it can be represented as below [10]:

$$D_{k,\ell} = \sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(s_k) N_{j,q}(t_\ell) \mathrm{P}_{i,j} \tag{1}$$

, where s_k 's and t_l 's are the chosen parameter values; $N_{i,p}(s_k)$ ($N_{j,q}(t_\ell)$) is the i-th (j-th) B-spline basis function of order p (q); and $P_{i,j}$'s ($0 \le i \le m$ and $0 \le j \le n$) are the control points. Once the numbers of sampling points *m* and *n* are selected, then the B-spline orders *p* and *q* are limited by (2):

$$p \ge \frac{3m+1}{m+1}, \quad q \ge \frac{3n+1}{n+1} \tag{2}$$

, that is, $p \ge 3$ and $q \ge 3$.

Solving (1), $\mathrm{P}_{i,j}$'s can be obtained, and the interpolated B-spline surface $\sum_2^I$ can be represented as follows:

$$\textstyle\sum_2^I(u,v) = \displaystyle\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) \mathrm{P}_{i,j} \tag{3}$$

, where u and v denote the surface parameters; *2* denotes the surface of gear tooth; and *I* denotes the surface obtained by interpolation.

Table 1 Basic data of the target gear tooth surface.

Gear data	
Normal module m_n	1.5
Diameter of base circle d_{b2}	119.618*mm*
Diameter of addendum circle d_{add2}	126.71*mm*
Diameter of root circle d_{r2}	119.33*mm*
Diameter of pitch circle d_{p2}	123.915*mm*
Normal pressure angle in pitch circle α_{pn2}	14.5°
Face width fw_2	18*mm*
Gear tooth number Z_2	79
Helix angle in pitch circle β_{p2}	17°
Normal circular tooth thickness s_{pn2}	2.32*mm*

Table 2 Data of gear tooth modifications of the target surfaces.

Parameter	a_t	a_t	a_t	a_t	a_t
$\sum_2^I$	6e-3mm	31.8°	6e-3mm	28.2°	6e-3mm

According to the data listed in Table 1 and 2, the result of interpolation is shown in Fig. 4, in which the solid lines denote the standard involute tooth surface (without modifications) while the doted lines denote the interpolated surface. It presents the double crowned (profile and lead modified) gear tooth surface, and the values in the figure denote the variations of circular tooth thickness between standard and double crowned surfaces.

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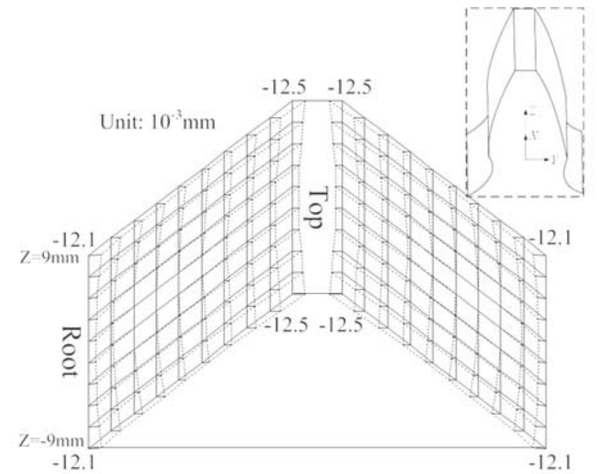

Fig. 4. Validations of the interpolated gear tooth surface.

III. Optimization of Topographic Errors of Gear Plunge Shaving Cutter

The tooth surface of the shaving cutter is usually finished last using a cone grinding wheel on the shaving cutter re-sharpening machine. Because the topographic accuracy of the plunge shaving cutter maps directly onto the work gear, it is important to identify the topographic error of the ground tooth surfaces $\sum_1^G$ in comparison to the theoretical $\sum_1^T$ where 1 denotes the surface of shaving cutter tooth; G and T denote the surfaces derived from the re-sharpening machine and the interpolated surfaces, respectively.

The basic meshing condition for the crossed helical gear set [5] is used to calculate the basic geometric data for the shaving cutter. It needs the following eight basic items: the tooth number Z_1, the normal circular tooth thickness s_{pn1}, the helix angle β_{p1} of the shaving cutter; the tooth number Z_2, the normal circular tooth thickness s_{pn2}, the helix angle β_{p2} of the work gear, and the normal module m_{pn} and pressure angle α_{pn}, of the shaving cutter and work gear. Fig. 5 [11] shows the coordinate system of the CNC shaving machine. Considering coordinate transformation

$$\begin{aligned} \mathbf{r}_1 &= [x_1 \quad y_1 \quad z_1 \quad 1]^T = \mathbf{M}_{12}(\phi_2)\mathbf{r}_2(R_2, Z_2) \\ \mathbf{n}_1 &= [n_{x1} \quad n_{y1} \quad n_{z1}]^T = \mathbf{L}_{12}(\phi_2)\mathbf{n}_2(R_2, Z_2) \end{aligned} \tag{4}$$

and meshing equation

$$f(R_2, Z_2, \phi_2) = \mathbf{n}_h \cdot \mathbf{v}_h^{(12)} = 0 \tag{5}$$

simultaneously, the theoretical tooth surface of shaving cutter $\sum_1^T$ can be derived from surface $\sum_2^I$. $\mathbf{M}_{12}$ and $\mathbf{L}_{12}$ are matrices for transforming position and unit normal vectors ($\mathbf{r}_2$ and $\mathbf{n}_2$) of surface $\sum_1^T$ from coordinate system S_2 (gear) to S_1 (shaving cutter), and $\mathbf{v}_h^{(12)}$ denotes the vector of relative velocity on auxiliary coordinate system S_h.

Likewise, as shown in Fig. 6 [11], the ground surfaces of shaving cutters $\sum_1^G$ can be obtained by coordinate transformations from S_g (grinding wheel) to S_s (shaving cutter)

$$\begin{aligned} \mathbf{r}_s &= [x_s \quad y_s \quad z_s \quad 1]^T = \mathbf{M}_{sg}\mathbf{r}_g \\ \mathbf{n}_s &= [n_{xs} \quad n_{ys} \quad n_{zs}]^T = \mathbf{L}_{sg}\mathbf{n}_g \end{aligned} \tag{6}$$

and the meshing equation

$$g(u_g, \theta_g, \phi) = \mathbf{n}_m \cdot \mathbf{v}_m^{(sg)} = 0 \tag{7}$$

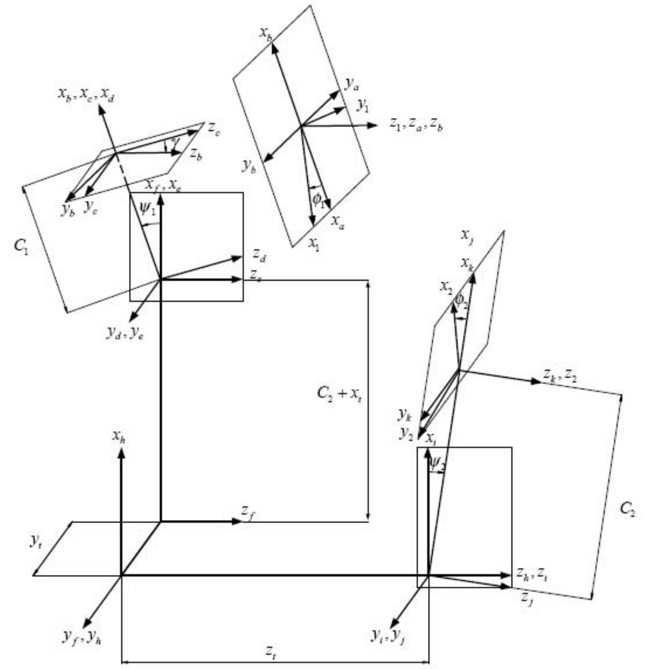

Fig. 5. Coordinate systems of gear shaving machine [11].

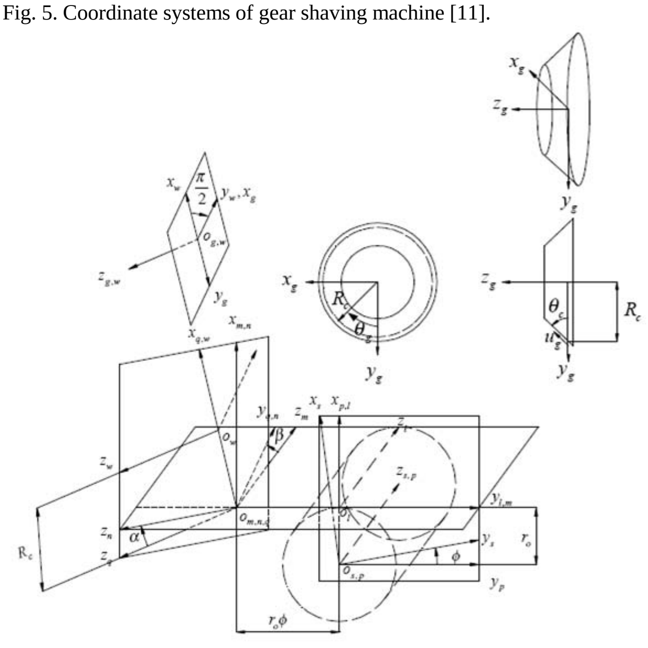

Fig. 6. Coordinate systems of shaving cutter re-sharpening machine [11].

For interpolated gear tooth surfaces $\sum_2^I$, the corresponding data of plunge shaving cutter and grinding wheel are presented in Table 3. Obtaining theoretical tooth surface $\sum_1^T$ as well as ground tooth surfaces $\sum_1^G$ through (4)-(7), the topographic errors are calculated by

$$e_{i,j}^{Topo} = r_{i,j}\left(\tan^{-1}\left(\frac{y_{i,j}^T}{x_{i,j}^T}\right) - \tan^{-1}\left(\frac{y_{i,j}^G}{x_{i,j}^G}\right)\right), \ i=1,2,\ldots,5, \ j=1,2,\ldots,9 \tag{9}$$

and illustrated in Fig. 7 by 5×9 grids of specified radiuses and

Z cross-sections, in which theoretical surfaces are presented in straight solid lines, while the corresponding ground ones are shown in dashed lines. From S.A.P to E.A.P. of the cutter tooth, errors are obvious that the grinding wheel needs to be dressed. Traditionally, it is modified back and forth for the desired accuracy, which is very time-consuming. In this section, optimization is performed by adjusting the cone angle of the grinding wheel θ_c and the profile of the grinding wheel.

TABLE 3 DATA OF CUTTER AND GRINDING WHEEL.

Plunge shaving cutter	
Normal circular tooth thickness s_{pn1}	2.464*mm*
Tooth number Z_1	139
Helix angle in pitch circle β_{p1}	20°
Face width fw_1	20*mm*
Diameter of start of active profile (SAP.)	225.922*mm*
Diameter of end of active profile (EAP.)	219.537*mm*
Operating center distance E_o	173.04*mm*
Operating crossed angle γ_o	3.002°
Grinding wheel and grinding machine	
Operating cone pitch radius R_c	350*mm*
Cone angle θ_c	10°
Pressure angle α	4.675°
Operating radius r_o	111.03*mm*

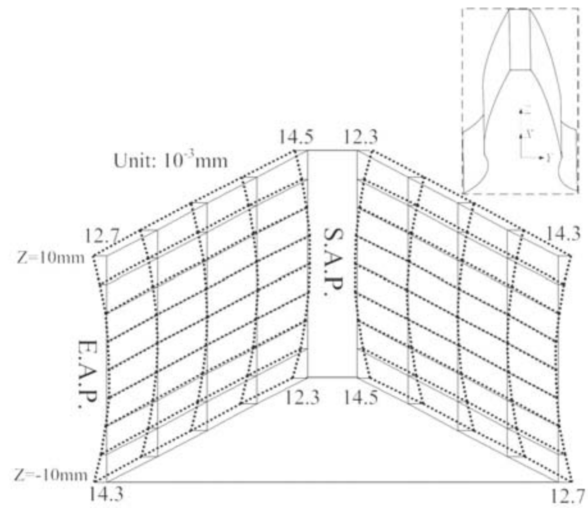

Fig. 7. Topographic errors between theoretical and ground shaving cutter tooth surfaces.

The grinding wheel is further parameterized as shown in Fig. 8. Coordinate system S_g' is attached to the unmodified profile. Within the effective length $L_A + L_B$ (L_A=4.2*mm*, L_B=5.5*mm*), the profile with four sections are defined by w_A, h_A, w_B and h_B. Represented in S_g', the four points A_2, A_1, B_1 and B_2 are fitted by a B-spline curve with order 4.

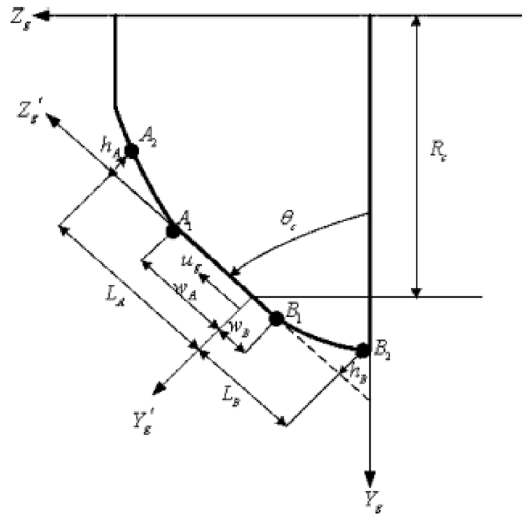

Fig. 8. Parameterized profile of grinding wheel for optimization.

TABLE 4 RESULTS OF OPTIMIZED PROFILE PARAMETERS.

	Initial design (*mm*)	Optimum design (*mm*)
w_A	0.1	0.561
h_A	0	0.056
w_B	0.1	0.538
h_B	0	0.235

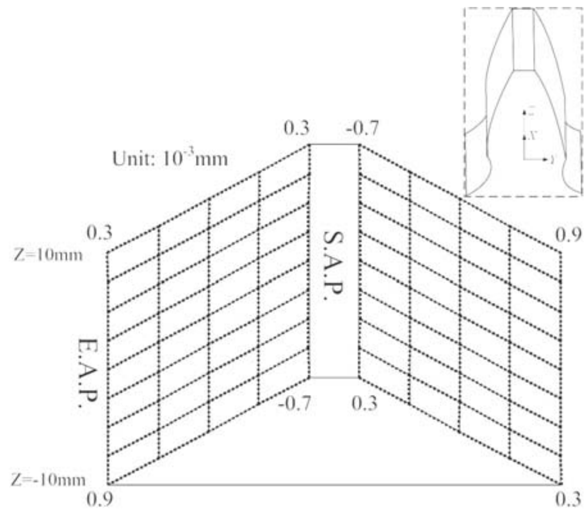

Fig. 9. Topographic errors between theoretical and ground shaving cutter tooth surfaces after optimization.

To improve the topographic errors between $\sum_1^T$ and $\sum_1^G$, the problem formulation is formulated:

find $\mathbf{x} = [\theta_c, w_A, h_A, w_B, h_B]$

that minimizes $\sum_{i=1}^{5}\sum_{j=1}^{9} e_{i,j}^{Topo}(\mathbf{x})$

subject to $e_{i,j}^{Topo} < 10^{-3}mm$, *i*=1,2,…,5, *j*=1,2,…,9,

and $0.5^\circ \le \theta_C \le 30^\circ$

$10^{-4} < w_A < L_A - 10^{-4}$; $0 < h_A < 3$; $10^{-4} < w_B < L_B - 10^{-4}$; $0 < h_B < 3$ (unit: *mm*)

The optimized profile parameters are presented in Table 4, and the cone angle converges to $\theta_c = 2.382^\circ$. The profile of the grinding wheel is considered straight sided initially. When it reaches the optimum, the profile is modified for conjugation to the shaving cutter. The topographic errors between $\sum_1^T$ and $\sum_1^G$ are shown in Fig. 9, where the errors are all controlled below 10^{-3}*mm*.

IV. DESIGN OPTIMIZATION OF THE GEAR PLUNGE SHAVING CUTTER SERRATIONS

Plunge shaving is characterized by a radial feed stroke without transverse feed. The serrations in consecutive shaving cutter teeth must be shifted longitudinally so that the cutting marks on the work gear move in a lengthwise direction, which can even the cutting residue and improve the surface roughness. As shown in Fig. 2, the design parameters for the serrations of the plunge shaving cutter are the serration pitch p_s and the serration shift *SF* between consecutive cutter teeth.

The trace of the plunge shaving cutter's cutting edge on the work gear during the cutting process is illustrated schematically in Fig. 10 [11]. At the beginning of a shaving cycle, the cutting edge presses into the work gear surface, moves on its tooth surface due to the relative motion, and then removes chips when contact stress reaches local ultimate stress. Therefore, the front cutting edge produces the desired cutting action.

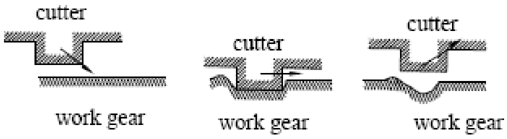

Fig. 10. Cutting path of the cutting edge on the work gear [11]

Besides, the cutting efficiency can be characterized by the "cutting-down ratio" r_{cd} , which is the ratio between the height of the second peak and that of the first peak, shown in Fig. 11. Smaller r_{cd} represents better cutting efficiency because most of the material can be removed in the first few cutting cycles.

According to the data of gear, shaving cutter and the grinding wheel listed in Tables 1 to 4, after optimizing the cutter tooth surface, the serrations can also be optimized to improve the cutting efficiency. The problem formulated as below:

find serration shift *SF*
that minimizes cutting-down ration r_{cd}
subject to $SF \times N_1 \geq p_s$

The value of *SF* goes from 0.243 *mm* to 0.376 *mm*. For the plunge shaving cutter with tooth number $N_1 = 137$ and serration pitch $p_s = 1.7$ *mm*, the cutting-down ratio has been improved form 0.318 to 0.242 (first peak= $3.1\mu m$, second peak= $0.7\mu m$).

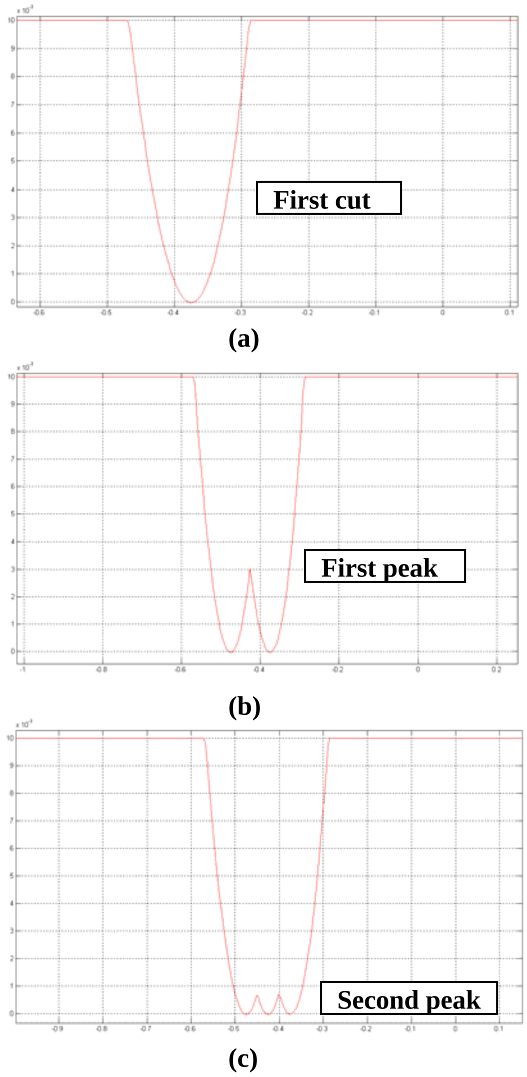

Fig. 11. Cutting residue (a) first cut (b) first peak (c) second peak.

V. CONCLUSION

For gear plunge shaving, the analytical descriptions of crowned gear and hence plunge shaving cutter have been constructed so that the grinding wheel can be optimized to minimized the topographic error. The cutting trace of plunge shaving cutter has also been analyzed so that the final real tooth forms can be predicted. The proposed mathematical model of design optimization can be use to calculate the optimum of grinding wheel profile and cutter serration, which can facilitate the design and manufacture of gear shaving cutter.

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Description of the changes:

1. Reference No. 11 is added to the reference list.
2. Reference No. 11 is added to the figure descriptions of Figs. 5, 6 and 10.

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