Thermal Spreading Resistance of Isoflux Hyperellipse on a Half-space

S.K. Bhullar and J. L. Wegner

Abstract- This paper aims to calculate the thermal spreading resistance for a hyperellipse on a half space, with uniform heat flux. The effect of shape and aspect ratio on a steady state averaged temperature and the centroid temperature based thermal spreading resistance is studied. The square root of source area and characteristic dimension are assumed as characteristic length scales to obtain the results. A compact correlation of averaged temperature based spreading thermal resistance is developed for hyperellipitical source area.

Keywords: aspect ratio, effect of shape, heat flux, thermal spreading resistance.

Nomenclature

x, y, z	Cartesian coordinates, m
$G(v, v)$	Function defined by eqn (14)
u	Dimensionless coordinate, (x/b)
v	Dimensionless coordinate, (y/b)
A	Contact area, m^2
A_0	Contact area in first quadrant m^2
a, b	Characteristic dimensions, m
T	Temperature, K
R_s	Spreading resistance, (K/W)
$\overline{R}_s$	Spreading resistance based upon average temperature, (K/W)
R_0	Spreading resistance based upon centroid temperature, (K/W)
m, n	Geometric parameter
q	Heat flux, (W/(m²))
Q	Total heat flow rate, W
$\sqrt{A}$	Square root of source area

Manuscript submitted on December 17, 2009.
S.K Bhullar and J. L. Wegner
Department of Mechanical Engineering,
University of Victoria, PO Box 3055,
Victoria, B.C. Canada V8W 3P6
E-mail: sbhullar@uvic.ca and jwegner@uvic.ca

Superscripts and Subscripts

n	Geometric parameter
a	Length scale along x-axis
$\sqrt{A}$	Square root of source area

Greek Symbols

α	Aspect ratio $(b/a) \le 1$
Γ	Gamma function
λ	Thermal conductivity, $(W/m)K$
ξ, η	Flux coordinates
μ	Dimensionless coordinate ξ/b
ν	Dimensionless coordinate η/b
ψ, ϕ	Spreading resistance parameters

1. Introduction

Thermal contact resistance plays an important role to evaluate the overall thermal behavior of systems in a wide range of fields, from the microelectronics to the nuclear industry. One of the major components of the contact resistance is the spreading/constriction resistance. It occurs whenever heat flows from a source to the sink with different cross-sectional areas. There is also a similar term called as constriction resistance which quite often understood similar to the spreading resistance. Constriction resistance occurs when heat flows from larger region to a narrow region and the spreading resistance occurs when heat flows from a small area to large area in contact [1]. Thermal spreading resistance theory finds widespread application in electronics cooling, both the board and at the end chip level and in heat sink applications. It also arises in thermal analysis of bolted joints and other mechanical connection resulting in discrete point of contact. Specifically one may encounter single or multiple thermal contacts of simple or arbitrary geometries on the surface of a half space. This paper concerns to study the thermal spreading resistance of an hyperellipitical contact.

2. Literature Review

Thermal spreading resistance is a problem commonly known in the thermal analysis of electronic packages. The early work was started by Kennedy [2] who investigated thermal resistance problem and derived analytical solutions for axi-symmetric models with uniform heat flux source on a finite cylinder. The studies on the thermal spreading resistance have a vast history. Several analytical solutions have been developed in the literature in order to calculate thermal spreading/constriction resistance according to contact shape, boundary conditions and length scale. A theoretical analysis is presented for predicting thermal constriction resistances of coaxial cylindrical contacts [3], a complete solution of the transient circular [4] and a circular annular contact area subjected to various boundary conditions [5]. Several studies are performed on a steady state thermal constriction resistance of a singly connected, planar contact area [6], a circular contact area on a circular flux tube[7], doubly-connected source area [8], thermal constriction resistance between smooth-sphere and rough flats in contact [9], a general solution for the thermal constriction resistance due to flux applied over circular portion of a compound disk [10], a transient thermal response of two semi-infinite bodies through a small circular contact area[11], spreading resistance of an isoflux rectangular flux channel [12] and a strip contact spot on a layer of material source for the heat-flux specified boundary conditions on the contact zone[13]. Recently a number of new solutions and application of thermal spreading resistance theory have been addressed in [14-15]. A review of thermal spreading resistance in compound and orthotropic systems is presented in [16]. Some correlation equations and important aspects of thermal spreading resistance theory are summarized in [17]. The other contributed work is a comparison of planar, axis-symmetric and 3-D spreading resistance [18], calculation of spreading resistance in heat sinks [19], thermal spreading resistance of rectangular sources and plates with non-unity aspect ratio [20], the spreading resistance between two parallel contacts [21] and a numerical modeling to understand the effects of thermal spreading resistance on the total resistance from junction to ambient for square and rectangular entities [22].

The first objective of this paper is to study the effect of shape and aspect ratio on a steady state thermal spreading resistances of a singly connected, planar contact area on an isotropic half space with uniform heat flux. For this purpose a thermal spreading resistance model is developed for an isoflux hyperellipse situated on a half-space. The elliptical geometry is chosen for the model because it provides more general geometrical configurations. They represent a finite-length plate when the aspect ratio $\alpha\left(=b/a\right)\rightarrow 0$ and a circular cylinder when the axis ratio $\alpha\rightarrow 1$. By increasing the aspect ratio α, we obtain narrower and narrower ellipses of contact, and at the limit $\alpha\rightarrow\infty$, we arrive at the case of contact of two cylinders with parallel axis. The surface of contact now a narrower rectangle. Thus, they cover a wide range of shapes. The details of calculations, a comparison of averaged temperature and centroid temperature based thermal spreading resistance using characteristic length scales as square root of source area and characteristic dimension of contact area are presented and discussed. Also, there exist a solution in Ref.[6] to calculate spreading resistance associated with averaged temperature for an isoflux hyperellipse situated on a half space. The second objective of this paper is to develop easy-to-use compact relationship for the solution provided in Ref.[6]. The accuracy of the compact correlation is verified through available data in literature.

3. Model Development

The family of contact areas is defined by a hyperellipse $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1,\ 0 \le n \le \infty$ with $b \le a$, situated on a insulated half space $z \ge 0$ and subjected to a uniform heat flux q with thermal conductivity λ. The hyperellipse result depends on two geometric parameters, $\alpha . n$. The perimeter n allows for the study on the effect of shape and aspect ratio $\alpha \left(= b/a\right)$ helps to study effect of shape on the spreading resistance. By varying these parameters one can control the forms of contact area, with $n = 2$ and $\alpha \le 1$, the spreading resistances for the circle and ellipse can be obtained. For $n < 2$ and $n > 2$ and $\alpha \le 1$, spreading resistance can be obtained for hypoellipse (which develop pointy corners in x and y directions) and hyperellipse (which increasingly resembles rectangles) respectively. The effect of the geometric parameters n upon the shape of the contact region in first quadrant is shown in Fig.1.

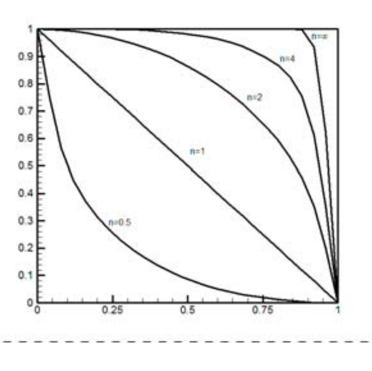

Fig.1. The effect of geometric parameter n, upon contact region in first quadrant

The thermal spreading resistance R_s is defined in [1] as the difference between the source temperature and thermal sink temperature divided heat transfer rate Q :

$$R_s = \frac{T_{source} - T_{\sin k}}{Q} \qquad (1)$$

The total heat rate entering the contact area, Q:

$$Q = 4qA_{0,} \qquad (2)$$

where, A_0, is the total area in the first quadrant:

$$A_0 = \frac{ab\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(1+\frac{2}{n}\right)}. \qquad (3)$$

where, $\Gamma(n)$ is the Gama function. The local temperature within the contact due to uniform - flux q entering the contact at the point $(\xi.\eta)$ is given by

$$T(x, y, 0) = \frac{q}{2\pi\lambda}\iint_A \frac{d\xi d\eta}{\sqrt{(\xi - x)^2 + (\eta - y)^2}} \qquad (4)$$

Following Yovanovich [6], the constriction resistance, in the special case of uniform flux, defined as the average temperature of the contact area divided by the total heat flow rate, is given:

$$\overline{R} = \frac{1}{qA^2}\iint_A T(x, y, 0) dA \qquad (5)$$

Substituting (4) into (5) we obtain the following expression for the constriction resistance:

$$\overline{R} = \frac{2}{\pi\lambda A^2}\iint_{A_0}\left[\iint_A \frac{d\xi d\eta}{\sqrt{(\xi - x)^2 + (\eta - y)^2}}\right] dxdy \qquad (6)$$

To evaluate the integral, first we are introducing dimensionless variables,

$$u = \frac{x}{a},\ v = \frac{y}{a},\ \mu = \frac{\xi}{b}, v = \frac{\eta}{b}. \qquad (7)$$

and integral evaluation for the integral appearing in (6) can be written as

$$I = b^3 \int_0^{+1} \int_0^{+u1} \int_{-1}^{+1} \int_{-\mu_1}^{+\mu_1} \frac{d\mu d\nu}{\sqrt{(\mu-u)^2+(\nu-v)^2}} dudv \tag{8}$$

$$u_1 = \left(\frac{1}{\alpha}\right)\left[1-v^n\right]^{\frac{1}{n}}, \mu_1 = \left(\frac{1}{\alpha}\right)\left[1-v^n\right]^{\frac{1}{n}} \tag{9}$$

$$\alpha = \frac{b}{a}$$

Further integral appearing in (8) can be simplified as: $I = b^3 \int_{v=0}^{+1} \int_{\nu=-1}^{1} G(\nu, v) d\nu dv$ (10)

$$G(\nu,v) = 2\mu_1 og(\nu - v) - (\mu_1 - u_1)$$

$$\text{where,} \times \log\left[\begin{matrix}(\mu_1 - u_1) \\ + \sqrt{(\mu_1-u_1)^2+(\nu-v)^2}\end{matrix}\right]$$

$$-(\mu_1+u_1)\log\left[-(\mu_1+u_1)+\sqrt{(\mu_1+u_1)^2+(\nu-v)^2}\right]$$

$$+\sqrt{(\mu_1-u_1)^2+(\nu-v)^2} - \sqrt{(\mu_1+u_1)^2+(\nu-v)} \tag{11}$$

Using Quadrature Rule of integration and dividing the intervals of integration of v and ν into M and N strips of equal width respectively, the integral in (10) can be written as

$$I = b^3 \Delta\nu\Delta v, \sum_{i=1}^{M} \sum_{j=1}^{N} G(\nu_j, v_i) \tag{12}$$

We have assumed that heat flux is uniform. In this case maximum temperature rise within a singly-connected contact area occurs at or near the centroid of the area. Following Ref. [6], an alternative definition of the spreading resistance based on quotient of the centroidal temperature and the total heat flow rate is:

$$R_{s,0} = \frac{T_0}{Q}$$

$$= \frac{2b}{\pi\lambda A} \int_0^{\frac{\pi}{2}} \frac{b}{\left[\sin^n\theta + \alpha^n \cos^n\theta\right]^{\frac{1}{n}}} \tag{13}$$

$$T_0 = \frac{2q}{\pi\lambda} \int_0^{\frac{\pi}{2}} \rho_0 d\rho \tag{14}$$

and $\rho_{0,}$ is the radius vector from the centroid to any point on the contact area contour making an angle θ with x-axis, for a hyper ellipse:

$$\rho_0 = \frac{b}{\left[\sin^n\theta + \alpha^n \cos^n\theta\right]^{\frac{1}{n}}}, \alpha \le 1 \tag{15}$$

The spreading resistance parameters $\psi = \lambda £ R_s$ and $\phi = \lambda £ R_0$ for averaged and centroid temperature based thermal spreading resistance are introduced for convenience and £ and is the arbitrary characteristic length scale, chosen as $£ = a$, characteristic dimension and $£ = \sqrt{A}$, the square root of the source area. Hence expressions for dimensionless spreading resistance are:

$$\psi_a(n,\alpha) = \frac{\alpha}{8\pi}\left[\frac{\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(1+\frac{2}{n}\right)}\right]^2 \times \Delta\nu\Delta v, \sum_{i=1}^{M}\sum_{j=1}^{N} G(\nu_j, v_i) \tag{16}$$

$$\psi_{\sqrt{A}}(n,\alpha) = \frac{1}{4\pi}\left[\frac{\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(1+\frac{2}{n}\right)}\right]^{\frac{3}{2}} \times \Delta\nu\Delta v \sum_{i=1}^{M}\sum_{j=1}^{N} G(\nu_j, v_i) \tag{17}$$

$$\phi_a(n,\alpha) = \frac{1}{2\pi}\frac{\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(1+\frac{2}{n}\right)} \times \int_0^{\frac{\pi}{2}} \frac{b}{\left[\sin^n\theta + \alpha^n \cos^n\theta\right]^{\frac{1}{n}}} d\theta \tag{18}$$

$$\phi_{\sqrt{A}}(n,\alpha) = \frac{1}{\pi}\left[\frac{\alpha\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(1+\frac{2}{n}\right)}\right]^{\frac{1}{2}} \times \int_0^{\frac{\pi}{2}} \frac{b}{\left[\sin^n\theta + \alpha^n \cos^n\theta\right]^{\frac{1}{n}}} d\theta \tag{19}$$

4. Numerical Results and Conclusion

Integrals appearing in (14), (15), (18) and (19) are evaluated by making use of Mathematica for different values of n. The accuracy of the integration technique is checked by comparing the computed results for a circular contact area with classical results and results obtained by Yovanovich [6] as shown in Table 1.

First, the results are obtained to see the effect of shape that depends on geometric parameter n and the effect of aspect ratio $\alpha\left(=b/a\right)\leq 1$), on thermal spreading resistances. To see effect of shape of source area on spreading thermal resistance numerical values are calculated for geometric parameter n, $0.5\leq n\leq\infty$ and aspect ratio $\alpha\left(=b/a\right)=1$. The selected values of the dimensionless constriction are shown in Table-2. It can be observed from the related graphs for shown in Fig.2 that there exists a constant relation between $\psi_{\sqrt{A}},\psi_a$ and $\phi_{\sqrt{A}},\phi_a$. Averaged temperature based thermal spreading resistance using characteristic dimension $£=\sqrt{A}$ increased when $0.5\leq n\leq 2$. It decreases with very high rate for same values of perimeter n, when it is calculated using $£=a$. Also, the dependence of the dimensionless resistance upon the source area is reduced considerably when $\sqrt{A}$ is selected as the characteristic dimensions of the system. It's clear that with the increasing n effect of thermal spreading resistance is diminishing for $\psi_{\sqrt{A}}$ and $\phi_{\sqrt{A}}$ but it is very small in the case of $\phi_{\sqrt{A}}$. Also for ϕ_a and ψ_a the result is not only opposite but the difference is large for both cases.

Overall it's concluded that length scale $£=\sqrt{A}$ is more appropriate than $£=a$, and the thermal spreading resistance $\phi_{\sqrt{A}}$ (centroid temperature based), is more effective than ($\psi_{\sqrt{A}}$ averaged temperature based). Second, the effect of aspect ratio on thermal spreading resistances is studied. In this case numerical data is calculated taking $0.02\leq\alpha\leq 1$ and $0.5\leq n\leq 2$ for averaged and centroid temperature based thermal spreading resistance, using $£=\sqrt{A}$ and a. The selected values of the dimensionless constriction are shown in Table-3 and Table 4 and, related graphs are shown Fig.3-4. It is noticed that both $\psi_{\sqrt{A}}$ and $\phi_{\sqrt{A}}$ increase slightly as α goes from 0.02 to 1 and for $\alpha<0.04$ and $\alpha<0.4$, the change is greater whereas the pattern of variation is opposite as well as ϕ_a and ψ_a are concerned. Third, a compact correlation for estimating $\psi_{\sqrt{A}}$ is developed as following:

$$\psi_{\sqrt{A}}\left(n,\alpha\right)=\frac{C_1\alpha}{\alpha+0.3},0.5\leq n\leq 40,\ 0.04\leq\alpha\leq 1 \quad (20)$$

The value of the coefficient C_1, appearing in correlation equation are given in Table 5 and Table 6, is produced using exact solution developed by Yovanovich [6] and derived compact relationship. The depicted plots are presented to show the difference between these results for geometric parameter n, $0.5\leq n\leq 40$, and aspect ratio α, $0.2\leq\alpha\left(=b/a\right)\leq 1$ in Figs. 5-8.

Table 1:The comparisons of results for circular contact

$n=2$, $\alpha=1$	Present	Ref. [6]	Classical
$\psi_{\sqrt{A}}$	0.4792	0.4787	0.4789
ψ_a	0.2703	0.2701	0.2702
$\phi_{\sqrt{A}}$	0.5641	0.5642	
ϕ_a	0.3183	0.3183	

ISBN: 978-988-17012-7-5

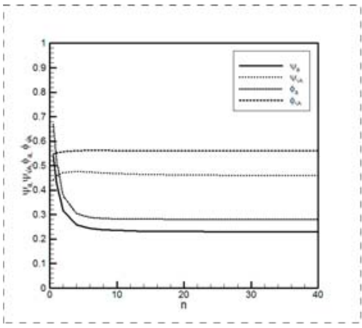

Fig.2. Effect of parameter n on centroid and averaged based thermal spreading resistance

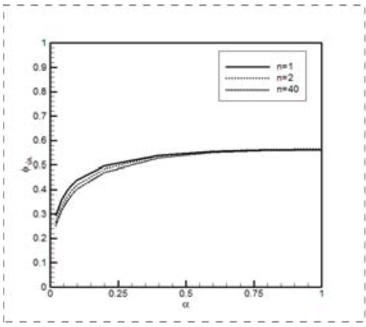

Fig. 3. Effect of aspect ratio $0.2 \le \alpha \le 1$ on thermal spreading resistance based on centroid temperature.

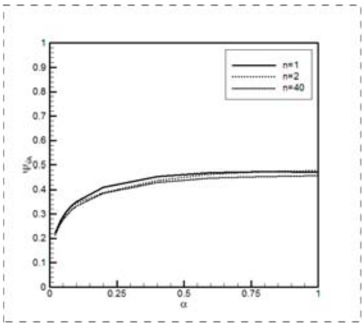

Fig. 4. Effect of aspect ratio $0.2 \le \alpha \le 1$,on thermal spreading resistance based on averaged temperature.

Table 2:Effect of shape on thermal spreading resistance

n	$\psi_{\sqrt{A}}$	ψ_a	$\phi_{\sqrt{A}}$	ϕ_a
0.5	0.5414	0.4421	0.6697	0.5468
1	0.3345	0.4730	0.3968	0.5611
2	0.2703	0.4792	0.3183	0.5642
4	0.2473	0.4762	0.2924	0.5631
8	0.2379	0.4706	0.2840	0.5619
10	0.2351	0.4669	0.2829	0.5617
20	0.2315	0.4621	0.2812	0.5613
40	0.2302	0.4601	0.2807	0.5612

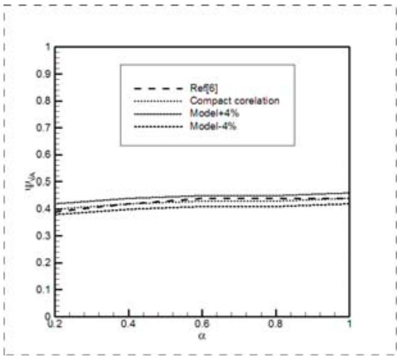

Fig. 5. Difference between solution in Ref. [6] and proposed compact correlation for $n = 0.5$, $0.2 \le \alpha \le 1$.

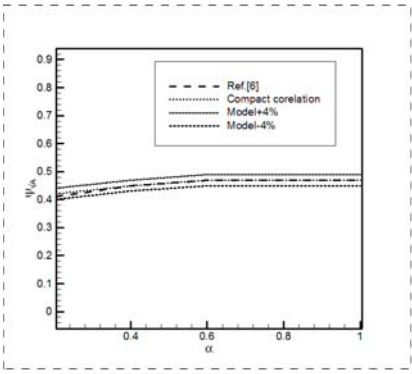

Fig. 6. Difference between solution in Ref. [6] and proposed compact correlation for $n = 1$ and $0.2 \leq \alpha \leq 1$.

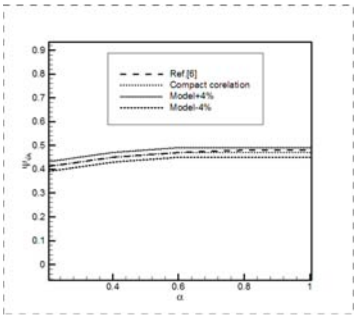

Fig. 7. Difference between solution in Ref. [6] and proposed compact correlation for $n = 2$ and $0.2 \leq \alpha \leq 1$.

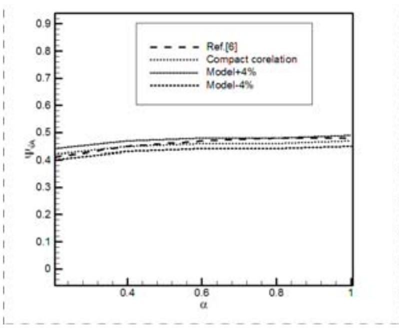

Fig. 8. Difference between solution in Ref. [6] and proposed compact correlation for $n = 4$ and $0.2 \leq \alpha \leq 1$.

Table 3: Centroid temperature based thermal spreading resistance, $\phi_{\sqrt{A}}$

α	$n=1$	$n=2$	$n=4$	$n=40$
0.02	0.294	0.269	0.258	0.253
0 04	0.356	0.331	0.319	0.313
0.06	0.393	0.370	0.358	0.352
0.08	0.419	0.398	0.386	0.380
0.1	0.439	0.420	0.409	0.403
0.2	0.496	0.484	0.476	0.471
0.4	0.539	0.536	0.532	0.528
0.6	0.554	0.555	0.553	0.551
0.8	0.560	0.562	0.561	0.559
1	0.561	0.564	0.563	0.561

Table 4: Averaged temperature based thermal spreading resistance, $\psi_{\sqrt{A}}$

α	$n=1$	$n=2$	$n=4$	$n=40$
0.02	0.223	0.223	0.222	0.215
0 04	0.275	0.275	0.273	0.264
0.06	0.309	0.308	0.306	0.295
0.08	0.333	0.332	0.329	0.317
0.1	0.352	0.350	0.348	0.335
0.2	0.409	0.406	0.403	0.388
0.4	0.455	0.451	0.448	0.432
0.6	0.470	0.469	0.467	0.450
0.8	0.474	0.477	0.475	0.457
1	0.473	0.479	0.478	0.461

Table 5: Coefficients for averaged temp. based spreading resistance	
n	0.45
0.5	0.49
1	0.49
2	0.48
4	0.48
16	0.46
∞	0.47

Table 6:A comparison of current study with Ref. [6] for $n = 0.5$			
A	Ref[6]	Correl.	%diff.
0.2	0.39	0.40	-2.56
0.4	0.42	0.42	0.87
0.6	0.44	0.43	2.06
0.8	0.44	0.43	2.19
1	0.44	0.44	1.85
$n = 2$			
A	Ref[6]	Correl.	%diff.
0.2	0.41	0.41	-2.26
0.4	0.45	0.45	0.62
0.6	0.47	0.47	1.59
0.8	0.48	0.47	1.59
1	0.48	0.47	1.13
$n = 4$			
A	Ref[6]	Correl.	%diff.
0.2	0.41	0.42	-1.00
0.4	0.45	0.45	1.63
0.6	0.47	0.46	2.53
0.8	0.48	0.46	2.50
1	0.48	0.47	2.03

Acknowledgement:-we are thankful to Prof. Majid Behrami for the valuable suggestions during the preparation of this work.

References

1. S. Lee, Van Au, Seaho Song and K.P.Moran, Constriction/Spreading Resistance Model for Electronics Packaging in Proc, 4th ASME/JSME, Thermal Eng. Joint Conference, Vol. 4, pp. 199-206, 1995.
2. D. P. Kennedy, Spreading Resistance in Cylindrical Semiconductor devices, Journal of Applied Physics, Vol. 31, pp. 1490-1497, 1960.
3. M. M.Yovanovich, General Expression for Circular Constriction Resistances for Arbitrary Flux Distribution, AIAA 13th Aerospace Sciences Meeting, Pasadena, California, pp. 381--396.
4. G. E. Shiender, A.B. Strong and M.M. Yovanovich, Transient Heat Flow from a thin circular Disc, Relative Transfer and Thermal Control , Vol. 49, pp. 419-426, 1976.
5. M. M. Yovanovich, G.E. Shiender and A.B. Strong, Thermal Constriction Resistance Due to a Circular Annular Contact, Thermophysics of Spacecraft and Outer Planet Entry Probes, Vol. 56, pp. 141-154, 1976.
6. M. M. Yovanovich, S.S. Burde and J.C. Thompson, Thermal Constriction Resistance of Arbitrary Planar Contacts With Constant Flux, AIAA Progress in Astronautics and Aeronautics , Heat Transfer and Spacecraft Thermal Control, Vol. 56, edited by A.M. Smith New York, pp. 127-139, 1976.
7. M. M. Yovanovich, Thermal Constriction Resistance for Annular Contacts on Circular Flux Tubes, AIAA Journal, Vol. 7, No. 6, pp. 822-824, 1976.
8. M. M. Yovanovich, K.A. Martin and G.E. Shiender, Constriction Resistance of

Doubly- Connected Contact Areas Under Uniform Flux, AIAA 14th Thermophysics conference pp. 1-5, June 4-6, 1979.

9. S.S. Burde and M.M. Yovanovich, Thermal Resistance at Smooth Sphere/ Rough Flat Contacts: Theoretical Analysis, Thermophysics and Thermocontrol Vol. 65, pp. 47-162, 1978.

10. M. M. Yovanovich, C. H. Tien and G. E. Schneider, General Solution of Constriction Resistance Within a Compound Disk, Heat Transfer Thermal Control, Heat Pipes, Vol.70, pp. 47-62,1979.

11. G.E. Shiender, A.B. Strong and M.M. Yovanovich, Transient Thermal Response of Two Bodies Communicating Through a Small Circular Contact, International Journal of Heat and Transfer, Vol. 20, pp. 301-308, 1977.

12. M. M.Yovanovich, Y. S., Muzychka, and J.R.Culham, Spreading Resistance of Isoflux Rectangles and Strips on Compound Flux Channels, 6th AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV, January 12-15, 1998.

13. Y. Fugian, V. Prasad and K. Imin, The thermal constriction resistance of a strip contact spot on a thin film, Journal of Applied physics, ISSN 0022-3727, Vol. 32, no8, pp. 930-936, 1999 .

14. A. K. Das and S.S.Sadhal, Thermal Constriction Resistance Between Two Solids for Random Distribution of Contacts, Heat Mass Transfer, 35, pp. 101--111, 1999.

15. M. M Yovanovich, Y.S. Muzychka and J. R Culham, Influence of Geometry and Edge Cooling on Thermal Spreading Resistance. AIAA Journal of Thermophysics and Heat Transfer. Vol. 20. no. 2, pp. 247-255. April-June, 2006.

16. Y. S. Muzychka, M. M Yokanovich and J. R. Culham. Thermal Spreading Resistance in Compound and Orthotropic Systems, Journal of Thermophysics And Heat Transfer, Vol. 18, No. 1, January-March 2004.

17. M. M. Yovanovich, Four Decades of Research on Thermal Contact, Gap and Joint Resistance in Microelectrics. IEEE Transactions on Components and Packaging Technologies, Vol. 28, no. 2, pp. 182-206. 2005.

18. D. J. Nelson and W. J Sayers, A Comparison of 2-D Planar. Axissymmetric and 3-D Spreading Resistances. Proceeding 8th Annual IEEE Semiconductor thermal Measurement and Management Symposium, Austin, TX, pp.62-68, 1992.

19. S. Lee, Calculating Spreading Resistance in Heat Sinks, Electronics Cooling Magazine, Vol.4. no.1. January 1998.

20. G. N. Ellison, Maximum thermal spreading resistance for rectangular sources and plates with non-unity aspect ratios, IEEE Transaction of Component and Packaging technologies, Vol. 26, no.2. pp. 434-459, June 2003.

21. S. Karmalkar, P.V. Mohan and B.P.Kumar, A unified compact model of electrical and thermal 3-D spreading resistance between eccentric rectangular and circular contacts, IEEE Electron Device Letters, Vol. 26. Issue 12, pp. 909-912, 2005.

22. A. Bhatt and J. Rhee, Thermal spreading resistance for square and rectangular entities, 11th International Symposium on Advanced Packaging Materials: Processes, Properties, and Interfa, IEEE Cat. No. 06TH8875, pp.4, 2006.