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Stackelberg Solution to Two-Level Linear Programming Models of Food Retailers and Brokerages Problems

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Abstract—**In Japan, farm products of fruits and vegetables are sent to consumers by the supply chain that manufactures pass through distributors, processing suppliers and retailers traditionally. The wholesale markets are of particular importance for the supply chain of the farm products' distribution. Under such a situation, it is necessary for retailers to take the sale policy which can support demand changes of consumers. In such cases, when retailers make a decision, brokerages as a transaction partner wield a very large influence over them. In this research, we focus on food retailers and brokerages problems, we formulate two-level programming problem maximizing both profits of a food retailer as an upper level decision maker and a brokerage as a lower level decision maker. And using an actual data, we derive a Stackelberg solution which is a reasonable solution in case that both decisions are made consecutively. Moreover we consider an order quantity and a purchase volume of each food maximizing both profits of a food retailer and a brokerage.**

Keywords: two-level programming problem, Stackelberg solution, food retailer and brokerage problem

1 Introduction

In Japan, farm products of fruits and vegetables are sent to consumers by the supply chain that manufactures pass through distributors, processing suppliers and retailers traditionally. The wholesale markets are of particular importance for the supply chain of the farm products' distribution. The evidence that produces (about 80% in vegetables and about 60% in fruits [6]) are distributed through wholesale markets indicates that wholesale markets as an efficient brokerage system to close manufactures and consumers have worked for a long time.

For Asian food retailers except Japan, the existence of stalls and wet markets is important. Many consumers believe such a traditional market sells items at lower prices and fresher than a food supermarket. And they understand that food supermarkets are location purchasing processing foods instead of perishables. In addition, eating and drinking are prosperous at stalls and food courts in Asian various countries. This eating out is very popular since there are many households of working together in Asian Chinese community. Consumers purchase processing foods instead of perishables in food supermarkets [1].

On the other hand, in Japanese consumers, eating out is not so popular and they usually purchase several kinds freshness farm products by a small quantity at a time. Thus retailers have to prepare many products in order to respond to consumer demands at all times. For such a purchasing format of consumers, when retailers supply a large quantity of freshness farm products characterized by a small quantity at a time, if there's no wholesale market, each retailer has to do business with all producers. Since there is a system intermediating between transactions such as a wholesale market, retailers can reduce the number of transactions and the transaction cost becomes lower. Furthermore, for diversification of recent consumer needs, retailers have to prepare more many products over wide quality effectively since the present [3]. Thus, corresponding to purchasing behaviour of Japanese consumer, retailers stocking products from a producer or a distributor work toward improving consumers' satisfaction and with the help of a distributor, retailers supply necessary quantity when consumers feel necessary. In this way, retailers don't supply products only in own companies and they order products from a brokerage and supply. such a sale format is adopted in many retailers, and it makes possible to supply the proper quantity of the appropriate product in just in time.

Under such a situation, it is necessary for retailers to take the sale policy which can support demand changes of consumers. In such cases, when retailers make a decision, brokerages as a transaction partner wield a very large influence over them.

Thus, since interdependence between retailers and brokerages, to be concrete, there is a relation between a purchase (the volume of shipment) of brokerages and an order quantity (a quantity of supply) from retailers, we formulate as a two-level programming problem. Since

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interdependence between retailers and brokerages, to be concrete, there is a relation between a purchase (the volume of shipment) of brokerages and an order quantity (a quantity of supply) from retailers, we formulate as a two-level programming problem. We consider the two-level programming problem of food retailers and brokerages as a Stackelberg problem [4], [5] that both decisions are made consecutively since retailers decide an order quantity first and brokerages decide a purchase.

In this research, we focus on food retailers and brokerages problems, we formulate two-level programming problem maximizing both profits of a food retailer as an upper level decision maker and a brokerage as a lower level decision maker. And using an actual data, we derive a Stackelberg solution which is a reasonable solution in case that both decisions are made consecutively. Moreover we consider an order quantity and a purchase volume of each food maximizing both profits of a food retailer and a brokerage.

2 Fomulation

In the present life style, consumers buy the manufacturing product and consume it. Therefore, if manufacturing and consumption are divided, the process of distributing products from manufacturing place to consuming place is needed. In this way, if manufacturers' place and consumers' place are far, it is necessary for products to distribute smoothly. In addition, it becomes important to supply a necessary product to a consumer when a consumer needs. Therefore, it is necessary to cover the difference of the distance or the hour between manufacturers and consumers and to distribute smoothly. Thus, brokerages purchase the product which a retailer needs from each manufacturer in each manufacturing place (wholesale market) and transport to sale place of a retailer. And a retailer purchases products from brokerages, and then keeps products in a warehouse, and sells products to a consumer when it is necessary. Such a selling form is adopted in Japan.

In this research, for such a situation, we assume distributing from a producer to a consumer. There is a major food retailer (upper level decision maker) in Tokyo (sale place). And it sells fruits and vegetables for fruits and vegetables branch of a supermarket. Here, we do modeling as follows:

Then a retailer purchases n kind foods (for example, fruits, vegetables) through a brokerage (lower level decision maker) and sells. Here, a brokerage purchases n kind foods in s wholesale markets in the whole country. Purchased foods are transported to a warehouse of a retailer by a brokerage. And a retailer pays this transportation cost to a brokerage and supplies to a consumer. Then we set decision variables of a retailer an order quantity x_{1j} of food j, $j = 1, 2, \ldots, n$. On the other hand, we set decision variables of a brokerage a purchase quantity x_{2qj} of food j, $j = 1, 2, \ldots, n$ in a wholesale market q, $q = 1, 2, \ldots, s$. If we assume warehouse capacity in a retailer is w and unit volume of each food is v_j [cm^3/kg], a constraint equation about volume of stock of a warehouse is as follows:

$$\sum_{j=1}^{n} v_j x_{1j} \leq W, \quad j = 1, 2, \ldots, n \tag{1}$$

In addition, the smallest order quantity D_j^L and the largest one D_j^U for each food j are settled. And the order quantity x_{1j} decided is ordered in range of a bound considering a quantity of stock and constrained as follows:

$$x_{1j} \geq D_j^L, \quad j = 1, 2, \ldots, n \tag{2}$$

$$x_{1j} \leq D_j^U, \quad j = 1, 2, \ldots, n \tag{3}$$

On the other hand, a brokerage accepts orders from a retailer and purchases foods. A brokerage has to purchase the foods which can support orders and has constraint as follows:

$$\sum_{q=1}^{s} x_{2qj} \geq x_{1j}, \quad j = 1, 2, \ldots, n \tag{4}$$

In addition, a brokerage has the fund constraint b_q purchasing in each wholesale market as follows:

$$\sum_{j=1}^{n} c_{22qj} x_{2qj} \leq b_q, \quad q = 1, 2, \ldots, s \tag{5}$$

Under the above-mentioned constraints, we consider the problem maximizing profits of both a retailer and a brokerage. In this research, we assume parameters that for a retailer, sale profit of food j is c_{11j}, the transportation cost from the wholesale market q for food j is c_{12qj}, and for a brokerage, shipping price from a brokerage to a retailer for food j is c_{21j}, purchase price in the wholesale market q for food j is c_{22qj}. As for the total sale profit of a retailer, it is $\boldsymbol{c}_{11}\boldsymbol{x}_1$ and the transportation cost is $\boldsymbol{c}_{12}\boldsymbol{x}_2$. In addition, as for the total shipping cost of a brokerage, it is $\boldsymbol{c}_{21}\boldsymbol{x}_1$ and the purchasing cost is $\boldsymbol{c}_{22}\boldsymbol{x}_2$. Therefore, the gross operating profit of a retailer is $\boldsymbol{c}_{11}\boldsymbol{x}_1 - \boldsymbol{c}_{12}\boldsymbol{x}_2$, and the one of a brokerage is $\boldsymbol{c}_{21}\boldsymbol{x}_1 - \boldsymbol{c}_{22}\boldsymbol{x}_2$. Then, this two-level linear programming problem maximizing both profits of a retailer and a brokerage is formulated two-level maximizing problem as follows:

$$\left.\begin{array}{ll}
\underset{\boldsymbol{x}_1}{\text{maximize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{11}\boldsymbol{x}_1 - \boldsymbol{c}_{12}\boldsymbol{x}_2 \\
\text{where } \boldsymbol{x}_2 \text{ solves} & \\
\underset{\boldsymbol{x}_2}{\text{maximize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{21}\boldsymbol{x}_1 - \boldsymbol{c}_{22}\boldsymbol{x}_2 \\
\text{subject to} & \sum_{j=1}^{n} v_j x_{1j} \leq W, \\
& x_{1j} \geq D_j^L, \quad j = 1, 2, \ldots, n \\
& x_{1j} \leq D_j^U, \quad j = 1, 2, \ldots, n \\
& \sum_{q=1}^{s} x_{2qj} \geq x_{1j}, \quad j = 1, 2, \ldots, n \\
& \sum_{j=1}^{n} c_{22qj} x_{2qj} \leq b_q, \quad q = 1, 2, \ldots, s \\
& \boldsymbol{x}_1 \geq \boldsymbol{0}, \boldsymbol{x}_2 \geq \boldsymbol{0}
\end{array}\right\} \tag{6}$$

decision variable

x_{1j} : order quantity for food j of a retailer (upper level decision maker)

x_{2qj} : purchase quantity for food j in the wholesale market q of a brokerage (lower level decision maker)

parameters

c_{11j} : sale profit of food $j, j = 1, 2, \ldots, n$

c_{12qj} : transportation cost from the wholesale market $q, q = 1, 2, \ldots, s$ for food $j, j = 1, 2, \ldots, n$

c_{21j} : shipping price for food $j, j = 1, 2, \ldots, n$

c_{22qj} : purchase cost for food j in the wholesale market $q, q = 1, 2, \ldots, s$

W : warehouse capacity

D_j^L : smallest order quantity for food $j, j = 1, 2, \ldots, n$

D_j^U : largest order quantity for food $j, j = 1, 2, \ldots, n$

b_q : the fund purchasing in each wholesale market $q, q = 1, 2, \ldots, s$

3 Application to actual data

3.1 data set

In this research, we assume that a retailer (upper level decision maker) sells 16 kinds of vegetables and fruits ($n = 16$) for about 10000 households. The transportation cost of a retailer is calculated based on highway toll and light oil cost (116[yen/L]) that a brokerage transports from each wholesale market to Tokyo by 8t truck and shown in Table 1. From Table 2, we set food retail price from 50% to 75% that cost ratio of general fruits and vegetables and its mean cost ratio is 62%. And unit volume v_j [cm^3/kg] of each food is estimate value. We assume that warehouse capacity W is 150 [m^2]×2[m] for consumers to satisfy freshness demand and purchasing frequency. Based on past data of stocking quantities, the smallest order quantity D_j^L is estimate for 10000 households and the largest one D_j^U is that we multiply the smallest one by 1.1-1.4 (Table 4).

In addition, we assume that the wholesale markets of a brokerage is placed in Sapporo, Sendai, Niigata, Kanazawa, Tokyo, Osaka, Hiroshima and Miyazaki ($s = 8$). And we set food shipping price c_{21j} [yen/kg] from a brokerage to a retailer 95% of a wholesale price in Tokyo (Table 2). Although, we set purchase price of a brokerage in each wholesale market mean price in each Central Wholsale Markt at March, 2008 and it is shown in Table 5. Moreover, Table 6 shows the fund b_q purchasing in each wholesale market.

For the set two-level linear programming problem, we will derive a Stackelberg solution by HJS method [2].

4 Conclusion

In this research, we focus on food retailers and brokerage problems, formulated two-level linear programming models. In near future, we will derive a Stackelberg solutin and report the result of the application to formulated problems.

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Table 1: c_{12qj} transportation cost from the wholesale market q for food j

	q	1	2	3	4	5	6	7	8
j	item	Sapporo	Sendai	Niigata	Kanazawa	Tokyo	Osaka	Hiroshima	Miyazaki
1	onion	12.4760199	2.8349361	2.8371225	3.8820996	0.20273	4.5538464	6.2258521	10.2734614
2	potato	7.984652736	1.814359104	1.8157584	2.484543744	0.1297472	2.914461696	3.984545344	6.575015296
3	cabbage	12.4760199	2.8349361	2.8371225	3.8820996	0.20273	4.5538464	6.2258521	10.2734614
4	Japanese white radish	2.694820298	0.612346198	0.61281846	0.838533514	0.04378968	0.983630822	1.344784054	2.219067662
5	Chinese cabbage	9.98081592	2.26794888	2.269698	3.10567968	0.162184	3.64307712	4.98068168	8.21876912
6	carrot	5.988489552	1.360769328	1.3618188	1.863407808	0.0973104	2.185846272	2.988409008	4.931261472
7	cucumber	2.49520398	0.56698722	0.5674245	0.77641992	0.040546	0.91076928	1.24517042	2.05469228
8	lettuce	59.88489552	13.60769328	13.618188	18.63407808	0.973104	21.85846272	29.88409008	49.31261472
9	tomato	4.99040796	1.13397444	1.134849	1.55283984	0.081092	1.82153856	2.49034084	4.10938456
10	spinach	39.92326368	9.07179552	9.078792	12.42271872	0.648736	14.57230848	19.92272672	32.87507648
11	eggplant	24.9520398	5.6698722	5.674245	7.7641992	0.40546	9.1076928	12.4517042	20.5469228
12	apple	9.98081592	2.26794888	2.269698	3.10567968	0.162184	3.64307712	4.98068168	8.21876912
13	banana	2.49520398	0.56698722	0.5674245	0.77641992	0.040546	0.91076928	1.24517042	2.05469228
14	strawberry	4.99040796	1.13397444	1.134849	1.55283984	0.081092	1.82153856	2.49034084	4.10938456
15	orange	3.74280597	0.85048083	0.85113675	1.16462988	0.060819	1.36615392	1.86775563	3.08203842
16	lemon	3.74280597	0.85048083	0.85113675	1.16462988	0.060819	1.36615392	1.86775563	3.08203842

Table 2: c_{11j}, c_{21j} c_{11j} : sale profit of food j [yen/kg] c_{21j} : shipping price of food j [yen/kg]

j	1	2	3	4	5	6	7	8
c_{11j}	60.167	47.636	98.800	54.467	86.277	76.950	111.150	107.667
c_{21j}	90.25	111.15	98.8	81.7	105.45	179.55	259.35	161.5
j	9	10	11	12	13	14	15	16
c_{11j}	186.712	117.664	293.032	131.977	139.650	295.767	98.727	225.409
c_{21j}	346.75	274.55	358.15	245.1	139.65	887.3	183.35	275.5

Table 3: v_j

j	1	2	3	4	5	6	7	8
v_j	5000	3200	5000	1080	4000	2400	1000	24000
j	9	10	11	12	13	14	15	16
v_j	2000	16000	10000	4000	1000	2000	1500	1500

Table 4: D_j^L, D_j^U

j	1	2	3	4	5	6	7	8
D_j^L	4000	4000	2000	5000	10000	2000	800	1500
D_j^U	5000	5000	2400	6000	14000	2500	1000	2000
j	9	10	11	12	13	14	15	16
D_j^L	3000	3000	1200	6000	12500	6000	4000	1000
D_j^U	4000	3600	1500	6600	14500	7500	4800	1300

Table 5: c_{22qj}

	q	1	2	3	4	5	6	7	8
j	item	Sapporo	Sendai	Niigata	Kanazawa	Tokyo	Osaka	Hiroshima	Miyazaki
1	onion	55	78	73	83	95	111	92	85
2	potato	57	87	90	105	117	110	81	106
3	cabbage	100	113	98	103	104	88	87	72
4	Japanese white radish	102	95	85	83	86	71	72	60
5	Chinese cabbage	104	115	114	113	111	97	104	88
6	carrot	156	187	169	178	189	189	179	151
7	cucumber	288	270	274	276	273	260	248	217
8	lettuce	229	168	186	188	170	173	138	93
9	tomato	349	394	312	429	365	317	300	249
10	spinach	339	284	335	296	289	287	257	242
11	eggplant	421	336	342	373	377	368	315	260
12	apple	221	250	231	226	258	274	265	249
13	banana	157	165	149	115	147	147	176	186
14	strawberry	926	867	743	872	934	939	693	782
15	orange	195	198	168	159	193	156	168	150
16	lemon	294	353	283	290	290	310	301	231

Table 6: b_q

q	1	2	3	4	5	6	7	8
b_q	2000000	1500000	1500000	1500000	1500000	2000000	1500000	2000000