A Hybrid of Modified Simplex and Steepest Ascent Methods with Signal to Noise Ratio for Optimal Parameter Settings of ACO

J. Ratanaphanyarat*, S. Chunothaisawat, N. Rungruengsattaya, S. Tunsiriroongruang, K. Sriwannusorn and P. Luangpaiboon

Abstract—**Metaheuristics are sequential processes that perform exploration and exploitation in the solution space aiming to efficiently find near optimal solutions with natural intelligence as a source of inspiration. One of the most well-known metaheuristics is called Ant Colony Optimisation, ACO. This paper is conducted to give an aid in complicatedness of using ACO in terms of its parameters: number of iterations, ants and moves. Proper levels of these parameters are analysed on eight noisy continuous non-linear continuous response surfaces. Considering the solution space in a specified region, some surfaces contain global optimum and multiple local optimums and some are with a curved ridge. ACO parameters are determined through Modified Simplex, MSM and Steepest Ascent methods, SAM, including their hybridisation. SAM was introduced to enhance a performance of MSM via the statistically significant regression analysis and Taguchi's signal to noise, S/N, ratio to recommend preferable levels of parameters. A series of computational experiments using each algorithm were conducted. Experimental results were analysed in terms of design points, best so far solutions, mean and standard deviation including S/N ratio. It was found that the results obtained from hybridisation were better than those using single algorithm itself. However, the average execution time of experimental run and number of design points using hybridisation were longer than those using a single method. Finally they stated a recommendation of proper level settings of ACO parameters for all eight functions that can be used as a guideline for future applications of ACO. This is to promote ease of use of ACO in real life problems.**

Index Terms—**Ant Colony Optimisation, Modified Simplex, Taguchi's Signal to Noise Ratio, Steepest Ascent and Response Surface Methodology.**

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I. INTRODUCTION

Optimisation algorithms can be categorised as being either conventional or approximation optimisation algorithms [1]. Conventional optimisation algorithms are usually based upon mathematical procedures. However, the applications of these methods might need exponentially computational time in the worst cases. This becomes an impractical approach, especially for solving a very large size problem.

Alternative approaches that can guide the search process to find near optimal solutions in acceptable computational time are therefore more practical and desirable. Metaheuristics iteratively conduct stochastic search processes inspired by natural intelligence. They can be categorised into three groups: physically-based inspiration such as Simulated Annealing [2]; socially-based inspiration for instance Taboo Search [3]; and biologically-based inspiration e.g. Ant Colony Optimisation [4], Artificial Immune System [5], Genetic Algorithm [6], Memetic Algorithm [7], Neural Network [8], Particle Swarm Optimisation [9], and Shuffled Frog Leaping [10]. These alternative approaches have been widely used to solve large-scale combinatorial optimisation problems [11]—[14].

Response Surface Methodology (RSM) is a bundle of mathematical and statistical techniques that are helpful for modeling and analysing problems. A response of our interest is influenced by several predictor variables. An objective is to optimise this response. For example, suppose that a process engineer wishes to find the levels of temperature (x_1) and pressure (x_2) that maximise the yield (y) of a process. The process yield is a function of levels of temperature and pressure; $y = f(x_1, x_2) + \varepsilon$ [15].

Where ε represents the level of noise (standard deviation) or error observed in the response y. If we denote the expected response by $E(y) = f(x_1, x_2) = \eta$, then the surface represented by; $\eta = f(x_1, x_2)$. So, it is called a response surface.

A response surface above describes how the yield of a process varies with changes in *k* independent variables. Estimation of such surfaces, and hence identification of near optimal settings for predictor variables is an important practical issue with interesting theoretical aspects. Many systematic methods for making an efficient empirical investigation of such surfaces have been proposed in the last fifty years. These are generally referred to as evolutionary operation (EVOP). RSM is used to improve the current operating conditions until the conditions of optimal yield are satisfied. In most RSM problems, a form of the relationship between the response and the independent variables is unknown. Thus, the first step in RSM is to find a suitable approximation for the true functional relationship between y and the set of its independent variables. Usually, a low-order polynomial in some region of the independent variables is employed [15]. If the response is well modeled by a linear function of the independent variables, then the approximating function is the first-order model.

Metaheuristics are more complicated due to constraints of the algorithm itself not of the question. These constraints or their parameters are needed to be initialised to optimise the outcome of the solution, or in other word, constraints directly affect the quality of the solution. So it is in turn inspiring an objective of this paper to examine the relation of constraints adjacent to the quality of solution of a chosen metaheuristic algorithm, Ant Colony Optimisation (ACO) in the context of Response Surface Methodology. A hybrid algorithm to determine the optimum of surfaces consists of two treatments; Modified Simplex Method (MSM) and Steepest Ascent Method (SAM).

Inspection and analysis are used to determine a recommendation on the proper levels of parameter settings for eight non-linear continuous mathematical models within three main classes; unimodal, multimodal and curve ridge including a combination of multimodal and curve ridge functions. Eight non-linear continuous mathematical models are considered being complicated optimisation problems when applied to real industrial processes.

This paper is organised as follows. Section II describes the selected metaheuristic; Ant Colony Optimisation (ACO) and its pseudo code. Sections III and IV are briefing about algorithms of Modified Simplex and Steepest Ascent, respectively. Sections V and VI are presenting Taguchi methodology and a hybrid algorithm of MSM and SAM, respectively. Section VII illustrates tested functions. Section VIII shows design and analysis of computational experiments for comparing the performance of the proposed methods. The conclusion is also summarised and it is followed by acknowledgment and references.

II. Ant Colony Optimisation Algorithm (ACO)

Ant algorithm was first proposed by Dorigo and his colleagues [4] as a multi-agent approach to optimisation problems, such as a travelling salesman problem (TSP) and a quadratic assignment problem (QAP). There is currently a lot of ongoing activity in the scientific community to extend or apply ant-based algorithms to many different discrete optimisation problems. Recent applications cover problems like a vehicle routing, a plant layout and so on. Ant algorithm is inspired by observations of real ant colonies. Ants are social insects and they live in colonies. Behaviour is direct more to the survival of the colony as a whole than to that of a single individual component of the colony. Social insects have captured the attention from many scientists because of a structure of their colonies, especially when compared with a relative simplicity of the colony's individual. An important and interesting behaviour of ant colonies is their foraging behaviour and in particular how ants can find shortest paths between food sources and their nest [9], [16].

While walking from food sources to the nest and vice versa, ants deposit on the ground a substance called pheromone, forming a pheromone trail. With ants ability to smell pheromone they tend to choose a path marked by strong pheromone concentrations with the higher probability. The pheromone trail allows the ants to find their way back to the food source and vice versa. It can be also used by other ants to find the location of the food sources found by their nest mates [10]. The pseudo code is used to briefly explain to all the procedures of ACO shown in Fig. 1.

```
Procedure ACO Metaheuristic()
While (termination criterion not satisfied) – (line 1)
    Schedule activities
      ants generation and starting point;
      makes path or step for each ant
      compare response function
      if no improvement of response function then
         communication with best ant response function
         make path or step from local trap to best ant
      else
         if ant found the better response function then
            go to line 5.
         else
            wait for best ant communication
         end if
      end if
    end schedule activities
end while
end procedure
```

Fig. 1 Pseudo Code of ACO Metaheuristic.

III. The Simplex & Modified Simplex Method (MSM)

The basic shape (design) is called the simplex [17]. The simplex design in a problem with k variables consists of k+1 design points (vertices) but it is not necessary to have a property of equidistance. There are many extensions on the rigid simplex algorithm. One of the well-known is a modified simplex method (MSM) of Nelder and Mead [18].

In the MSM an expansion or contraction of the reflection is allowed at each step. Although there are many possible stopping criterions for simplex algorithms, this study follows Nelder and Mead and includes the standard deviation of the estimated yields at the vertices of the simplex. Various stopping rules and one based on the sample range were also tried on the literatures, but they appeared to offer no advantage over the stopping rule based on the standard deviation of process yields.

The simplex design is first applied at an arbitrary point within the safe region of operation. The response is measured for each of the design points. In a maximisation process with three variables or a tetrahedron simplex, the vertex corresponding to the lowest yield (W) is identified and reflected in the opposite hyper-face to obtain (R) via the centroid ($\overline{P}$). The centroid obtained by other vertices in the simplex consists of V_H, V_S, and V_{SH}, or vertices of highest yield, second least yield and second highest yield, respectively [19], [20]. The new design point can be extended (E) in the direction of more favourable conditions, contracted (C- or C+) if a move is taken for least favourable conditions, and shrunk toward best vertex if a contracted vertex is still the least but not less than the rejected trial condition (Fig. 2). The next run is carried out with variables set at values corresponding to this new design point. This MSM terminates, and the finishing strategy is applied. An idea of MSM's logical decision is shown in Fig. 3.

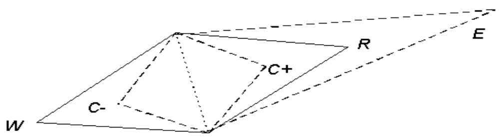


Fig. 2 *Different Simplex Moves from the Rejected Trial Condition (W). R = Reflection, E = Expansion, C+ = Positive Contraction and C- = Negative Contraction*.

```
Procedure of MSM ()
While (termination criterion not satisfied) – (line 1)
   Schedule activities
      Reflection of least yield W is processed
      Compute R and f(R)
      Compare response function
      if f(R) is highest then
           extension E will be processed
      else
          if R and f(R) continue to be the least then
                reflect backward to prior point
                 recalculate W and f(W)
             or
                contraction C or shrinking S will be processed
                recalculate f(C) or f(S)
             else
              go to line 3.
          end if
      end if
   end schedule activities
end while
end procedure
```

Fig. 3 Pseudo Code of MSM.

IV. Steepest Ascent Method (SAM)

The procedure of SAM is that a hyperplane is fitted to the results from the initial 2^k factorial designs. The data from these design points are analysed. If there is an evidence of main effect(s), at some chosen level of statistical significance and no evidence of curvature, at the same level of significance, the direction of steepest ascent on the hyperplane is then determined by using principles of least squares and experimental designs. The next run is carried out at a point, which has some fixed distance in this direction, and further runs are carried out by continuing in this direction until no further increase in yield is noted. When the response first decreases and no improvement of two more verified yields, another 2^k factorial design will be carried out, centered on the preceding design point. A new direction of steepest ascent is estimated from this latest experiment. Provided at least one of the coefficients of the hyperplane is statistically significantly different from zero, the search continues in this new direction (Fig. 4). Once the first order model is determined to be inadequate, the area of optimum is identified via a second order model or a finishing strategy.

```
Procedure of SAM ()
While (termination criterion not satisfied) – (line 1)
   Schedule activities (when Regression verification criteria not satisfy)
        Determine significant first order model from the factorial design points
     Schedule activities
      Move along the steepest ascent's path with a step length (Δ)
      Compute response functions
        if new response function is greater than the preceding then
           move ahead with another Δ
         else
          calculate two more response function to verify the descending trend
              if
                  one of which response function turn out to be greater than
                  preceding coordinate's response function then
                     use the biggest response function to continually move along
                     the same path
                 else
                    use closest preceding point as a centre for new 2^3 design
              end if
          end if
      end schedule activities
   end schedule activities
end while
end procedure
```

Fig. 4 Pseudo Code of SAM.

V. Taguchi Methodology

This method is created to propose understanding and alternative resolutions to manufacturing qualities. The fundamental concepts are a consequence of variations. The three main statistic contribution theories are loss function, off-line and design of experiments.

Consequence of qualities in this case is marked by Signal to Noise ratio (S/N), which will present sensitivity of response to noises or uncontrollable factors. This ratio is used to point out stability of the design system and quality of chosen design's factors. A philosophy of *off-line* quality control, designing products and processes are insensitive to parameters outside the design engineer's control. Taguchi's robust design of experiment with an advantage of S/N is encompassing both internal and external arrays within the improving assessment process [19].

The S/N provide a standard index for data comparison, while having noises contributing the sets of n data. Consequently, the method will be more comprehensive to extend the process design to quality control. Signal to noise ratio consists of;

$$S/N = -10\,\mathrm{Log}_{10}[(\sum_{i=1}^{n} 1/y_i^2)/n] \quad \text{(Maximisation)}$$

$$S/N = -10\,\mathrm{Log}_{10}[(\sum_{i=1}^{n} (y_i^2))/n] \quad \text{(Minimisation)}$$

VI. HYBRID OF MSM AND SAM

Iterative strategies of MSM and SAM have S/N as a moving trigger rather an ordinary yield. Parameters are 8 unit^3 of the volume of the factorial design; 5 units of the step length; 10% of the significance level for tests of significance of slopes; Ants, Moves and Iterations. If conditions were satisfied by SAM, MSM will be then continued. The iterations replicate until the termination criteria is at the satisfaction state. Whilst continually checking stopping criteria in section VIII, following steps below would be carried out;

Step 1a: Perform a 2^3 design at a random centre point go to *Step 2*, or,
Step 1b: Perform a 2^3 design at a maximal S/N point from preceding design.
Step 2: Fit a regression plane to S/N so that the fitted model has the form, $\hat{y} = \beta_0 + \beta_1 Iterations + \beta_2 Ants + \beta_3 Moves.$
Step 3: Test whether there is evidence that either β_1, β_2 or β_3 is different from zero at the 10% level of significance.
Step 4: If the result is significant, move one step along the path of steepest ascent (the fitted regression line).
Step 5: Perform k+1 design points at the highest S/N vertex from the last SAM design plus one movement.
Step 6: Fit a regression plane to the k+1 design points plus the centre coordinate.
Step 7: Test for a significance level of the design vertices plus one centre coordinate.
Step 8: If the result is significant, a reflection or an extension will be carrying out, while omitting the regression plane and applying ordinary MSM motions.
Step 9: Return to *Step 1b*.

Stopping Criteria for a Hybrid Algorithm;

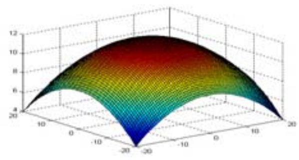
- Parameter default rule – when the coordinates escape from the first quadrant of ACO parameters or the upper or lower limit, or,
- Dispersion rule – when the best four S/N's meet the preset standard deviation (SD); 0.0005 for no noise operation, 0.005 for noise equal to one and 0.05 for noise equal to three, and,
- Regression verification rule – when a significance level of the regression of SAM or MSM is more than 0.1.

VII. TESTED FUNCTIONS

In this paper, eight non-linear continuous mathematical functions were used to test performance measures of the related methods whilst searching for proper ACO parameter settings.

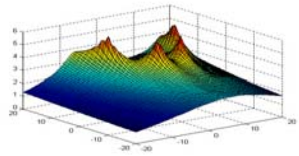
A. Parabolic Function

$$f(x_1, x_2) = 12 - (x_1^2 + x_2^2/100)$$



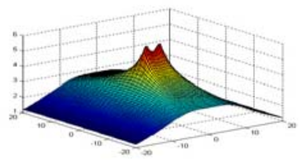
B. Branin Function

$$f(x_1, x_2) = 5 - \log_{10}[(x_2 - (5.1/4\pi^2)\, x_1^2 + ((5/\pi)\, x_1 - 6)^2 + ((10 - (5/4\pi))\cos(x_1)) + 10]$$



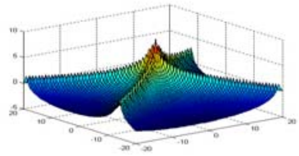
C. Camelback Function

$$f(x_1, x_2) = 10 - \log_{10}[x_1^2 (4 - 2.1\, x_1^2 + (1/3)\, x_1^4) + x_1 x_2 + 4\, x_2^2 (x_2^2 - 1)]$$



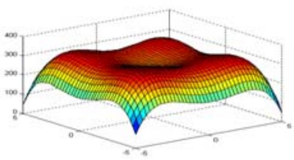
D. Goldstein-Price Function

$$f(x_1, x_2) = 10 + \log_{10}[1/\{(1 + (1 + x_1 + x_2)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)) * (30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))\}]$$



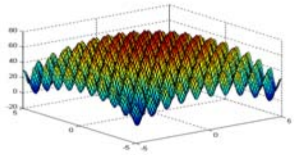
E. Styblinski Function

$$f(x_1, x_2) = 275 - [((x_1^4 - 16x_1^2 + 5x_1)/2) + ((x_2^4 - 16x_2^2 + 5x_2)/2) + 3]$$



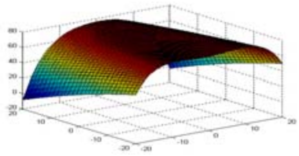
F. Rastrigin Function

$$f(x_1, x_2) = 80 - [20 + x_1^2 + x_2^2 - 10(\cos(2\pi x_1) + \cos(2\pi x_2))]$$



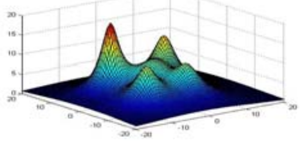
G. Rosenbrock Function

$$f(x_1, x_2) = 70\ [[[\,20 - \{(1 - x_1/-7)^2 + ((x_2/6) + (x_1/-7)^2)^2\}]$$
$$+150]/170] + 10$$



H. Shekel Function

$$f(x_1, x_2) = 100\ \ [1/(9 + (x_1 - 4)^2 + (x_2 - 6)^2) +$$
$$1/(20 + (x_1 + 0)^2 + (x_2 - 0)^2) + 1/(14 + (x_1 - 8)^2 + (x_2 + 3)^2)$$
$$+ 1/(11 + (x_1 - 8)^2 + (x_2 - 8)^2) + 1/(6 + (x_1 + 6)^2 + (x_2 - 7)^2)$$



VIII. Experimental Design And Analysis

In this work, a computer simulation program was developed using Matlab 2006v.7.3B, and EVOPtimiser v.1.1.0. A Laptop computer DV2000 HP Pavilion was used for computational experiments. The proposed method is designed to use S/N as improving trigger, rather than ordinary yield.

If the range of factorial design points from SAM or simplex design points from MSM led to significant effect of $\beta_1, \beta_2,$ *or* β_3 (Table I). The design shape will move one step forward. The precedent is followed by analysing the current design shape plus one move and a maximal design point will be used as a vertex for the next design method.

Countering the latter if P-value exceeds the preset value of significance level, there's no effect of regression coefficients. Accordingly to noises, operating points from either simplex or factorial design may need to duplicate and measure yields to satisfy Regression verification rule. If Dispersion rule is not satisfied but Regression verification rule is met, a replication from the last three realisations will be chosen from the least significance level without a consideration of the preset value. The algorithm does proceed to the next design method and the only chosen one will be attributed to the prior-best-four calculation.

It is also stated that ACO's parameters have to only be positive integers. Consequently the process will confront with round-up error that would probably create a premature stop. On the early phase, MSM are more efficient for some surfaces. When levels of noises increase in the system, computational time taken is also longer due to complexities of ACO algorithm [22]. Some non-linear continuous functions have effects of keen peak, zig-zag and multi local optimum. These effects and the nature of the algorithm can terminate the final results uncertainly. However, the results did show the high level of the spread of yields. The performance of the MSM algorithm was then enhanced via SAM and Taguchi's S/N.

When there is no noise (Table II) the performance of pure MSM brought the same level of average yields. However, the hybrid algorithm seems to be better in terms of the number of design points. The performances of the hybrid continue to be more preferable when levels of noises increase in the system, (Tables III and IV). Number of design points is taken more when compared.

From Table V, preferable levels of parameters found by the pure MSM and hybrid algorithms are determined and are set to be suggested levels for ACO's parameters, to promote an ease of use in all classes of equations. Under a consideration of recommended levels of its parameters, those may bring the benefit to solve industrial processes via ACO when the nature of the problems can be categorised as unimodal, multimodal or curve ridge including the mixed nature of multimodal and curve ridge response surface.

An extension on super modified simplex method could be applied to enhance the performance of MSM [17] when computational processes exceed the upper or lower limit (Parameter default rule). For SAM, a polynomial of higher degree function may be used if there is an evidence of curvature in the system. Hybrid algorithm would be more efficient in terms of Best So Far (BSF) solutions if the preset SD was set at lower level than what had been presented. Number of motions by each method could be enlarged to speed up the process improvement. Moreover, it tends to increase chances of moving directly toward the optimal direction by a regression path of SAM. For MSM the enlargement would probably lead to an increase in capability of local searches.

TABLE I
Analysis of Variance (ANOVA) and Regression Coefficients and their Significance for Parabolic Function without Noise.

Sources of Variation	*Df*	*SS*	*MS*	*F*	*P-value*
Regression	3	0.93	0.31	11.03	0.021001
Residual	4	0.11	0.03		
Total	7	1.05			

	Coefficients	*t Stat*	*P-value*
Iterations, β_1	0.118	1.992	0.117
Ants, β_2	0.085	1.427	0.227
Moves, β_3	0.309	5.205	0.006

TABLE II
Experimental Results Obtained from Related Methods on each Tested Function without Noise.

Function Name	MSM					Hybrid				
	Average BSF and Round-up Parameters					Average BSF and Round-up Parameters				
	Actual Yield	Iterations	Ants	Moves	Design Points	Actual Yield	Iterations	Ants	Moves	Design Points
Branin	5.922	7	12	10	49	5.922	9	14	8	25
Camelback	28.252	8	14	14	55	26.286	10	7	11	156
Goldstein-Price	8.901	9	15	19	47	8.901	5	6	14	82
Parabolic	12.000	6	7	6	31	12.000	6	7	7	39
Rastrigin	100.000	7	8	13	33	100.000	7	4	13	66
Rosenbrock	80.000	3	4	5	21	80.000	6	4	8	24
Shekel	18.981	7	14	12	27	18.980	4	8	9	40
Styblinski	353.332	6	5	9	40	353.332	5	10	7	24

TABLE III
Experimental Results Obtained from Related Methods on each Tested Function with Noise Standard Deviation of 1.

Function Name	MSM					Hybrid				
	Average BSF and Round-up Parameters					Average BSF and Round-up Parameters				
	Actual Yield	Iterations	Ants	Moves	Design Points	Actual Yield	Iterations	Ants	Moves	Design Points
Branin	5.922	7	20	11	36	5.922	14	8	13	114
Camelback	30.280	13	11	13	31	36.526	11	11	19	122
Goldstein-Price	8.901	10	14	9	35	8.901	11	11	16	105
Parabolic	12.000	8	8	10	43	12.000	6	7	8	24
Rastrigin	99.644	4	4	6	46	99.913	5	4	8	24
Rosenbrock	80.000	13	16	12	26	80.000	6	6	7	24
Shekel	18.981	10	11	23	28	18.979	6	6	8	50
Styblinski	353.332	11	9	12	35	353.332	6	9	9	24

TABLE IV
Experimental Results Obtained from Related Methods on each Tested Function with Noise Standard Deviation of 3.

Function Name	MSM					Hybrid				
	Average BSF and Round-up Parameters					Average BSF and Round-up Parameters				
	Actual Yield	Iterations	Ants	Moves	Design Points	Actual Yield	Iterations	Ants	Moves	Design Points
Branin	5.922	10	6	13	39	5.922	7	7	12	40
Camelback	17.537	4	3	7	27	29.096	8	6	13	65
Goldstein-Price	8.901	7	15	11	29	8.868	4	5	7	24
Parabolic	12.000	9	8	11	37	12.000	5	6	8	24
Rastrigin	99.840	6	5	6	36	100.000	6	5	11	23
Rosenbrock	80.000	6	4	8	30	79.879	3	2	3	24
Shekel	18.981	5	8	10	25	18.981	9	8	9	25
Styblinski	353.332	6	11	13	33	353.332	9	4	10	40

TABLE V
Recommended Levels of Parameter Settings without Noise (N=0) and with Noise (N=*x*).

Function Name	Recommended Levels of Parameters		MSM	Hybrid
Branin	N=0	(9,14,8)*		✓
	N=*x*	(11,8,13)		✓
Camelback	N=0	(8,14,14)	✓	
	N=*x*	(9,9,16)		✓
Goldstein-Price	N=0	(5,6,14)		✓
	N=*x*	(9,15,10)	✓	
Parabolic	N=0	(6,7,7)		✓
	N=*x*	(6,7,8)		✓
Rastrigin	N=0	(7,4,13)		✓
	N=*x*	(6,5,10)		✓
Rosenbrock	N=0	(3,4,5)	✓	
	N=*x*	(10,10,10)	✓	
Shekel	N=0	(7,14,12)	✓	
	N=*x*	(8,10,17)	✓	
Styblinski	N=0	(5,10,7)		✓
	N=*x*	(8,7,10)		✓

Note: (a,b,c)*: a = (Average) Iterations, b = (Average) Ants, c = (Average) Moves

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