On Limitations of the Prioritization Methods in Analytic Hierarchy Process: A Study of Transportation Selection Problems

Kevin Kam Fung Yuen*

Abstract— The most appropriate prioritization method is still one of the unsettled issues of the Analytic Hierarchy Process, although many studies have been made and applied. Interestingly, many transportation evaluation and selection models using AHP apply only Saaty's Eigenvector method. Many theoretical studies have found that this method may produce rank reversals and have proposed various prioritization methods as alternatives. Some methods have been proved to be superior to the Eigenvector method. However, a literature review shows that these methods seem not to attract the attention of researchers in this research area. In this paper, a Prioritization Operator Recommendation Model is proposed to address this issue. Eight important prioritization methods are reviewed. A Mixed Prioritization Operators model is developed to select a vector which has the most appropriate prioritization operators for generic evaluation and selection problems in transportation. To verify this new model, a case study is revised using the proposed model. The contribution is that MPOs are useful for solving prioritization problems in the AHP.

Index Terms—, Analytic Hierarchy Process, Multi Criteria Decision Making, Rank reversal, Transportation Selection

I. INTRODUCTION

In Multiple Criteria Decision Making, the criteria are usually classified as being of two kinds: quantitative criteria which can be measured by number such as amount of time, labor, cost, and price; qualitative criteria such as quality and qualitative weights which cannot be objectively and directly determined, but which are usually determined by subjective judgment. The Analytic Hierarchy Process [13] is the popular tool for the subjective judgment of the qualitative data. In the transportation evaluation and selection problems, the AHP can be applied to the qualitative analysis.

For the evaluation issue, AHP can be used for determining the weights of multi-criteria, which are further incorporated into other models. For example, Tanadtang et al. [18] adopted AHP to measure the weights of criteria for evaluating transportation demand management (TDM) alternatives. Sohn[16] proposed a model for the Seoul Metropolitan Government to eliminate some useless overpasses among 22 existing overpasses, and the weights of this model adopted AHP methodology. In selection problems, Saaty [12, 15] proposed applying AHP in different scenarios in transport planning. Ferrari [7] proposed a method for choosing from among alternative transportation projects. Kulak and Kahraman [9] proposed a crisp AHP model and a fuzzy AHP model to solve transportation company selection problems. The methodology of a fuzzy AHP model also can be found in [21-23]. A Crisp AHP is a special case of a fuzzy AHP.

The AHP includes three major processes: assessment, prioritization, and synthesis. In the assessment, verbal judgments are given by decision makers for pairwise comparisons. The verbal judgment is usually on a 9 point verbal scale represented by numbers: 1 for equal importance, 2 for weak importance, and finally 9 for extreme importance. For pairwise comparisons, a_{ii} is a numeric point to estimate the relative importance of object i over object j, and $A = \{a_{ij}\}$, $0 < a_{ii} = a_{ii}^{-1}$, i, j = 1, 2, ..., n is a pairwise comparison matrix. Thus a pairwise comparison matrix is also called a reciprocal matrix. The reciprocal matrices of all assessments are formed by transforming the linguistic labels to numerical values. In the prioritization process, a local priority vector $W = \{w_1, \dots, w_n\}, \sum_{i=1}^n w_i = 1$ is generated from a reciprocal matrix A by a Prioritization Operator (PO), i.e. $PO: A \rightarrow W$. In the synthesis stage, these local priority vectors W's are aggregated as a global priority vector $V = \{v_1, \dots, v_n\}$ by an aggregation operator $Agg : \{W\} \rightarrow V$.

These processes are likely to induce three fundamental problems: (i) selection of numerical scales in stage one; (ii) selection of prioritization operators (or methods) in stage two, and (iii) selection of aggregation operators in stage three. These three problems probably create rank reversals. Problem (i) is addressed by [19,24]. This research focuses on the solution of rank reversals due to problem (ii) using the most appropriate approximate prioritization operator.

Although there are many POs (illustrated in section 2), actually the best prioritization operator relies on the content of a pairwise matrix, and none of prioritization methods performs better than the others in every inconsistent case. The application in this study and Refs.[10,17,20] also verify this issue. Thus, it is most appropriate to propose a framework to select the most appropriate prioritization operator for each reciprocal matrix among sufficient PO candidates with an objective measurement method. This study proposes a Mixed Prioritization Operators strategy for this measurement

Kevin Kam Fung Yuen is with Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hung Hom, KLN, Hong Kong (e-mail:kevinkf.yuen@gmail.com; ise.kevinyuen@polyu.edu.hk)

method.

The structure of this article is as follows: Section 2 reviews the Prioritization Operators. Section 3 proposes a measurement model to measure the POs, and Mixed POs are proposed. Section 4 revises the application adopted from Kulak and Kahramna [9] using Mix POs strategy. In section 5 a conclusion to this paper is drawn.

II. PRIORITIZATION OPERATORS

Eight essential prioritization operators are reviewed as follows:

2.1 Eigenvector(EV)

The Eigenvector operator for intuitive justification is proposed by Saaty[13]. EV derives the principal eigenvector λ_{max} of *A* as the priority vector w by solving following Eigen system.

$$Aw = \lambda_{\max} w$$
, and $\sum_{i=1}^{n} w_i = 1$ (1)

A is consistent if and only if $\lambda_{\max} = n$, and is not consistent if and only if $\lambda_{\max} > n$ where $\lambda_{\max} \ge n$.

The Normalization operator was introduced in Saaty[13]. The following methods (2.2-2.5) are named according to their calculation steps since Saaty[13] has not given them appropriate names.

2.2 Normalization of the Row Sum (NRS)

NRS sums up the elements in each row and normalizes by dividing each sum by the total of all the sums, thus the results now add up to unity. NRS has the form:

$$a'_{i} = \sum_{j=1}^{m} a_{ij} \quad i = 1, 2, ..., n$$

$$w_{i} = \frac{a'_{i}}{\sum_{i=1}^{n} a'_{i}} \quad i = 1, 2, ..., n$$
 (2)

2.3 Normalization of Reciprocals of Column Sum (NRCS)

NRCS takes the sum of the elements in each column, forms the reciprocals of these sums, and then normalizes so that these numbers add up to unity, e.g. to divide each reciprocal by the sum of the reciprocals. It is in this form:

$$a'_{i} = \frac{1}{\sum_{i=1}^{n} a_{ij}} \quad j = 1, 2, ..., n$$

$$w_{i} = \frac{a'_{i}}{\sum_{i=1}^{n} a'_{i}} \quad i = 1, 2, ..., n$$
(3)

2.4 Arithmetic Mean of Normalized Columns (AMNC)

AMNC was also called the Additive Normalization method in [10]. The new name is relatively clear, in that it describes its calculation process. Each element in A is divided by the sum of each column in A, and then the mean of each row is taken as the priority w_i . It has the form:

$$a'_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}$$
 $i, j = 1, 2, ..., n$, and

$$w_i = \frac{1}{n} \sum_{j=1}^{n} a'_{ij} \quad i = 1, 2, \dots, n$$
(4)

The AHP applications use this method due to the simplicity of its calculation process.

2.5 Normalization of Geometric Means of Rows (NGMR) / Logarithm Least Squares (LLS)

NHMR multiplies the *n* elements in each row and takes the *n*th root, and then normalizes so that these numbers add up to unity. It is in the form:

$$w'_{i} = \prod_{j=1}^{n} a_{ij}^{1/n}, \ i = 1, 2, \dots, n$$

$$w_{i} = \frac{w'_{i}}{\sum_{i=1}^{n} w'_{i}}, \quad i = 1, 2, \dots, n$$
(5)

Although it is more complex than other three normalization methods, it is recommended by some authors [e.g. 1, 4] since this method produces the same result as LLS, which is in the form:

Min
$$\sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij} - \frac{w_i}{w_j} \right)^2$$
 (6)

Subject to $\sum_{i=1}^{n} w_i = 1, \ w_i > 0, i = 1, 2, ..., n$

Saaty and Vargas [14] made comparisons among EV, LLS, and DLS. EV and LLS were discussed previously. DLS is as follows.

2.6 Direct Least Squares / Weighted Least Squares (DLS/WLS)

This method is used to minimize the sum of errors of the differences between the judgments and their derived values. The Direct Least Squares, which was proposed in [5], is in the form:

$$\operatorname{Min} \qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij} - \frac{w_i}{w_j} \right)^2 \tag{7}$$
Subject to $\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1, 2, ..., n$

Subject to $\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1, 2, ..., n$

The above non-linear optimization problem has no special tractable form or closed form and thus is very difficult to be solved [5]. For efficient computation with a closed form, Chu at el. [5] modified the objective function and proposed the Weighted Least Squares (WLS) in the form:

Min
$$\sum_{i=1}^{n} \sum_{j=1}^{n} (w_i - a_{ij}w_j)^2$$

Subject to $\sum_{i=1}^{n} w_i = 1, \ w_i > 0, i = 1, 2, ..., n$ (8)

2.7 Fuzzy Programming (FP)

The FP is proposed by Mikehailov [11], which has the form: max μ

Subject to
$$\mu d_{j}^{+} + R_{j}W^{T} \ge d_{j}^{+},$$

 $\mu d_{j}^{-} - R_{j}W^{T} \ge d_{j}^{-}, \quad j = 1, 2, ..., m, \quad 1 \ge \mu \ge 0$
 $\sum_{i=1}^{n} w_{i} = 1, \quad w_{i} > 0, \quad j = 1, 2, ..., n$
(9)

 $R_j \in R^{m \times n} = \{a_{ij}\}$ is the row vector. The values of the left and right tolerance parameters d_j^- and d_j^+ represent the admissible interval of approximate satisfaction of the crisp equality $R_j W^T = 0$. The measure of intersection of μ is a natural consistency index of the FP. Its value however depends on the tolerance parameters. For the practical implementation of the FP, it is reasonable that all these parameters be set equal to each other. The limitation of this method is that parameters d_j^- and d_j^+ are undetermined in [11]. This leads to infinite candidate values for them. [11] set $d_i^- = d_i^+ = 1$ in his example.

2.8 Goal Programming (GP)

Bryson [2] proposed a goal programming operator (GP) with the form:

Min $\ln \Theta = \sum_{i=1}^{n} \sum_{j>i}^{n} \left(\ln \delta_{ij}^{+} + \ln \delta_{ij}^{-} \right)$

Subject to $\ln w_i - \ln w_j + \ln \delta_{ij}^+ - \ln \delta_{ij}^- = \ln a_{ij}, \quad \forall (i, j) \in IJ$ where $IJ = \{(i, j) : 1 \le i < j \le n\}$; $\ln \delta_{ij}^+$ and $\ln \delta_{ij}^-$ are non-negative. (10)

Ideally the objective value should be 0 when $\ln \delta_{ij}^+ = \ln \delta_{ij}^- = 0$, i.e. $\delta_{ij}^+ = \delta_{ij}^- = 1$.

When priorities are used as rank, the exact values of the rank are not so important so long as the ranks are preserved. However, when the priorities are used as weights, not only is the rank preservation important, but also the exact values of the priorities of the weights are essential. A little variation in the value of the priorities might possibly result in a different solution. When there are many prioritization operators, it is necessary to develop a measurement model to determine which operator is the most appropriate one. This is addressed in the next section.

III. MEASUREMENT MODEL AND MIXED PRIORITIZATION OPERATORS STRATEGY

The measurement models measure the validity of the prioritization operators. Thus they can be used for the comparing different POs. Two variance methods are reviewed and a new variance model, which is used for Mix POs strategy, is developed.

To measure the distribution of the variance, one approach is to use Root Mean Square Variance which has the form

$$RMSV(A,W) = \sqrt{\frac{1}{n \times n} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij} - \frac{w_i}{w_j}\right)^2}$$
(11)

A is a pairwise matrix $\{a_{ij}\}$, W is a priorities vector of a prioritization operator, and $w_i, w_j \in W \in \{W_1, \dots, W_K\} = \{W\}$.

If $\frac{1}{n \times n}$ is taken out, the new form is Euclidean Distance,

which was used by [8]. For easier interpretation of the result, it is more appropriate to use the average of the value. Thus RMSV is preferred.

However, a limitation of RMSV is that the weights for the penalty are not justified. For example, the penalty of the

$$\begin{pmatrix} w_i > w_j & a_{ij} < 1 & a_{ij} \neq \frac{w_i}{w_j} \end{pmatrix} = True$$

same as the one of the condition
$$\begin{pmatrix} w_i > w_j & a_{ij} > 1 & a_{ji} \neq \frac{w_j}{w_j} \end{pmatrix} = True.$$

To determine the variance associated with weights, Minimum Violation [8], $MV(A,W) = \sum_{i} \sum_{j} I_{ij}$, can be used as weight determination. However, one mistake is that the condition $\{w_i < w_j \& a_{ij} > 1\}$ is not appropriated in the piecewise function I_{ij} . In addition, as the value of MV depends on the size (n^2) of the matrix (usually a larger sized matrix leads to a higher value of MV), the revised Mean value of MV is more appropriate for measuring POs. Thus, MMV has the form:

$$MMV(A,W) = \frac{1}{n^2} \left(\sum_{i} \sum_{j} I_{ij} \right)$$

where $I_{ij} = \begin{cases} 1 & w_i > w_j \& a_{ij} < 1 \\ 1 & w_i < w_j \& a_{ij} > 1 \\ 0.5 & w_i = w_j \& a_{ji} \neq 1 ; \\ 0.5 & w_i \neq w_j \& a_{ji} = 1 \\ 0 & Otherwise \end{cases}$
(12)

A limitation of MMV is that it takes care of the penalty scores only, and ignores the actual variance values.

To combine the advantage of Root Mean Square Variance and Mean Minimum Violation, as well as offset their shortages, this paper proposes the Weighted Root Mean Square Variance method, which is expressed as:

$$\sigma = WRMSV(A, \{W\}) = \sqrt{\sum_{i} \sum_{j} \frac{Y_{ij}}{n \times n}}, \text{ where}$$

$$Y_{ij} = \begin{cases} \beta_1 \left(a_{ji} - \frac{w_j}{w_i}\right)^2, w_i > w_j \& a_{ij} < 1 \\ or \ w_i < w_j \& a_{ij} > 1 \\ \beta_2 \left(a_{ji} - \frac{w_j}{w_i}\right)^2, w_i = w_j \& a_{ji} \neq 1 \\ or \ w_i \neq w_j \& a_{ji} = 1 \\ \beta_3 \left(a_{ji} - \frac{w_j}{w_i}\right)^2 , otherwise \end{cases}$$

$$\rho, \beta_1 \ge \beta_2 \ge \beta_3 = 1 \tag{13}$$

 $\beta_1, \beta_2, \beta_3$ are the penalty weights. RMSV is the special case of WDV if $\beta_1 = \beta_2 = \beta_3 = 1$. In MMV, $\beta_1 = 1, \beta_2 = 0.5, \beta_3 = 0$. However, by default setting of WRMSV, $\beta_1 = 2, \beta_2 = 1.5, \beta_3 = 1$ are defined as $\beta_3 = 0$ can

cancel the variance when $\left(w_i > w_j \& a_{ij} > 1 \& a_{ij} \neq \frac{w_i}{w_j}\right)$ is

true.

Next, WRMSV can be used to develop a Mix POs strategy. For a set of the priorities vectors $\{W\}$ of the prioritization candidates, a set of WRMSV is the form:

 $WRMSV(A, \{W\}) = \{\sigma_1, \dots, \sigma_k, \dots, \sigma_K\}, \text{ K is the cardinal}$ number of $\{W\}$ (14)

Less variance reflects better fitness. For a set of POs, \overline{P} , the best PO is expressed as:

$$p^* = p_{\gamma}$$
, where the best PO index
 $\gamma = \underset{k \in \{1, 2, \dots, K\}}{\operatorname{arg\,min}} \{\{\sigma_1, \dots, \sigma_K\}\}, \text{ and the best PO } p^* \in \overline{P}$ (15)

Thus, Mixed POs use a vector of best PO for a set of pairwise matrices $\{A\}$ is the form:

 $\overline{P}^* = \{p^*_1, \dots, p^*_t, \dots, p^*_T\}, \text{ T is the number of a set of}$ pairwise matrices $\{A\}$. (16)

In the next section the validity of the MPOs approach is illustrated.

IV. APPLICATION

In this application, the transportation company selection problem from Kulak and Kahramna [9] is revised by the proposed method. One company is selected from four candidates using five criteria: cost, defect rate, tardiness rate, flexibility and documentation ability. The details of the problem are shown in the Appendices.

In this comparative analysis, a set of eight POs are used: $\overline{P} = \{p_1, ..., p_8\} = \{\text{EV}, \text{NRS}, \text{NRCS}, \text{AMNC}, \text{NGNR/LLS}, \text{WLS}, \text{FP}, \text{GP}\}$. Table 1 shows the priorities and WRMSV for six pairwise matrices using eight prioritization operators. The best prioritization operator is the one with the least WRMSV. The result of using the method proposed by Kulak and Kahranna [9] is the same as the result obtained when using P4 (AMNC). The Saaty's Eigen vector method is P₁. It can be observed that neither of these two, in the six matrices is selected as the best PO. The best PO is the one with the fonts of bold, italic and underlined in Table 1. In the six matrices, the best POs are P_2 , P_2 , P_8 , P_6 , and P_6 . It can be concluded that the best PO is case dependant as no PO can outperform other POs in all matrix cases.

Table 1: The priorities and WRMSV for six pairwise matrices using eight POs

W_1	C_1	C_2	C ₃	C_4	C ₅	WRMSV	C_1	B_1	B_2	B_3	B_4	WRMSV
P ₁	0.513	0.108	0.156	0.195	0.027	2.129	P_1	0.099	0.284	0.099	0.518	0.107
\underline{P}_2	<u>0.398</u>	<u>0.159</u>	<u>0.188</u>	<u>0.229</u>	<u>0.026</u>	<u>1.529</u>	<u>P</u> ₂	<u>0.099</u>	<u>0.293</u>	<u>0.099</u>	<u>0.508</u>	<u>0.085</u>
P_3	0.563	0.102	0.156	0.146	0.032	2.070	P_3	0.100	0.273	0.100	0.527	0.136
P_4	0.494	0.114	0.163	0.199	0.029	1.851	P_4	0.099	0.284	0.099	0.518	0.104
P_5	0.503	0.111	0.162	0.198	0.027	2.133	P_5	0.099	0.284	0.099	0.518	0.105
P_6	0.553	0.110	0.167	0.136	0.034	1.949	P_6	0.101	0.275	0.101	0.523	0.120
P_7	0.488	0.130	0.216	0.130	0.036	1.605	P_7	0.102	0.275	0.095	0.528	0.172
P ₈	0.504	0.101	0.168	0.202	0.025	2.343	P ₈	0.100	0.300	0.100	0.500	0.087
C_2	B_1	B_2	B_3	B_4		WRMSV	C_3	B_1	B_2	B_3	B_4	WRMSV
\mathbf{P}_1	0.565	0.055	0.262	0.118		0.887	P_1	0.087	0.311	0.549	0.053	1.073
P_2	0.507	0.053	0.296	0.144		0.855	P_2	0.098	0.386	0.464	0.052	0.742
P_3	0.605	0.063	0.224	0.109		0.855	P_3	0.089	0.237	0.613	0.060	1.344
P_4	0.558	0.057	0.263	0.122		0.809	P_4	0.092	0.319	0.533	0.055	0.922
P_5	0.564	0.055	0.263	0.118		0.893	P_5	0.090	0.313	0.543	0.054	1.000
P_6	0.593	0.068	0.227	0.111		0.745	P_6	0.089	0.252	0.597	0.061	1.231
P_7	0.583	0.095	0.221	0.101		0.917	P_7	0.167	0.278	0.500	0.056	1.270
<u>P</u> 8	<u>0.521</u>	<u>0.063</u>	<u>0.313</u>	<u>0.104</u>		<u>0.585</u>	<u>P</u> 8	<u>0.085</u>	<u>0.427</u>	<u>0.427</u>	<u>0.061</u>	<u>0.665</u>
C_4	\mathbf{B}_1	B_2	B_3	B_4		WRMSV	C ₅	\mathbf{B}_1	B_2	B_3	B_4	WRMSV
\mathbf{P}_1	0.249	0.054	0.592	0.105		1.111	P_1	0.078	0.522	0.200	0.200	0.471
P_2	0.279	0.050	0.538	0.134		1.258	P_2	0.076	0.489	0.217	0.217	0.450
P_3	0.223	0.063	0.625	0.089		0.916	P_3	0.084	0.539	0.189	0.189	0.450
P_4	0.251	0.056	0.584	0.109		1.024	P_4	0.079	0.519	0.201	0.201	0.453
P_5	0.253	0.053	0.590	0.104		1.143	P_5	0.078	0.520	0.201	0.201	0.475
<u>P</u> ₆	<u>0.227</u>	<u>0.069</u>	<u>0.615</u>	<u>0.089</u>		<u>0.792</u>	<u>P</u> 6	<u>0.088</u>	<u>0.534</u>	<u>0.189</u>	<u>0.189</u>	<u>0.409</u>
\mathbf{P}_7	0.239	0.054	0.616	0.091		1.150	P_7	0.085	0.540	0.188	0.188	0.441
P ₈	0.216	0.043	0.648	0.093		2.019	P ₈	0.063	0.563	0.188	0.188	1.000

	\mathbf{P}_1	P_2	P_3	P_4	P ₅	P_6	\mathbf{P}_7	P_8	MPOs
A_1	0.176	0.204	0.167	0.18	0.179	0.17	0.162	0.196	0.193
A_2	0.225	0.222	0.224	0.225	0.224	0.229	0.252	0.233	0.237
A_3	0.286	0.302*	0.272	0.288	0.289	0.271	0.289	0.27	0.315*
A_4	0.313*	0.271	0.336*	0.306*	0.308*	0.33*	0.296*	0.301*	0.256
Sum(WRMSV)	5.777	4.919	5.770	5.163	5.749	5.247	5.555	6.699	4.065*

Table 2: The synthesis results of eight POs and a Mixed POs method

Table 2 shows the results of a synthesis of eight POs and a Mixed POs (MPOs) method. p_1, \ldots, p_8 apply a pure PO strategy to derive the local priorities, and then synthesize the result. The result shows that p_1, p_3, \dots, p_8 support the fact that the transportation company 4 is the best candidate whilst p_2 and the MPOs method show that company 3 is the best. To measure the results, the WRMSVs of all pairwise matrices for all methods are summed up. It can be found that P2 and MPOs are with the least two summation of WRMSVs. In this example, summation of WRMSV, more than five possible produce inaccurate results. Although NRS (P₂) is the most straightforward, its accuracy probably is better than other methods. This result is supported by MPOs. As MPOs apply the best PO regarding cases, the result of MPOs are much better than the pure PO method.

The results of many of the transportation studies in AHP stated in the introduction section are dubious as the studies only applied a pure PO strategy using the Eigenvalue method without comparison with other prioritization operators. The Mixed POs strategy is recommended, especially for those quantitative and qualitative researches with subjective measurement weights. The literature reviews also find most of the studies did not incorporate the data of the pairwise matrix to derive the weights which are further propagated with other criteria measurement data. Thus some research findings are questionable.

V. CONCLUSION

The most appropriate prioritization method is still one of the unsettled issues of Analytic Hierarchy Process theory, although many studies have been done and applied. Interestingly, many studies of transportation evaluation and selection models using AHP use only a single prioritization method: Saaty's Eigenvector method. Many theoretical studies have found this method to be unsatisfactory and have proposed various prioritization methods as alternatives. Some have proved to be superior to the Eigenvector method. However, a literature review shows that these methods seem not to attract the attention of researchers in this research area.

In this paper, a Mix Prioritization Operators strategy is proposed to address this issue. Eight important prioritization operators are reviewed: Eigenvector method, Normalization of the Row Sum, Normalization of Reciprocals of Column Sum, Arithmetic Mean of Normalized Columns, Row Geometric Mean or Logarithm Least Squares, Weighted Least Squares, Fuzzy Programming, and Goal Programming. The Weighted Root Mean Square Variance method, which combines the advantages and offsets the disadvantages of Root Mean Square Variance and Mean Minimum Violation, is proposed for the selection of the best PO for solving AHP Transportation problems.

To ensure the validity and usability of this new model, one case study selected from Kulak and Kahramna [9] is used to compare the outcomes of the Pure PO strategy and the Mixed POs strategy. The research result shows that a single pure PO strategy possibly produces unreliable results as it may have high Weighted Root Mean Square Variance. On the other hand, the MPOs strategy can produce more convincing results than the pure PO strategy since the multiple best prioritization operators are used to minimize the approximate variance. The contribution of this paper is that the MPOs model may be used as the guidelines for developing an AHP transportation model, especially for the important unsettled issue regarding the prioritization problem in AHP.

APPENDIX

The following two tables are the summary of the details of the transportation company selection problem using crisp AHP, which is adopted from Kulak and Kahramna [9].

Table A1: Criteria and alternatives

Criteria	Description	Labels	Alternatives	Labels
ТС	Transportation Cost	C ₁	Transport Company 1	B_1
DR	Defective rate	C_2	Transport Company 2	B_2
TR	Tardiness Rate	C_3	Transport Company 3	B_3
F	Flexibility	C_4	Transport Company 4	B_4
DA	Documentation Ability	C_5		

Table A2	. The Pair	wise matrice	38							
С	TC	DR	TR	F	DA	C_1	\mathbf{B}_1	B_2	B_3	B_4
TC	1.00	5.00	3.00	5.00	9.00	B_1	1.00	0.33	1.00	0.20
DR	0.20	1.00	0.50	0.50	7.00	B_2	3.00	1.00	3.00	0.50
TR	0.33	2.00	1.00	0.50	7.00	B_3	1.00	0.33	1.00	0.20
F	0.20	2.00	2.00	1.00	8.00	B_4	5.00	2.00	5.00	1.00
DA	0.11	0.14	0.14	0.13	1.00					
C_2	\mathbf{B}_1	B_2	B_3	B_4		C_3	B_1	B_2	B_3	B_4
B_1	1.00	7.00	3.00	5.00		B_1	1.00	0.20	0.20	2.00
B_2	0.14	1.00	0.20	0.33		B_2	5.00	1.00	0.33	7.00
B_3	0.33	5.00	1.00	3.00		B_3	5.00	3.00	1.00	7.00
B_4	0.20	3.00	0.33	1.00		B_4	0.50	0.14	0.14	1.00
C_4	B_1	B_2	B_3	B_4		C ₅	B_1	B_2	B ₃	B_4
B_1	1.00	5.00	0.33	3.00		B_1	1.00	0.20	0.33	0.33
B_2	0.20	1.00	0.14	0.33		B_2	5.00	1.00	3.00	3.00
B_3	3.00	7.00	1.00	7.00		B_3	3.00	0.33	1.00	1.00
B_4	0.33	3.00	0.14	1.00		B_4	3.00	0.33	1.00	1.00

Table A2: The Pairwise matrices

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