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A Tabu Search Approach based on Strategic Vibration for Competitive Facility Location Problems with Random Demands

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Abstract— **This paper proposes a new location problem of competitive facilities, e.g. shops and stores, with uncertain demands in the plain. By representing the demands for facilities as random variables, the location problem is formulated to a stochastic programming problem, and it is reformulated to three deterministic programming problems: expectation maximizing problem, probability maximizing problem, and satisfying level maximizing problem. . After showing that one of their optimal solutions can be found by solving 0-1 programming problems, their solution method is proposed by improving the tabu search algorithm based on strategic vibration. The efficiency of the solution method is shown by applying it to numerical examples of the facility location problems.**

Keywords: facility location, competitiveness, stochastic programming, 0-1 programming, tabu search

1 Introduction

Competitive facility location problem (CFLP) is one of optimal location problems for commercial facilities, e.g. shops and stores, and the objective of a decision maker (DM) for the CFLP is mainly to obtain as many demands for her/his facilities as possible. Mathematical studies on CFLPs are originated by Hotelling [7]. He considered the CFLP under the conditions that (i) customers are uniformly distributed on a line segment, (ii) each of DMs can locate and move her/his own facility at any times, and (iii) all customers only use the nearest facility. CFLPs on the plain were studied by Okabe and Suzuki [12], etc. As an extension of Hotelling's CFLP, Wendell and McKelvey [20] assumed that there exist customers on a finite number of points, called demand points (DPs), and they considered the CFLP on a network whose nodes are DPs. Based upon the CFLP proposed by Wendell and McKelvey, Hakimi [5] considered CFLPs under the conditions that the DM locates her/his facilities on a network that other competitive facilities were already located. Drezner [3] extended Hakimi's CFLPs to the CFLP on the plain that there are DPs and competitive facilities. As extension of their CFLPs, CFLPs with quality or size of facilities are considered by Uno et al. [16], Fernández et al. [4], Bruno and Improta [2], and Zhang and Rushton [21], CFLPs with fuzziness are considered by Moreno Pérez et al [10], and CFLPs based on maximal covering are considered by Plastria and Vanhaverbeke [13].

In the above studies of CFLPs, the demands of customers for facilities is represented as definite values. Wagnera et al. [19] considered facility location models with random demands in a noncompetitive environment. For the details of location models with random demands, the reader can refer to the study of Berman and Krass [1].

In this paper, we proposes a new competitive facility location problem with random demands by extending Drezner's location model that introduces Huff's attractive function [8]. Then, the location problem can be formulated as a stochastic programming problem. The problem is reformulated to the three deterministic programming problems: expectation maximizing problem, probability maximizing problem, and satisfying level maximizing problem. Because the problems are nonlinear programming problems, it is difficult to find a strict optimal solution of the problems directly. We show that the problems can be reformulated as 0-1 programming problems, and propose their solution method improving the tabu search algorithm with strategic vibration, which Hanafi and Freville [6] proposed for multidimensional knapsack problems. For details of the tabu search algorithms, the readers can refer to the book of Reeves [14]. We apply it to numerical examples of the CFLPs with random demands, and show its efficiency by comparing to other solution algorithms.

The remaining structure of this article is organized as follows. In Section 2, we formulate the CFLP with random demand as a stochastic programming problem. Since it is difficult to solve the formulated problem directly, we show

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that one of its optimal solutions can be found by solving a 0-1 programming problem in Section 3. In Section 4, we propose an efficient solution method based upon tabu search algorithms by utilizing characteristics of the CFLPs. We show the efficiency of the solution method by applying to numerical examples of the CFLPs with random demands in Section 5. Finally, in Section 6, concluding comments and future extensions are summarized.

2 Formulation of CFLP with random demands

In the proposed CFLPs, we assume that all customers only exist on DPs in plain R^2. For convenience sake, by aggregating all customers on the same DP, we regard one DP as one customer.

There are n DPs in R^2, and let $D = \{1, \cdots, n\}$ be the set of indices of the DPs. Let m be the number of new facilities that the DM locates, and k be the number of competitive facilities which have been already located in R^2. The sets of indices of the new facilities and the competitive facilities are denoted by $F = \{1, \ldots, m\}$ and $F_C = \{m+1, \cdots, m+k\}$, respectively.

Let $\boldsymbol{u}_i \in R^2$ be the site of DP $i \in D$, and $\boldsymbol{x}_j \in R^2$ and $q_j > 0$ be the site and quality of facility $j \in F \cup F_C$, respectively. Then, attractive power of facility j for DP i is represented as the following function introduced by Huff [8]:

$$a_i(\boldsymbol{x}_j, q_j) \equiv \begin{cases} \dfrac{q_j}{||\boldsymbol{u}_i - \boldsymbol{x}_j||^2}, & \text{if } ||\boldsymbol{u}_i - \boldsymbol{x}_j|| > \varepsilon, \\ \dfrac{q_j}{\varepsilon^2}, & \text{if } ||\boldsymbol{u}_i - \boldsymbol{x}_j|| \le \varepsilon, \end{cases} \tag{1}$$

where $\varepsilon > 0$ is an upper limit of the distance that customers can move without any trouble. It is assumed that all customers only use one facility with the largest attractive power, and if the two or more attractive powers are the same, they use the facility in reverse numerical order of the indices of facilities; that is, in the order of competitive facilities and new facilities.

Let $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)$ be the location of the new facilities. Then we use the following 0-1 variable for representing whether DP i uses new facility $j \in F$:

$$\varphi_i^j(\boldsymbol{x}) = \begin{cases} 1, & \text{if DP } i \text{ uses the new facility } j, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Let $\bar{w}_i$ be the random variable meaning the buying power (BP) of DP i. New facility $j \in F$ can obtain the BP $\bar{w}_i$ if $\varphi_i^j(\boldsymbol{x}) = 1$. The objective of the DM is maximizing the sum of BP that all the new facilities obtain. Then, the CFLP with random demand is formulated as the following stochastic programming problem:

$$\left.\begin{array}{ll} \text{maximize} & f(\boldsymbol{x}) = \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m} \bar{w}_i \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in R^{2m} \end{array}\right\} \tag{3}$$

For finding an optimal solution of (3), we consider the following three deterministic programming problems: (i) expectation maximizing problem

$$\left.\begin{array}{ll} \text{maximize} & E[f(\boldsymbol{x})] \\ \text{subject to} & \boldsymbol{x} \in R^{2n} \end{array}\right\} \tag{4}$$

(ii) probability maximizing problem

$$\left.\begin{array}{ll} \text{maximize} & Pr[f(\boldsymbol{x}) \ge f_0] \\ \text{subject to} & \boldsymbol{x} \in R^{2n} \end{array}\right\} \tag{5}$$

where f_0 means a given satisfying level of obtaining BP, and (iii) satisfying level maximizing problem

$$\left.\begin{array}{ll} \text{maximize} & f_0 \\ \text{subject to} & Pr[f(\boldsymbol{x}) \ge f_0] \ge \alpha \\ & \boldsymbol{x} \in R^{2n} \end{array}\right\} \tag{6}$$

where α is a given satisfying level of probability that the DM can obtain BP level f_0.

Problems (4), (5), and (6) are nonconvex nonlinear programming problems and we need to find at least one optimal solution for each of the problems. However, for most CFLPs [3, 16], it is difficult to find an optimal solution by using general analytic solution methods with derived functions of the objective function, Kuhn-Tucker conditions, etc. Moreover, Uno and Katagiri [17] and Uno et al. [18] show that it is also difficult to find an optimal solution of such CFLPs by using heuristic solution methods for nonlinear programming problems, e.g. genetic algorithm for numerical optimization for constrained problem (GENOCOP) [9]. In the next section, we show that the above three problems can be reformulated as 0-1 programming problems.

3 Reformulation to combinatorial optimization problems

In the location model introduced in the previous section, if the new facilities are located, then the values of (2) for all facilities and DPs are given. On the other hand, we propose the following solution method:

1. Decide the set of DPs that she/he wants to obtain their BPs preferentially by giving the values of (2) for all facilities and DPs, and
2. Find the location of the new facilities such that (2) for each facility and DP is equal to or more than the given value.

For DP $i \in D$, the largest attractive power among all competitive facilities is denoted as follows:

$$a_i^C \equiv \min_{j \in F_C} \{a_i(\boldsymbol{x}_j, q_j)\}. \tag{7}$$

From (1), the set of DPs that new facility $j \in F$ cannot obtain their BPs wherever it is located is represented as follows:

$$D_j^{\triangle} = \{i \in D \mid \sqrt{q_j/a_i^C} \leq \varepsilon\}. \tag{8}$$

Then, the set of DPs that there is at least one location of facility j which can obtain their BPs is denoted by $D_j = D \backslash D_j^{\triangle}$. For new facility j, let $\bar{D}_j \subseteq D_j$ be the set of DPs that the DM wants to obtain their BPs by locating it preferentially. Let

$$l_{ij} = \begin{cases} 1, & \text{if } i \in \bar{D}_j, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Then, $\bar{D}_j$ can be represented as 0-1 vector $\boldsymbol{l}_j = (l_{1j}, \cdots, l_{nj})$. For new facility j and vector $\boldsymbol{l}_j$ given by the DM, we consider the following problem with an auxiliary variable $r_j \geq 0$:

$$\left.\begin{array}{rl} \text{minimize} & r_j^2 \\ \text{subject to} & ||\boldsymbol{x}_j - \boldsymbol{u}_i||^2 \leq \dfrac{q_j}{a_i^C} \cdot r_j, \\ & \qquad\qquad \forall i \in \{\bar{i} | l_{\bar{i}j} = 1\}, \\ & \boldsymbol{x}_j \in R^2,\ r_j \geq 0. \end{array}\right\} \tag{10}$$

Let $(\boldsymbol{x}_j^{\boldsymbol{l}_j}, r_j^{\boldsymbol{l}_j})$ be an optimal solution of (10). Then, the following theorem plays an important role to find an optimal location.

Theorem 1 *For new facility $j \in F$, let $\bar{D}_j$ be the set of DPs given by the DM, $\boldsymbol{l}_j$ be the 0-1 vector corresponding to $\bar{D}_j$. Then, if $r_j^{\boldsymbol{l}_j} < 1$, the new facility j can obtain all DPs in $\bar{D}$ by locating it at $\boldsymbol{x}_j^{\boldsymbol{l}_j}$.*

PROOF: *For the constraint of (10) and $r_j^{\boldsymbol{l}_j} < 1$, the following relation is satisfied for all DPs in $\bar{D}_j$:*

$$||\boldsymbol{x}_j^{\boldsymbol{l}_j} - \boldsymbol{u}_i||^2 < \frac{q_j}{a_i^C}. \tag{11}$$

Then, $a_i^C < q_j/||\boldsymbol{x}_j^{\boldsymbol{l}_j} - \boldsymbol{u}_i||^2$ is satisfied. From (1), this equation means that the attractive power of new facility j is more than that of competitive facilities if the site of facility j is $\boldsymbol{x}_l^{\boldsymbol{l}_j}$. □

Since (10) is a convex programming problem, (10) can be solved by using the solution algorithms for convex programming problems, such as successive quadratic programming (SQP) method; for the details of the SQP method, the reader can refer to the book of Nocedal and Wright [11].

Theorem 2 *Let $L = (\boldsymbol{l}_1, \ldots, \boldsymbol{l}_m)^T \in \{0,1\}^{mn}$ and $\boldsymbol{x}^L = (\boldsymbol{x}_1^{\boldsymbol{l}_1}, \ldots, \boldsymbol{x}_m^{\boldsymbol{l}_m})$. Then, there exists L such that $\boldsymbol{x}^L$ is an optimal solution of (4), (5), and (6).*

PROOF: *Let $\boldsymbol{x}^*$ be an optimal solution of (4). We define the 0-1 matrix $\bar{L} \in \{0,1\}^{mn}$, each of whose element for $i \in D, j \in F$ is that $\bar{l}_{ij} = \varphi_i^j(\boldsymbol{x}^*)$. Then, from Theorem 1, $\boldsymbol{x}^{\bar{L}}$ is also an optimal solution of (4) because $\varphi_i^j(\boldsymbol{x}^L) = \varphi_i^j(\boldsymbol{x}^*)$ for all i, j and $r_j^{\boldsymbol{l}_j} < 1$ for all j. This means that $\bar{L}$ is one of the matrices satisfying the condition of the theorem. This is also satisfied for the cases of (5) and (6).* □

Let $\boldsymbol{r}^L = (r_1^{\boldsymbol{l}_m}, \ldots, r_m^{\boldsymbol{l}_m})$ and $\boldsymbol{1} = (1, \ldots, 1)$. From Theorem (2), finding an optimal solution of (4), (5), and (6) can be formulated as the following 0-1 programming problems respectively:

$$\left.\begin{array}{rl} \text{maximize} & E[f(\boldsymbol{x}^L)] \\ \text{subject to} & \boldsymbol{r}^L < \boldsymbol{1}, \\ & L \in \{0,1\}^{mn} \end{array}\right\} \tag{12}$$

$$\left.\begin{array}{rl} \text{maximize} & Pr[f(\boldsymbol{x}^L) \geq f_0] \\ \text{subject to} & \boldsymbol{r}^L < \boldsymbol{1}, \\ & L \in \{0,1\}^{mn} \end{array}\right\} \tag{13}$$

$$\left.\begin{array}{rl} \text{maximize} & f_0 \\ \text{subject to} & \boldsymbol{r}^L < \boldsymbol{1}, \\ & Pr[f(\boldsymbol{x}^L) \geq f_0] \geq \alpha, \\ & L \in \{0,1\}^{mn} \end{array}\right\} \tag{14}$$

Because the number of solving (10) is 2^{mn}, it is NP-hard to find a strict optimal solution of the above three problems. In the next section, we propose an efficient solution method to find an approximate optimal solution of the problems.

4 Tabu Search method based on strategic vibration

First, we introduce an important theorem showing a similarity between the 0-1 programming problems in the previous section and multidimensional knapsack problems. Let $\boldsymbol{l}_j^{k+} := \boldsymbol{l}_j + \boldsymbol{e}^k$, where $\boldsymbol{e}^k$ is the k-th unit vector, and $L_j^{k+} := (\boldsymbol{l}_1, \ldots, \boldsymbol{l}_j^{k+}, \ldots, \boldsymbol{l}_m)^T$.

Theorem 3 *Let $L \in \{0,1\}^{mn}$ be the matrix that $\bar{l}_{ij} = \varphi_i^j(\boldsymbol{x}^L)$ for all i, j and $\boldsymbol{x}^L$ is an optimal solution of (12), (13), or (14). Then, $r^{\boldsymbol{l}_j^{k+}} \geq 1$ if $l_{kj} = 0$ for any k, j.*

PROOF: *We assume that there exists k, j such that $r^{\boldsymbol{l}_j^{k+}} < 1$. Then, from Theorem 2, facility j can obtain more values of objective function being located at $\boldsymbol{x}^{\boldsymbol{l}_j^{k+}}$ than $\boldsymbol{x}^{\boldsymbol{l}_j}$. This contradicts that $\boldsymbol{x}^L$ is an optimal solution.* □

From Theorem 3, an optimal solution of the three 0-1 programming problems exists on the neighborhood of the constraints. This is similar to the multidimensional knapsack problems whose optimal solution exists on the neighborhood of its constraints.

Moreover, if the matrix L which has many elements that $l_{ij} = 1$, there are many constraints of (10) for obtaining many BPs. Then, if L and L_j^{k+} hold that $\boldsymbol{r}^L < 1$ and $\boldsymbol{r}^{L_j^{k+}} < \mathbf{1}$, $\boldsymbol{x}^{L_j^{k+}}$ is mostly superior to $\boldsymbol{x}^L$. Similarly, the multidimensional knapsack problems has the character that their objective values are improved or not changed if an element of solutions is changed from 0 to 1.

From these two characteristics, we think that solution methods for multidimensional knapsack problems are also efficient for (12), (13), or (14). For multidimensional knapsack problems, Hanafi and Freville [6] proposed the tabu search algorithm with strategic vibration. We propose to improve the solution method by utilizing characteristics of CFLP.

The tabu search is one of the local search methods. In our solution method, we define moves from a current solution, denoted by L^{now}, as the increase or decrease of its one element. The neighborhood of a current solution in (12), (13), or (14) is represented as a set of all solutions which can transfer by only one move from the solution. In the tabu search including our solution method, the next searching solution from L^{now}, denoted by L^{next}, is basically chosen to the best solution for given criteria, e.g. objective value, in the neighborhood of a current solution. However, if we use such a search without modification, a circulation of certain chosen moves occurs on the way of search and then it can only find one local optimal solution. For preventing such a circulation, if a move is chosen in the search, the tabu constraint for its opposite move is activated for given terms, called the tabu term and denoted by T_1. Then the activated move are forbidden to choose in T_1 terms, called tabu, even if such a move makes the objective value of (12), (13), or (14) are the best in all solutions in a neighbor-hood. Such tabu moves are memorized in the tabu list for the search.

The tabu search method has advantage for searching in local areas. In generally, there are generally many local optimal solutions of (12), (13), or (14) existing on the neighborhood of the constraints widely, because of the above former characteristic of CFLP. We introduce the strategic vibration to the tabu search for searching various local optimal solutions efficiently. Then, our proposing solution method is described as follows:

Tabu search algorithm with the strategic vibration

Step 0: Generate the initial searching solution L^{now}, and initialize the tabu list and other variables. If $\boldsymbol{r}^{L^{\text{now}}} < \mathbf{1}$, then go to Step 4.

Step 1: Move L^{now} to L^{next} by decreasing an element of L^{now} so as to decrease $\boldsymbol{r}^{L^{\text{now}}}$ as much as possible. This step is repeated until it is satisfied $\boldsymbol{r}^{L^{\text{now}}} < \mathbf{1}$.

Step 2: Move L^{now} to L^{now} so as to improve the objective value of (12), (13), or (14). This step is repeated at given certain terms, denoted by T_2.

Step 3: Move L^{now} to L^{now} by decreasing an element of L^{now} so as to decrease $\boldsymbol{r}^{L^{\text{now}}}$ as much as possible. This step is repeated until $\boldsymbol{r}^{L^{\text{now}}}$ is less than a certain vector, denoted by $\boldsymbol{r}^{\text{low}}$.

Step 4: Move L^{now} to L^{now} by increasing an element of L^{now} so as to improve the objective value of (12), (13), or (14). This step is repeated until it is not satisfied $\boldsymbol{r}^{L^{\text{next}}} < \mathbf{1}$.

Step 5: Do the same operations as Step 2.

Step 6: Move L^{now} to L^{now} by increasing an element of L^{now} so as to improve the objective value of (12), (13), or (14). This step is repeated until $\boldsymbol{r}^{L^{\text{now}}}$ is more than a certain vector, denoted by $\boldsymbol{r}^{\text{upp}}$.

Step 7: If the given terminal conditions are satisfied, then this algorithm is terminated. The obtaining approximate solution is the best solution about the objective value of (12), (13), or (14) in all searched solutions.

Otherwise, return to Step 1.

5 Numerical experiments

In this section, we show the efficiency of the solution algorithm in the previous sections by applying to three examples of the CFLPs. In these examples, the numbers of DPs are $n = 30, 40, 50$. The sites of DPs $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n$ are given in $[0, 100] \times [0, 100]$ randomly, and their random BPs $\bar{w}_1, \ldots, \bar{w}_n$ are represented as random variables each of which have three scenarios whose probabilities are 0.5, 0.3, and 0.2, and BP of each DP for each scenario is given in $[5, 12]$ randomly. We give fifteen competitive facilities, that is $k = 15$, and for competitive facility $j \in F_C$, its site $\boldsymbol{x}_j$ is given in $[0, 100] \times [0, 100]$ randomly. In this plain, the decision maker locates one facility, that is $m = 1$. For (13) and (14), we give $f_0 = 30 + n$ and $\alpha = 0.8$.

Next, we give parameters about our solution method; for the meanings of parameters of tabu searches, the reader can refer to the book of Reeves [14]. We set the tabu term $T_1 = n/2 - 10$. The terminal condition in Step 7 is that the iteration of the tabu search algorithm is false until 10 times. At Step 2, let $T_2 = 10$. At Step 3 and 6, let $r_1^{\text{low}} = 0.3$ and $r_1^{\text{upp}} = 3$.

For showing the efficiency of our solution method, we compare its computational results to that of the genetic algorithm; for details of the genetic algorithms, the readers can refer to the studies of Sakawa et al. [15]. We set generation gap $G = 0.9$, population size $N_{GA} = 150$, and terminal generation $T_{GA} = 2000$. Probabilities of crossover, mutation, and inversion are $p_C = 0.9$, $p_M = 0.01$, and $p_I = 0.03$, respectively.

We apply the tabu search and the genetic algorithm to three examples of the CFLPs, where each of these algorithms is implemented 20 times for each example by using DELL Optiplex GX620 (CPU: Pentium(R) 4 2.33GHz, RAM: 512MB). The computational results of solving the CFLPs are shown in Tables 1-6. From Tables 1-6, the tabu search can obtain better solutions for (12), (13), and (14) than those of the genetic algorithm with shorter computational times. This means that our solution method is efficient for the CFLPs with random demands.

Table 1: Computational results by the taboo search algorithm with the strategic vibration for (12)

n	30	40	50
Best	64.87	67.79	86.77
Mean	64.87	67.79	86.77
Worst	64.87	67.79	86.77
CPU times (sec)	9.83	20.73	48.28

Table 2: Computational results by the genetic algorithm for (12)

n	30	40	50
Best	64.87	67.79	86.77
Mean	64.87	66.30	84.78
Worst	64.87	64.54	70.07
CPU times (sec)	47.86	52.93	69.14

Table 3: Computational results by the taboo search algorithm with the strategic vibration for (13)

n	30	40	50
Best	0.7	0.5	0.7
Mean	0.7	0.5	0.7
Worst	0.7	0.5	0.7
CPU times (sec)	10.41	22.73	53.70

Table 4: Computational results by the genetic algorithm for (13)

n	30	40	50
Best	0.7	0.5	0.7
Mean	0.7	0.5	0.64
Worst	0.7	0.5	0.5
CPU times (sec)	47.86	59.20	77.79

Table 5: Computational results by the taboo search algorithm with the strategic vibration for (14)

n	30	40	50
Best	45.86	48.20	58.96
Mean	45.86	48.20	58.96
Worst	45.86	48.20	58.96
CPU times (sec)	11.47	29.20	48.81

Table 6: Computational results by the genetic algorithm for (14)

n	30	40	50
Best	45.86	48.20	58.96
Mean	45.66	47.23	56.27
Worst	44.85	44.85	50.10
CPU times (sec)	54.11	61.70	80.28

6 Conclusions and future researches

In this paper, we have proposed a new CFLP on the plain with random demands. We have formulated the CFLP as a stochastic programming problem, and for finding an optimal solution of the problem, the three deterministic programming problems: expectation maximizing problem, probability maximizing problem, and satisfying level maximizing problem are reformulated. Because these problems are difficult to find a strict optimal solution of the problem directly, we have shown that the problems can be reformulated as 0-1 optimization problems. Since the combinatorial optimization problems are NP-hard, we have proposed an efficient solution method based upon the tabu search algorithm with the strategic vibration by utilizing characteristics of the CFLPs. The efficiency of the solution method is shown by applying to several examples of the CFLPs.

These three reformulated deterministic programming problems have the characteristic that the more the new facilities can obtain BPs, the more their objective values improve. However, if the CFLPs with random demands are reformulated to deterministic programming problems with considering risk, e.g. variance or VaR, these problems do not necessarily have such a characteristic. Therefore, to propose an efficient solution method for the deterministic programming problems is a future study.

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