Newly Designed Quasi-Cyclic Low Density Parity Check Codes

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Abstract— **This paper presents the construction of large girth Quasi-Cyclic low density parity check (QC-LDPC) codes. The row groups are paired two times the row weight which has cut down hardware implementation cost and complexity as compared to the connection of individual columns and rows. The construction of newly obtained codes gives a class of efficiently encodable quasi-cyclic LDPC codes.**

Index Terms—- **Fractional bandwidth, girth, QC-LDPC, PBNJ.**

I. Introduction

Low density parity check codes [1] have acquired considerable attention due to its near-capacity error execution and powerful channel coding technique with an adequately long codeword length.

The performance of LDPC codes has been investigated in [2-4], at many events of interests and are encountered to outperform turbo codes with good error correction.

Nevertheless, LDPC decoding performance increases with increasing code size with column weight of two, likened to the codes with a linear increase in size having column weight larger than two [1]. Notwithstanding the slow enhancement, column weight of two codes has been revealed their potentiality in such coverings as partial response channels [5-6]. LDPC codes chained with Reed-Solomon codes outperform for burst errors in magnetic recordings due to less computation.

Despite the fact that LDPC codes performance has been exposed to be superior but still face up their hardware accomplishment. This is mainly for the reason of their large sizes and complex random row-column connections.

In order to reduce hardware implementation complexity, structured codes have been explicated by confining code constructions. However, the performance of structured codes is not as much compatible compared to random codes at large code sizes [7]. The decoding performance usually increases by large girths. Iterations are interlinked with the girth size since it ascertains number of iterations before a message propagates back to its original node. Therefore the performance enhancement of structured codes is dependable by increasing their girths. In work [8] codes are constructed from graphical models with girth 16 and 18 having row-weights of 4 and 3. Their proposed method fails to cater an easy way of constructing codes for high row-weights and elaborating codes.

A novel methodology for constructing structured generalized LDPC (G-LDPC) codes is presented in [9].The proposed design in [9] results in QC-GLDPC codes based on the insertion of powerful constraint nodes in an LDPC code bipartite graphs. Their proposed QC-GLDPC codes are limited in code length and rates.

In this paper we construct large girth QC-LDPC codes and investigate their performance for short to longer block lengths.

II. Overview Of Low Density Parity Check

Shannon has defined the capacity of a communication channel, which anticipates the rates at which the information consistently communicates over a noisy channel [10]. He has recommended that the capacity is attainable with beneficial channel codes.

The separation principle, which is explicated in [11] significantly, modifies the complexity in communication system design. Nevertheless, the separation principle is an asymptotic result.

The foremost idea of error-correcting codes is to put in redundancy that is correlated to the information to be transmitted. Accordingly, the receiver can feat the correlation between the information bits and the redundancy bits and then correct or detect errors. The codes are classified into two major categories, explicitly, block codes and convolutional codes. Hamming codes, Bose- Chaudhuri-Hocquenghem (BCH) codes; Reed-Solomon (RS) codes [12-13] and newly rediscovered LDPC codes are the example of block codes.

Block codes like Hamming, BCH and RS codes have structures but with limited code length. A bounded-distance decoding algorithm is usually employed in decoding block codes except LDPC codes, in general it is hard to use soft decision decoding for block codes.

The achievement of Turbo codes directed to the rediscovery of LDPC codes by MacKay in [3]. They have recognized, seemingly independently of the work of Gallager, the advantages of linear block codes which own sparse (low-density) parity check matrices. They were initially explicated by Gallager in the 1960s [1]. LDPC codes were neglected for a long time since their computational complexity for the hardware technology was high at the time.

LDPC codes are a category of linear codes which caters near capacity performance on a large collection of data transmission and storage channels whilst concurrently

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accommodating executable decoders. LDPC codes are rendered with probabilistic encoding and decoding algorithms. In earlier, sparse random parity check matrices were being used which established promising distance properties [1].

LDPC codes are designated by a parity check H matrix comprising largely 0's and has a low density of 1's. More precisely we can articulate that LDPC codes have very few 1's in each row and column with large minimum distance. In specific, a (n, j, k) low-density code is a code of block length n and source block length k. The number of parity checks is delimitated as $m = n - k$. The parity check matrix weight (number of ones in each column or row) for LDPC codes can be either regular or irregular. LDPC can be regular if the number of ones is constant in each column or row and gets irregular with a variable number of ones in each column or row.

A regular LDPC code is a linear block code whose parity-check matrix H constitutes exactly J 1's in each column and exactly $k = j\left(n/m\right)$ 1's in each row, with the code rate $R = 1 - j/k$. Low density codes are not that much optimal in the fairly contrived sense of understating the probability of decoding error for a known block length, and it can be illustrated that the minimum rate being employed by them is bounded below channel capacity. Nevertheless, the simple decoding scheme innovation more than remunerates for these disadvantages.

III. Construction Of Large Girth QC-LDPC Codes

LDPC code construction requires parameters such as row and column weights, rate, girth and code length. LDPC codes are classified into two types of construction. The first one is random constructions which have tractability in design and construction but lack row-column connections regularity, which increases decoder interconnection complexity. The second type of construction is structure constructions with regular interconnection patterns but frequently bring forth a class of codes limited in rate, length and girth which shows the performance degradation compared to random codes longer than 10000. Structure connections generally reduce hardware complexity and the cost of encoders and decoders.

In this work, we have developed the large girth QC-LDPC codes by educing the basic idea from the search algorithm proposed in [14]. Some more constraints have been employed in this work by keeping the column weight $j = 2$ and row weight dependable on group size.

1. Rows in the set could be chosen randomly or sequentially, preferable choice is random since random searches will result in a variety of codes.
2. Formula has been derived in equation (1) to generate the row weight *k* times.

   $$k = \sum_{i=1}^{s} r_i \qquad (1)$$

   where s represents the size of group and the parameter r stands for number of rows.
3. The rows are evenly divided with respect to the size of group. In each group the number of rows should be *k* connection times. The row groups are paired in such a way that each group appears *k* times so there are *2k* row group pairs.
4. The reliance of row weight on group size will keep the obtained codes regular else the groups with different number of appearances will bring irregular codes.
5. Rows in the second and following groups are placed in descending order which will satisfy the least desired distance to search in each group.
6. The regular codes will make it easy to construct the parity check matrix due to uniform distribution of 1's and 0's from the column formation.
7. By employing array dispersion technique we use a prime field to construct an array of circulant permutation matrices. The null space of any subarray of constructed array results in QC-LDPC codes. QC-LDPC codes have encoding advantage over conventional LDPC codes and their encoding can be carried out by shift register with complexity linearly proportional to the number of parity bits of the code.

The row groups are paired two times the row weight which has cut down hardware implementation cost and complexity as compared to the connection of individual columns and rows. The complexity of directing within groups reckons on the transposition employed to connect rows and columns between groups. This modifies handling when messages are communicated between functioning nodes.

IV. Results

The focal intention of the algorithm is to generate high-rate LDPC codes specified a particular code length. The allowable maximum number of iteration for decoder is set to 60 in PBNJ environment.

In this work, the two girth-twelve QC-LDPC codes with a group size 16 are utilized as a forward error correction codes.

LDPC codes with column weight $j = 2$, have respective advantage, since they have lower computation complexity and storage complexity, their encoders and decoders are merer to employ. They have ameliorated block error statistics properties which have been mentioned by Song et al. in [6]. When chain them with forward error correction codes such as Reed Solomon codes, these properties make LDPC codes with $j = 2$ anticipating for data storage and other practical applications.

Figure 1 shows the bit error plots for the proposed large girth QC-LDPC codes with different block lengths. The performance of the newly obtained codes is compared with the constructed randomly codes as well as the proposed QC-LDPC codes in [15]. As shown in Figure 1, the large girth QC-LDPC codes perform significantly better than the randomly constructed LDPC codes as well as the codes proposed in [15] for short to longer block lengths.

The substantial BER performance gains for large girth QC-LDPC codes is obtained by the complexity of directing within groups reckons on the transposition employed to connect rows and columns between groups. In addition, they demonstrate that larger gains available by increasing block length.

The error probability of newly obtained code is shown in Figure 2. At the BER of 10^{-6}, this code achieves a 6.5dB coding gain over the uncoded BPSK and performs 0.7dB from the Shannon-limit. This code also has a very low error-floor for BER and block error rate (BLER). The curves show that large girth robust the system performance.

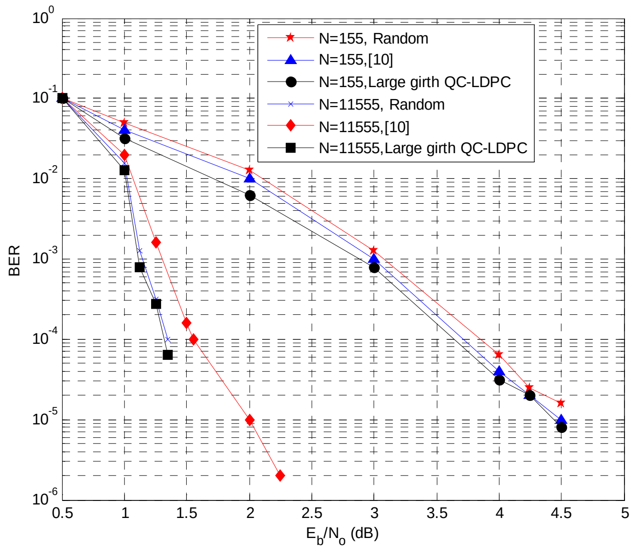

Figure 1. BER of Girth-twelve QC-LDPC codes with column weight 2

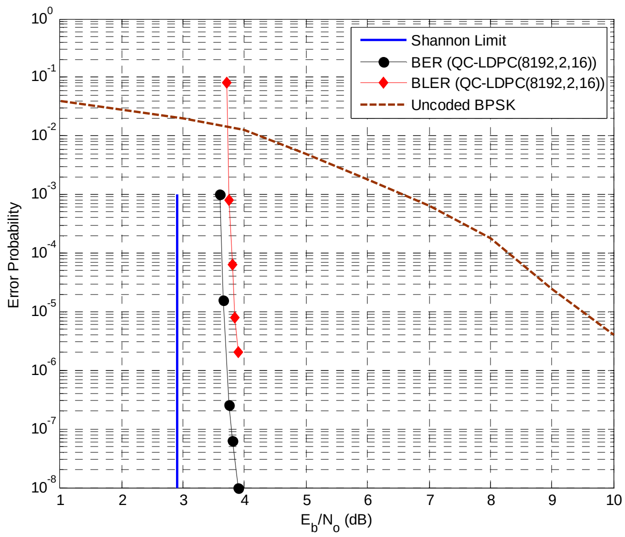

Figure 2. The performance of two girth-twelve QC-LDPC codes with rate 0.87.

V. Conclusion

This paper introduces new QC-LDPC codes of column weight two with a diversity technique. The potentiality of the newly codes has been established by plotting graph between E_b / N_o versus error probability. The development of two girth-twelve QC-LDPC codes has very low error floor. This furnishes information on the design of robust system. The newly constructed LDPC codes have both theoretical as well as practical implications.

References

[1] R. G. Gallager, Low-Density Parity-Check Code. Cambridge, MA: MIT Press, 1963.

[2] C. Berrou, A.Glavieux, P.Thitimajshima, “Near Shannon limit error correcting coding and decoding: turbo-codes,” Proc. of IEEE ICC'93, (Geneva, Switzerland), pp. 1064-1070, 1993.

[3] D. J. Mackay, “Good error-correcting codes based on very sparse matrices. IEEE Trans. Inform. Theory, vol. 45, pp. 399-431, 1999.

[4] S. Chung, G. D. Forney, J. J. Richardson, and R. Urbanke, “On the design of low-density parity-check codes within 0.0045db of the Shannon limit,” IEEE Communication Letters, vol. 5, pp. 58–60, February 2001.

[5] H. Song, J. Liu, and B. V. Kumar, “Large girth cycle codes for partial response channels,” IEEE Transactions on Magnetics, vol. 40, pp. 2362–2371, July 2004.

[6] H. Song, J. Liu, and B. V. K. Vijaya Kumar, “Low complexity ldpc codes for magnetic recording,” in IEEE Globecom 2002, Taipei, Taiwan, R.O.C., Nov 2002.

[7] R. Tanner, D. Sridhara, and T. Fuja, “A class of group-structured ldpc codes,” Proceedings of ISTA, Ambleside, England, 2001.

[8] H. Zhang and J. M. Moura, “The design of structured regular lpdc codes with large girth,” GLOBECOM, pp. 4022–4027, 2003.

[9] G. Liva, W.E. Rayan and M. Chiani, “Quasi-Cyclic Generalized LDPC Codes with Low Error Floors,” IEEE Transaction on Communications vol.56. No.1 pp 49-57, 2008.

[10] C. E. Shannon, “A mathematical theory of communication,” *Bell Systems Technology Journal*, vol. 27, pp. 379 - 423, 1948.

[11] T. M. Cover and J. A. Thomas, Elements of Information Theory, New York: John Wiley & Sons, 1991

[12] S. Lin and D. Costello, *Error-Control Coding: Fundamentals and Applications*, Prentice-Hall, 2nd edition, 2004.

[13] S.B. Wicker and S. Kim, *Fundamentals of Codes, Graphs and Iterative Decoding*. Kluver International Series in Engineering and Computer Science, 2003

[14] J. Campello, D. S. Modha, and S. Rajagopalan, “Designing LDPC codes using bit-filling,” in Proceedings of IEEE International Conference on Communications (ICC ’01), vol. 1, pp. 55–59, Helsinki, Finland, June 2001.

[15] C.M. Huang, J.F Huang, and C.C. Yang, “Construction of Quasi-Cyclic LDPC Codes from Quadratic Congruences” *IEEE COMMUNICATIONS LETTERS*, Vol. 12, no. 4, APRIL 2008, pp.313-315.