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A New Two-dimensional Empirical Mode Decomposition Based on Classical Empirical Mode Decomposition and Radon Transform

Zhihua Yang[1] and Lihua Yang[2] *

Abstract—**This paper presents a new two-dimensional Empirical Mode Decomposition Based on classical Empirical Mode Decomposition and Radon transform. The proposed method avoids to search for the maxima and minima in a two-dimensional function and produce two-dimensional envelopes, which are necessary in existed methods and usually difficult. Experiments show encouraging results.**

Keywords: empirical mode decomposition (EMD), two-dimensional empirical mode decomposition (BEMD), Radon transform

1 Introduction

Empirical mode decomposition is a method to decompose signals proposed by N.E.Huang et. al in 1998. It can extract adaptively the oscillatory modes at each time from a complex signal, namely it can decompose the signal into a finite (often less) number of *intrinsic mode functions (IMFs)*. With Hilbert transform, the IMFs yield instantaneous frequencies as functions of time, that give sharp identifications of embedded structures. The final presentation of the results is a time-frequency-energy distribution, designated as the Hilbert spectrum and the new method for signal processing is called as Hilbert-Huang transform(HHT)[1]. Being different from Fourier decomposition and wavelet decomposition, EMD has no specified "basis". Its "basis" is adaptively produced depending on the signal itself, which brings not only high decomposition efficiency but also sharp frequency and time localization. A key point is that the signal analysis based on HHT is physically significant. Because of its excellence, HHT has been utilized and studied widely by researchers and experts in signal processing and other related fields. In recent years, more and more works on HHT theory are reported such as [5, 6, 7, 2, 8]. Its application have spread from earthquake research, ocean science, biomedicine, speech signal analysis to image analysis and processing [9, 10, 11, 12, 13, 14, 4, 15, 16].

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[1]Information Science and technology School, Guangdong University of Business Studies, Guangzhou 510320, P. R. China. [2]School of Mathematics and Computing Science, Sun Yat-sen University, Guangzhou 510275, P. R. China.Corresponding author. Email: mcsylh@mail.sysu.edu.cn, Tel: (8620)84115508, Fax: (8620)84111696.

However, the EMD is just applicable to one-dimensional signals, which makes it is difficult to use HHT in many applications such as image processing, pattern recognition and so on. To apply HHT to a two-dimensional signal, one has to convert it into one or more one-dimensional signal(s) by means of various methods[14, 15]. However, many information will be lost inevitably when a two-dimensional signal is converted into one-dimensional ones no matter which method is employed. So a thorough resolution is to develop an available two-dimensional EMD algorithm. Some researchers have focused on this problem[10, 3], however, up to now, all of the existed methods attempt to spread, almost parallel, the EMD algorithm from one to two dimension. That is to say the maxima and minima of a two-dimensional signal should be found at first in order to produce the upper and lower envelopes. However, in the case of two dimensions, to determine the maxima and minima is often difficult due to the possible ridges, saddle points and so on. It means that there are many essential difficulties in the existed methods. Thus, to develop an available two-dimensional EMD algorithm, some new ideas and technologies should be employed.

Radon transform was presented in 1917. However, it wasn't attended enough before the FFT was developed. Now, it has became an important technology in medical imaging, remote sensing imaging and many other applications. In this paper, a new two-dimensional EMD algorithm combining classical EMD and Radon transform will be presented. The proposed method avoids to search for the maxima and minima in a two-dimensional function , which are necessary in existed methods and usually difficult. Experiments show encouraging results.

The rest of this paper is organized as follows: The Radon transform will be introduced briefly in Section 2; a new two-dimensional EMD algorithm is given in Section 3; Section 4 includes the experimental results and analysis; Section 5 is the conclusion of the paper.

2 The Radon transform

The Radon transform is actually a projection transform. Let $f(x, y)$ is a function defined in the plane (x, y), then its Radon transform is

$$P_\alpha(u) = \int_{-\infty}^{+\infty} f(x,y)dv \tag{1}$$

where α is the angle between u and x, as shown in fig.1. x, y and u, v satisfy

$$\begin{cases} x = u\cos\alpha - v\sin\alpha \\ y = u\sin\alpha + v\cos\alpha \end{cases} \tag{2}$$

The Fourier transform of the function $f(x,y)$ can be written as

$$F(\Omega_1,\Omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j(\Omega_1 x+\Omega_2 y)}dxdy \tag{3}$$

then the values of function $F(\Omega_1,\Omega_2)$ at the line with the angle of α with Ω_1 and passing the original point are

$$F_\alpha(\Omega_1,\Omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j\Omega(x\cos\alpha+y\sin\alpha)}dxdy \tag{4}$$

where $\Omega_1 = \Omega\cos\alpha, \Omega_2 = \Omega\sin\alpha$.

Let us change the integration variable from (x,y) to (u,v), then the equation (4) can be written as

$$\begin{aligned} F_\alpha(\Omega_1,\Omega_2) &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(u\cos\alpha - v\sin\alpha,\right. \\ &\quad u\sin\alpha + v\cos\alpha)dv]\, e^{-j\Omega u}du \\ &= \int_{-\infty}^{\infty} P_\alpha(u)e^{-j\Omega u}du \end{aligned} \tag{5}$$

The equation (5) suggests that $F_\alpha(\Omega_1,\Omega_2)$ can be obtained from applying the Fourier transform to $P_\alpha(u)$ and $F(\Omega_1,\Omega_2)$ can be formed by all $F_\alpha(\Omega_1,\Omega_2)$ in each angle. Then the $f(x,y)$ can be reconstructed by the inverse Fourier transform of $F(\Omega_1,\Omega_2)$.

3 Two-dimensional EMD algorithm based on one-dimensional EMD and Radon transform

In the one-dimensional case, an IMF is the approximation of a mono-component signal. The similar idea could be applied to two-dimensional case. That is to say an two-dimensional IMF, denoted as BIMF, should be the approximation of a two-dimensional mono-component signal too. Thus a BIMF should be a signal which satisfies that only one oscillatory mode can be included at each local and its oscillatory is symmetric with respect to the local zero mean. Some examples of ideal BIMF are given in Fig.2(a-c). To be convenient for comparison, a non-BIMF is given too. One have no difficulty noticing that more than one oscillatory modes are included in $f_4(x,y)$ nearby 'A' and 'B',as shown in Fig.2(d).

A great deal of experiments suggest that the Radon transforms of a two-dimensional IMF in any angle must be an one-dimensional IMF. For example, Fig.3(a-d) are the Radon transforms of the signals shown in Fig.2(a-d) when the angle are $0°, 5°, \cdots, 175°$ respectively. We can see that the Radon transforms in these angles are really one-dimensional IMFs when the signals are two-dimensional IMFs. On the contrary, there are always some angles in which the Radon transform aren't one-dimensional IMFs as marked by bold lines in Fig.3(d).

The experiments imply if we can decompose the Radon transforms of a two-dimensional signal in the angle of α into a finite number of one-dimensional IMFs, for simplicity, we denoted them by imf_i^α, then the ith two-dimensional IMF, denote by $bimf_i$, can be produced by reconstructing the one-dimensional IMFs,imf_i^α, $(0 \le \alpha \le 180°)$.

Finally, we give the two-dimensional EMD algorithm as following:

Algorithm 3.1 *Let $f(x,y)$ be a two-dimensional signal and $\alpha = 0, N = int(\frac{180}{\triangle\alpha}) - 1$, where $\triangle\alpha$ is the step span of angle.*

step 1 *If $\alpha < 180 - \triangle\alpha$, go to step 2, else go to Step 5;*

Step 2 *Calculate the Radon transform of $f(x,y)$ in the angle of α, denote by $P_\alpha(u)$;*

Step 3 *Apply the EMD to $P_\alpha(u)$ to produce imf_i^α $(i = 1, 2, \cdots, M_\alpha)$;*

Step 4 *Let $\alpha = \alpha + \triangle\alpha$ and go to Step 1*

Step 5 *Let $M = max(M_\alpha), \alpha = 0, \triangle\alpha, \cdots, N\triangle\alpha$;*

Step 6 *If $M_\alpha < M$, let $imf_i^\alpha = imf_{i-M+M_\alpha}^\alpha, i = M, M-1, \cdots, M-M_\alpha+1; imf_i^\alpha = 0, i = 1, \cdots, M-M_\alpha$, else go to Step 7;*

Step 7 *Let $R^{(i)} = (imf_i^0, imf_i^{\triangle\alpha}, \cdots, imf_i^{k\triangle\alpha}, \cdots, imf_i^{N\triangle\alpha}), (i = 1, 2, \cdots, M)$*

Step 8 *Apply inverse Radon transform to $R^{(i)}$ to form $bimf_i, (i = 1, 2, \cdots, M)$*

4 Experimental Results and Analysis

Fig.4(a) is the Lena image. We apply Algorithm 3.1 on it and the decomposition results are shown in Fig.4(b-h). It should be noticed that each pixels have been added by 128 to be convenient for showing in (b-g). One has no difficulty seeing that the images become blur more and more from $bimf_2 - bimf_6$. The high frequency details which are usually the contour information are included mainly in $bimf_2 - bimf_4$ and the next bimfs are the lower

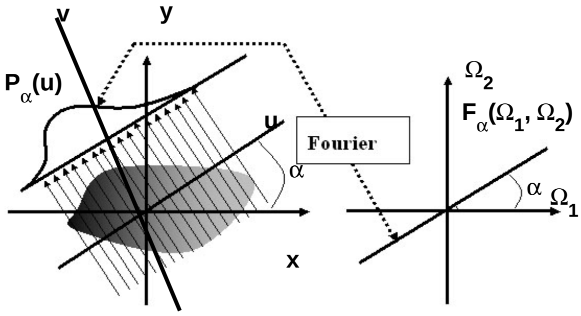

Figure 1: A sketch map of the Radon transform

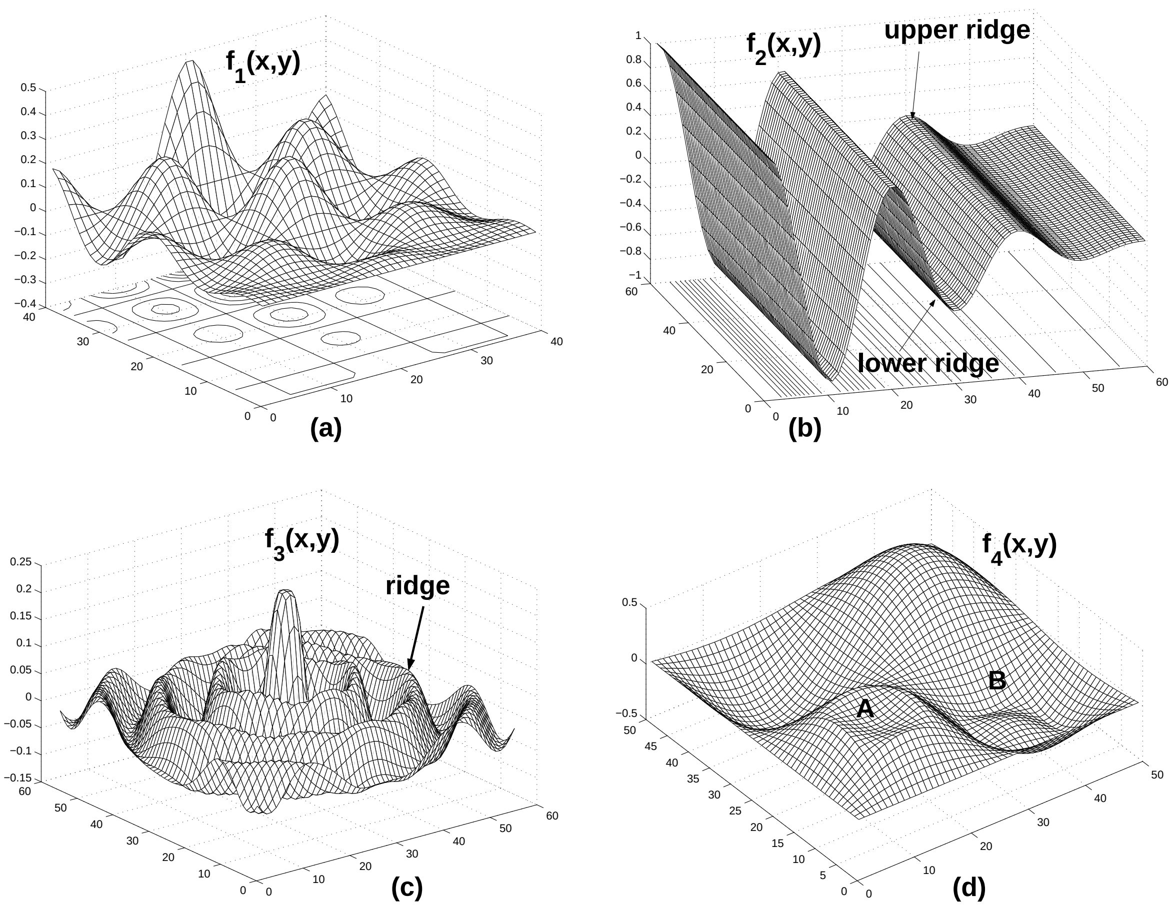

Figure 2: Some examples of two-dimensional IMF

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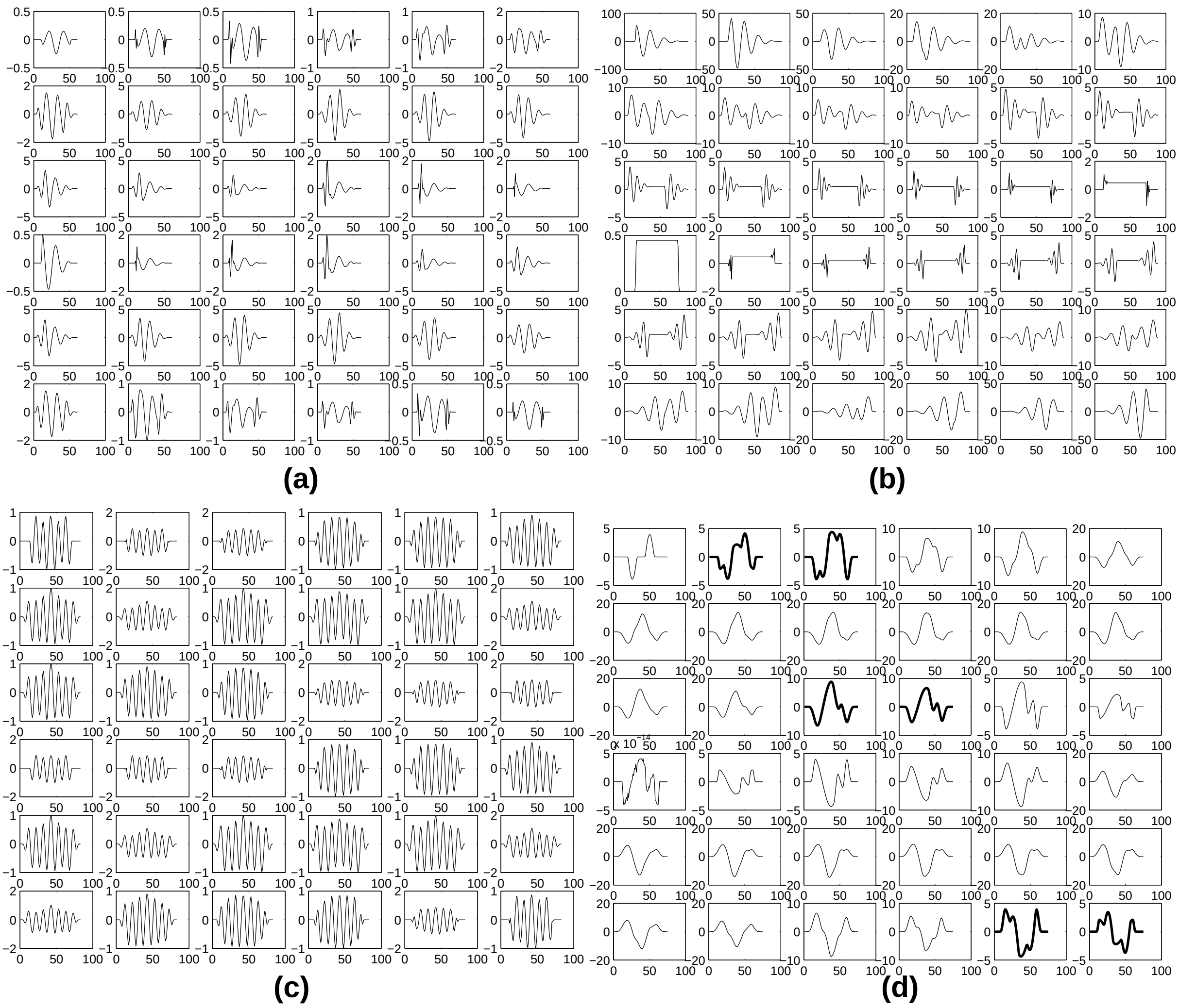

Figure 3: Some examples of the Radon transform of two-dimensional IMFs

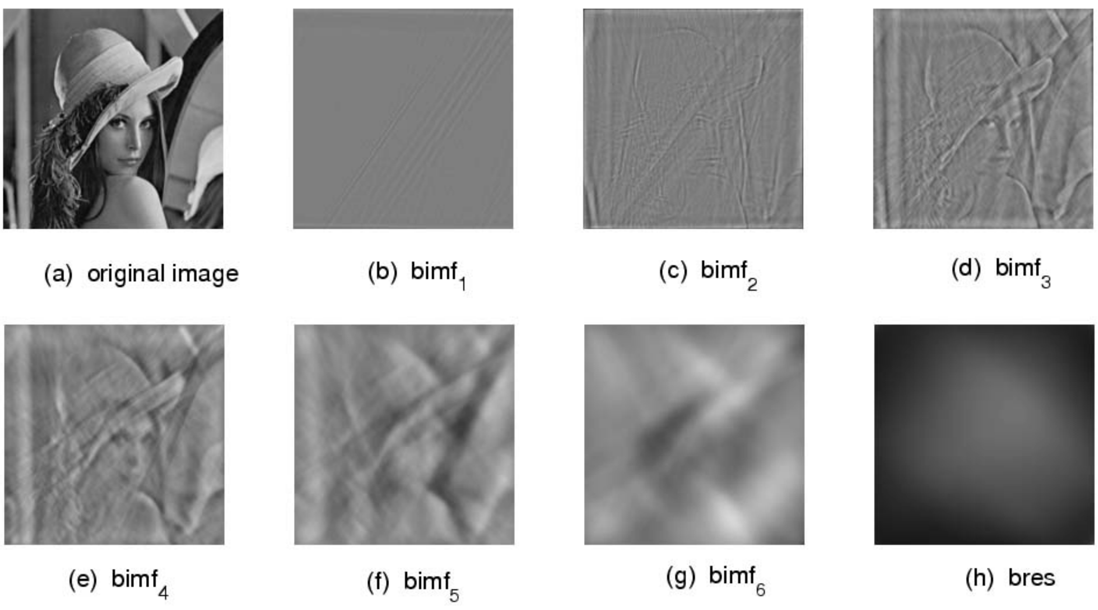

Figure 4: Lena image is decomposed into 6 bimfs and a residue image.

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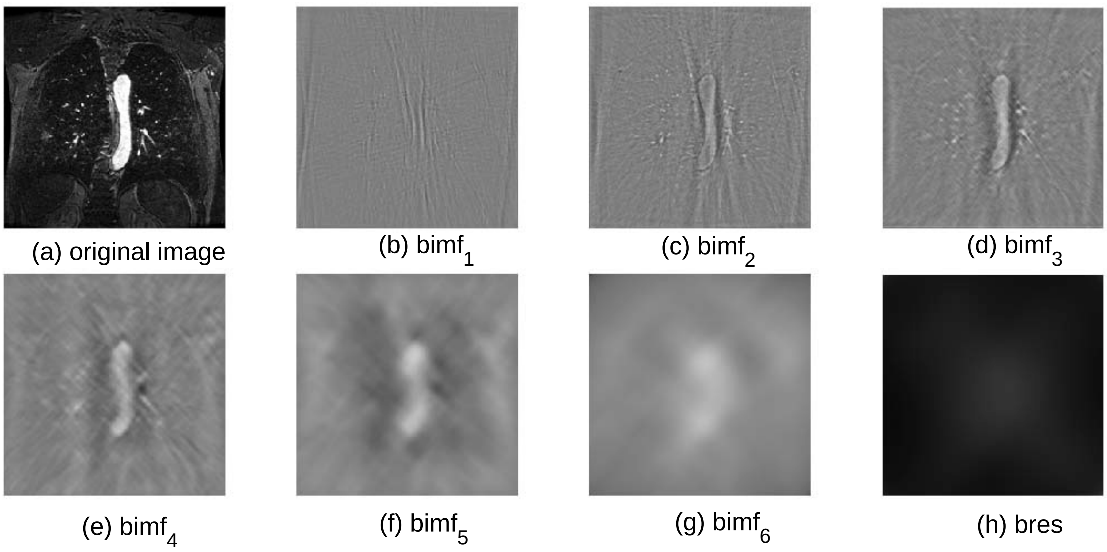

Figure 5: A MR image is decomposed into 6 bimfs and a residue image.

frequency information. The $bimf_1$ corresponds generally to noise and the image tendency is contained in $bres$.

Fig.5(a) is a MR image. Applying Algorithm 3.1 on it, we obtain six $bimfs$ and a $bres$. An encouraging result can be explicitly seen that the details in multi-scales have been depicted by different $bimfs$, as shown in $bimf_2 - bimf_6$, respectively.

Fig.6(a) is a synthetical texture. It is decomposed into 6 $bimfs$ and a residue image and similar results can be received too.

5 Conclusion

Many signals in applications are two-dimensional signals. So an available two-dimensional EMD algorithm is very important. The existed methods attempt to spread, almost parallel, the EMD algorithm from one to two dimension. It means that many essential difficulties are inevitable. In this paper, a new two-dimensional EMD algorithm based on classical EMD and Radon transform is developed. The proposed method avoids to search for the maxima and minima in a two-dimensional function and produce two-dimensional envelopes. Experiments show encouraging results.

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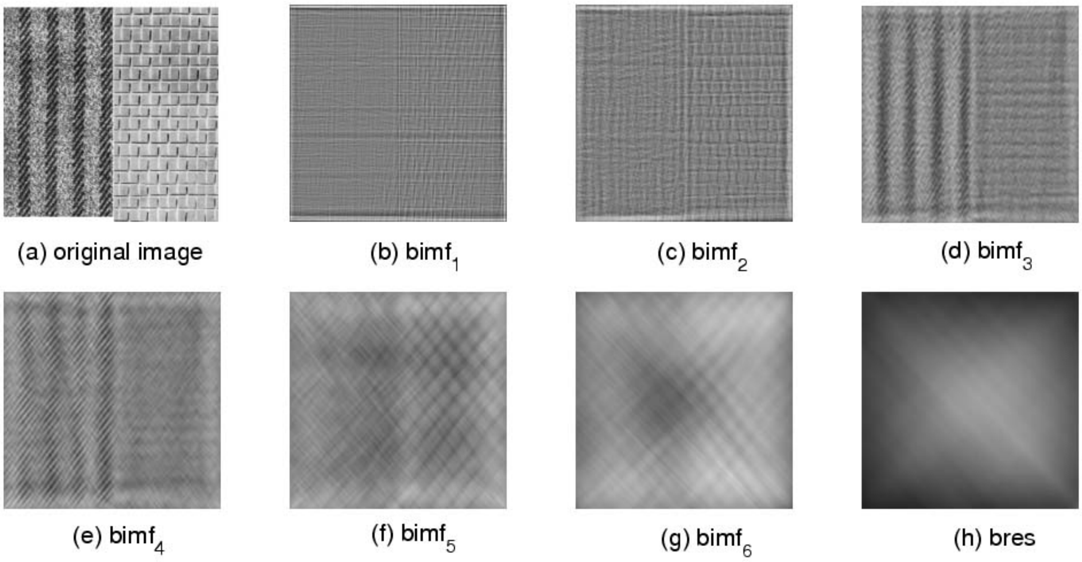

Figure 6: Texture image synthesized by D1(left side) and D11(right side) is decomposed into 6 bimfs and a residue image.

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