Lower Bound of Crossing and Some Properties of Sub-graph of Complete Graph Related to VLSI Design

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Abstract

In this paper, the lower bound of crossings of the complete graph K_{2m+2} for m≥2 have been studied. Various properties of some non-planar graphs of the complete graph K_{2m+2} have been discussed. The application of some sub-graphs of the complete graph for the existence of various forms of hyper graphs has been discussed. An algorithm has been developed to find out the graph from hyper graph.

Keywords :Algorithm ,Crossing, Clique, Complex triangle, Hamiltonian graph, Hyper graph ,Non-planar graph.

1. Introduction

The thickness and crossing of non-planar graph have been found to be very important in case of printed circuit board. The application of crossings has been playing a very important role for VLSI design technology. It has been found that the wire crossings in a circuit layout on silicon chips give some problems for power distribution when it is designed. Hence it is necessary to minimize the wire crossings between two wires. The wires are nothing but the edges of a non-planar graph joining some modules by nets, which are used for circuit layout process.
Again, it has been found that the circuit partitioning in VLSI design has been occupying a most important part in design purpose.

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It has also been known that partitioning of a circuit can have many levels and they are IC level, Sub IC level, board level and system level. There are some circuit partitioning results in VLSI design technology which are found as graphs and hypergraphs.But,the procedure from which one can obtain a graph from a hypergraph has not been found completely. It has been observed that the good partitioning can improve circuit performance and reduce the layout cost.The properties of graph and hypergraph and their application for VLSI design technology have been studied by M.Sarrafzadeh,C.K.Wong [12], Yeap,K.H, M.Sarrfzeh[13], Makedon, F. S.Tragoudas [14], Imai,H, T.Asano [15] and Ihler.E,D.Wangner and F.Wagner [16].

The representation of hyper edge of a hyper graph with the edge of a graph is not known completely. Various kinds of non-planar graphs and their crossing numbers have been studied by Kalita, B [2]-[3], Ajtai. M.V, Chvatal. M.M.,Neuborn and E. Szemeredic [4], Erdos, P, R.K.Guy [5], Guy. R. K [6], Kleitman, D. J [7], Pach,J.,G.Toth [8] Szekely, L. A [9] and Douglas,B [10]. Some structures of simple non-isomorphic Hamiltonian sub-graphs of complete graph K_{2m+3} for m ≥2 have been discussed by kalita [17]. The complete graph K_n and the bipartite graph $K_{m, n}$ are the known regular simplest non-planar graphs . The upper bound of crossings of the complete graph K_{2m+2} for m ≥2 and of the bipartite graph $K_{m,n}$ had already been established by the theorems 11.28 and 11.29 [1]. There is no general formula to know the crossing number of arbitrary non-planar graphs. Recently the maximum and the minimum number of crossings

have been discussed by Kalita [2] for the set H of non-planar graphs , where
H= { H_k (2m+3, 6m+6) / m ≥2, k=1 to 4m-2}. This set H of non-planar graphs has been constructed by Kalita from the TEEP graphs [3]. Other properties relating to isomorphism of graphs, existence of edge-disjoint Hamiltonian circuits had also been discussed by Kalita for the graphs of the set H of non –planar graphs. Another class of non-planar graphs has also been constructed from the TEEP graphs and thickness and crossings of them have been studied by Dutta, A , Kalita, B. and Baurah, H.K. [11]. It has been found that if G is a graph having n vertices and m edges and if K is the maximum number of edges in a planar sub-graph of G , then the crossings satisfy the inequality ≥ m-k . [10].

In this paper, we are going to find the lower bound of crossings of the complete graphs K_{2m+2} for m ≥ 2 . In addition to this, some properties relating to Hamiltonian graph , graphical partition, existence of clique and perfect matching have been studied .The values of m indicate the modules and the edges of the graph are nothing but nets , when we use the graph in case of VLSI design purpose. Various types of hyper graphs have been discussed.

The paper is organized as follows: The section 1 focuses some previous works of crossing and some works of graph for VLSI design . In section 2, the notation and terminology is included. Section 3 explains some structures of non-planar graphs for studying the lower bound of crossings.Section 4 contains the theoretical explanation of some graphs. In section 5, some applications of crossings and partitioning problem related with the rectangular floor planning are discussed . An algorithm is included in section 6 to find out the graphs from hyper graphs .Conclusion is included in section 7.

2. Notation and Terminology

The notation and terminology have been considered from the standard references [1-17] .The notation G (v, e) denotes the graph G of v vertices and e edges. The symbols l(G) and M(G) indicate the lower and upper bound of crossing number of the graph G.

3. Some Structure of Non-Planar Graphs

We consider the complete graph K_{2m+2} for m ≥ 2. We know that the complete graphs are non-planar as it generally does not satisfy the condition of planarity e ≤3n-6 where n are the number of vertices and e are the number edges of the graph. Let us now explain the construction process of some non-planar graphs to know the lower bound of crossings of the complete graph K_{2m+2} for m ≥ 2 [discussed later]. It has been found [3] that one can construct the planar graph of the form H (2m+2,6m) from the complete graph K_{2m+2} for m ≥2. Fig. 1 is one of such graph for m=2 which is obtained from the complete graph K_{2m+2} .

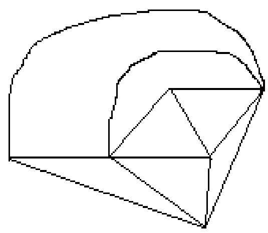

Figure-1. A graph H (6, 12) for m=2.

Now, if we connect two vertices of minimum degrees by one edge of the graph H (2m+2, 6m) , then definitely this graph H will be a non –planar graph and there will exist one crossing of edges between two vertices. Continuing the process of

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connection of edges between two vertices of minimum degrees of the graph H (2m+2, 6m) for m ≥ 2 till one edge left out only to become the graph H as complete. Since the complete graph K_{2m+2} has $2m^2$ +3m +1 edges for m ≥2 and therefore the final non-planar graph obtained from the graph H (2m+2, 6m) will be the form as K (2m+2, $2m^2$ +3m) for m ≥2. Hence to know the lower bound of crossings of the complete graph K_{2m+2} we study first the crossings of the non-planar graph K (2m+2, $2m^2$+3m) for m ≥2. Besides, we study various properties of the non-planar graph K (2m+2,$2m^2$ +3m) for m ≥2.

4. Theoretical Explanation

Various theorems are discussed in this section.

4.1.Theorem: The Graph K(2m+2, $2m^2$+3m) has two vertices of degree 2m and all other vertices are of degree 2m+1.

Proof: We know from the construction process of the graph K (2m+2, $2m^2$+3m) for m ≥2 that only one edge is left out from the complete graph K_{2m+2} . The complete graph K_{2m+2} for m ≥2 is regular graph of degree 2m+1. Since one edge is left out from the complete graph hence we have to leave two vertices without connection by one edge and therefore two vertices are of degree 2m (2m+1-1) for m ≥2. The following fig 2 is shown for m=2.

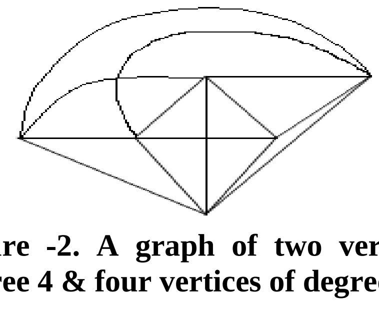

Figure -2. A graph of two vertices of degree 4 & four vertices of degree 5.

We thus found the other vertices are of degree 2m+1 for m ≥2.

4.2. Theorem: The graph K (2m+2, $2m^2$+3m) for m ≥2 is Hamiltonian and it has two / three / four…..edge disjoint Hamiltonian circuits for m ≥ 2 .

Proof: From the theorem 4.1, we have that the graph K(2m+2, $2m^2$+3m) has two vertices of degree 2m and all other vertices are of degree 2m+1 for m ≥2. On the other hand the complete graph K_{2m+2} for m ≥2 is Hamiltonian graph and the graph K (2m+2, $2m^2$+3m) is a sub-graph of the complete graph. Since the graph K(2m+2,$2m^2$+3m) for m ≥2 has two vertices of degree 2m for m ≥2 hence we can go two / three/ four ………times to get the m number of disjoint Hamiltonian circuits for m ≥ 2 connecting all the vertices of the graph with these two vertices of minimum degree. This process will give the two/three/four……..................edge disjoint Hamiltonian circuits for m ≥2,and this completes the proof.

4.3. Theorem: If $L(K_{2m+2})$ is the lower bound and $M(K_{2m+2})$ is the upper bound of crossings of the complete graph K_{2m+2} for m ≥2 then the difference of upper and lower bounds is 2n+1 for n≥0 with simultaneous changes of m ≥2 , where the value of $M(K_{2m+2})$ is known as ¼ [2m+2/2] [2m+1/2] [2m/2] 2m-1/2] for m ≥2.

That is $M(K_{2m+2}) - L(K_{2m+2})$ =2n+1 for n ≥ 0 with simultaneous changes of m ≥ 2

Remark: After knowing the difference of upper and lower bound for the values of n ≥ 0 for simultaneous changes of m ≥ 2, one can find the value of Lower bound .

Proof: It has been found [1] that the upper bound $M(K_{2m+2})$ of K_{2m+2} has been already established as ¼[2m+2/2] [2m+1/2] [2m/2] [2m-1/2] for m ≥2. Therefore we have to show that the difference of $M(K_{2m+2})$ and $L(K_{2m+2})$ is 2n+1 for n ≥ 0 for simultaneous changes of m≥2. We

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prove it by induction method. The result is true for n=0 and m =2 which indicates that the lower bound of $L(K_{2m+2})$ is one and the upper bound $M(K_{2m+2})$ is two and hence the difference is 1.[That is the crossing of non-planar graph $K(2m+2, 2m^2+3m)$ for m=2 is one and the crossing of K_6 , the complete graph of vertex six is two]. If we suppose that the value of n=1 and then we have the value of m=3 and hence the values of lower and upper bounds are respectively found as 15 and 18 which shows the difference of them is 3 and this is also true. Now suppose n=k and m =k+2 for k ≥ 0. Then the difference of upper and lower bounds is 2k+1 and the upper bound will be ¼[(2(k+1)+2)/2][(2(k+1) +1)/2] [2(k+1)/2] [(2(k+1)-1)/2]. Now we shall show that the result is true for n=k+1 and m =(k+1)+2 that is m=k+3. We see that the values of n and m started from 0 and 2 respectively for their simultaneous values and when we put for n=k+1 and m=(k+1)+2, then the starting value of n and m will be 2(k+1) +1=2k+2+1=2k+3 for k ≥0 and m=k+2 for k ≥2 ,which clearly shows that the result is true for all values of n ≥0 for simultaneous changes of m ≥2.

4.4. Theorem: The graph $K(2m+2, 2m^2+3m)$ for m ≥2 is one of the graphical partition of the partition of the number of the form 26n+2 for n≥1. (simultaneous changes of m ≥2 and n≥1)

Proof: We know that the sum of the degrees of the graph $K(2m+1, 2m^2+3m)$ is equal to $2(2m^2+3m)$ for m ≥2 and this the basic property of graph theory. It can be shown that the number 26n+2 for n ≥1 and $2(2m^2+3m)$ for m ≥2 are equivalent and this immediately follows the proof .

4.5. Theorem: There exist a maximum clique K_{2m+1} in the graph $K(2m+2, 2m^2+3m)$ for m ≥2 .

Proof: The complete graph K_{2m+2} for m ≥2 is a regular graph of degree 2m+1 . The graph $K(2m+2, 2m^2+3m)$ is a sub-graph of K_{2m+2} for m ≥2 having two vertices are of degree 2m and all other vertices are of degree 2m+1 [by the theorem 4.1].

If possible , we suppose that there does not exist clique K_{2m+1} for m ≥2 in the graph $K(2m+2, 2m^2+3m)$ for m≥ 2, then certainly without loss of generality we must have the cliques as K_{2m} or as K_{2m+2} for m ≥2. But the existence of cliques as K_{2m} and K_{2m+2} in the graph $K(2m+2,2m^2+3m)$ is not valid for m ≥2,as we see that the graph K_4 (minimum clique) existed in $K(2m+2,2m^2+3m)$ and K_6 does not satisfy for m=2]. Hence the existence of minimum cliques as K_{2m} is possible but K_{2m+2} is impossible for the graph $K(2m+2, 2m^2+3m)$ for m ≥2. Hence there exist a maximum clique K_{2m+1} for m ≥2 in the graph $K(2m+2, 2m^2+3m)$.

4.6. Theorem: There always exist perfect matching of the size m+1 (m+1 edges) in the graph $K(2m+2, 2m^2+3m)$ for m ≥2 .

Proof: We know that the perfect matching exist in the complete graph K_{2m+2} for m ≥ 2 . The graph $K(2m+2, 2m^2+3m)$ for m ≥ 2 is the sub-graph of K_{2m+2} and only one edge is deleted from the complete graph K_{2m+2} to obtained the graph $K(2m+2, 2m^2+3m)$,hence all other edges are incident with the vertices except two vertices. Therefore the m+1 edges form the perfect matching for m ≥2.

5.Application of crossing and partitioning of a circuit found in VLSI design

Here we denote the vertices of the graph $K(2m+2, 2m^2+3m)$ for m ≥2 as modules with different nets and the design will help for the fabrication of circuit. The graph $K(2m+2, 2m^2+3m)$ for m ≥ 2 is a circuit and this type of circuit is obtained from the circuit shown below [figure-3 for m=2].

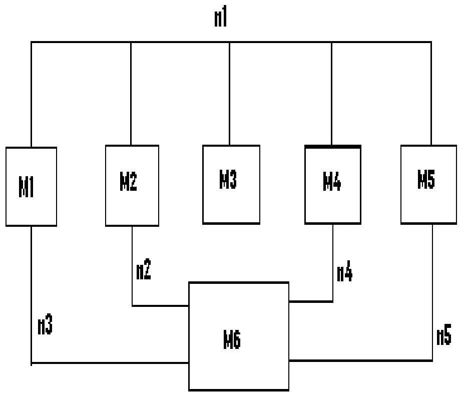

Figure-3. A circuit of six modules and five nets.

The fig.3 shown above is a circuit formed by six modules M1, M2, M3, M4, M5 and M6 connected by five nets n1, n2, n3, n4 and n5 . This type of circuit can be considered for the graph K(2m+2, $2m^2$+3m) for m ≥2 in case of VLSI design. Hence we have the following theorem related with the above circuit. We consider the following theorems 5.1 to 5.7 and the proposition 5.3 which have the relation with VLSI design in case of rectangular floor plan.

5.1. Theorem: There always exist a Graph of the type K (2m+2, $2m^2$ +3m) for m≥2 from the circuit having 2n+3 number of nets for n ≥1 with simultaneous changes of m ≥2.

Proof: We prove the theorem by contradiction process. If possible we suppose that the graph of the given form, that is K (2m+2, $2m^2$+3m) for m ≥2 is not obtained from the circuit given above (figure-3 for m=2) and if we add one more net to the number 2n+3 for n ≥1 of given nets, then immediately we have a connection by one edge with the vertices of degree 2m for m ≥2 and then the graph will take the form as complete graph K_{2m+2} . Since our graph K (2m+2, $2m^2$+3m) for m ≥2 is just the preceding graph of the complete graph K_{2m+2} and only one edge is necessary to form the complete graph and hence equivalently we must have the number of nets is 2n+4 for n ≥1 which is impossible. There fore the number of nets must be less than by one of 2n+4 for n ≥1 . This completes the proof.

5.2. Theorem: There exist a 1-uniform, 2-uniform, 3-uniform…….. hyper graphs in the graph K(2m+2, $2m^2$+3m) for m ≥2.

Proof: We know that a hyper graph is a graph which consists of a collection of vertices and the number of edges are considered from the subsets of the vertices . From the graph K (2m+2, $2m^2$+3m) for m ≥2 , we have different sub-sets of different sizes with null set from the set of vertices { 2m+2/m ≥ 2 }. [We do not consider the null set for the proof of the theorem as the null set has no meaning in circuit layout and floor planning in VLSI design technology] . Hence, definitely there exist one element sub –set, two elements sub-set, three elements sub-set, four elements sub-sets ……….etc from the set of vertices { 2m+2 / m ≥ 2} and they will form different types of hyper-graphs with the vertex set of K (2m+2, $2m^2$+3m) .

5.3.Proposition : The number of complex triangles are found as two, nine , twenty etc of graph K(2m+2, $2m^2$+3m) for m ≥ 2.(a complex triangle is defined as a circuit of length three which does not form a region of the graph)

Proof: We Know the maximum and minimum numbers of crossings of the complete graph K_{2m+2} for m ≥ 2 by theorem 4.3. Hence, when we consider the rectangular floor planning, then the circuit K (2m+2, $2m^2$+3m) for the modules m ≥ 2 definitely overlap some rectangles which is nothing but the crossing of edges of the graph. It is found that the sub-graph $K^{//}$ (2m+2, 6m) for m ≥ 2 of the graph K(2m+2, $2m^2$+3m) is planar [3] and then only rectangular floor planning with out overlap is found for such type of graph .One of the graph of the rectangular floor plan is shown in fig. 4 for m =2.

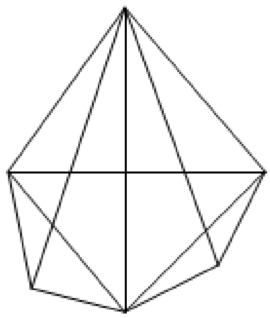

Figure-4 . A graph of rectangular floor plan.

It is also found that only two, nine , twenty ,…………..edges should be deleted from the graph K (2m+2, $2m^2$+3m) for m ≥2 to get the graph $K^{//}$ (2m+2, 6m) which give the rectangular floor plan without overlap as shown above figure-4. Therefore there are two, nine, twenty, ……..complex triangle in the graph K(2m+2, $2m^2$+3m) for m ≥2.

Remark: The deletion of complex triangle from a floor plan (not rectangular) is not easy. The above proposition gives the number of complex triangles as two, nine, twenty ….etcs. I invite the researchers to forward the proof of the proposition in general way.

5.4. Theorem: The one element sub-set of the set of vertices {2m+2/ m ≥2} of the graph K(2m+2, $2m^2$+3m) must have a hyper graph with one vertex of degree 2m+1 and two vertices are of degree 2 and all other vertices are of degree one.
Proof: It is true that there are 2m+2 number of vertices in the graph K (2m+2, $2m^2$+3m) for m ≥2 and definitely there will have one element sub-set of the set {2m+2} of vertices . If we suppose m=2 then from the vertex set { V_1, V_2, V_3, V_4, V_5, V_6 } we have one element sub-sets {V_1 }, {V_2 }, {V_3 }, {V_4}, {V_5},{ V_6} and they will be the edges of the hyper graph (by definition). Now considering the six vertices we can connect the vertices of the hyper graph keeping the hyper graph as simple by the obtained six edges (one element sub-set as shown above) as shown in fig. 5.

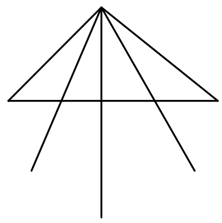

Figure-5. A hyper graph of six hyper edges.

Since there are 2m+2 number of vertices of the graph K(2m+2, $2m^2$+3m) for m ≥2 hence there are 2m+2 number of one element sub-sets and they will form the number of edges of the hyper graph for m ≥2, and the pattern of hyper graph shown in above figure -5 will follow for m ≥2.

5.5. Theorem: The two –element's disjoint sub-set of the set of vertices {2m+2 / m ≥2} of the graph K(2m+2, $2m^2$+3m) for m ≥2 must have a hyper graph with degree one.
Proof: We consider the number of two element's disjoint sub-set of the set of vertices { 2m+2/ m ≥2} . These two element's disjoint sub-sets form a set of edges of a hyper graph having 2m+2 number of vertices. We see that the set of vertices {2m+2/ m ≥2}, forms the number of two element's disjoint sub-set of various sizes of three, four, five elements ……….etc for m ≥2. [when we suppose m=2, then the set of vertices as shown by the above theorem 5.4 forms three edge

disjoint sub-sets as $\{V_1,V_2\},\{V_3,V_4\}, \{V_5,V_6\}$ and they are the edges of the hyper graph etc] . When we consider the set of two element's disjoint sub-set for m=2 as three edges and connect all the vertices 2m+2 , then we see that there is only possibility to obtain a hyper graph of degree one . Similarly for other cases of the value of m ≥3 the theorem is true.

5.6.Theorem: The two-element's non-disjoint sub-set of the vertex set {2m+2/ m ≥2} form a hyper graph as tree.

Proof: It is found that the two element's non-disjoint sub-set of the vertex set {2m+2/ m ≥2} can be form by 2m+1 ways for m ≥2. These 2m+1 ways of two element's sub-set for m ≥2 is nothing but the edges of the required hyper graph whose vertex set is {2m+2/ m ≥ 2}. Hence we have a graph of 2m+2 vertices and 2m+1 edges for m ≥2 and this is a tree .Hence the proof.

5.7 Theorem: The three element's sub-set of the vertex set { 2m+2/m≥2} form a hyper graph with 2n+2 edges for n≥1 for simultaneous changes of m ≥2.

Proof: It is easy to show that the element of the set {2m+2/m≥2} forms 2n+2 number of sub-set consisting of three elements of it for n ≥1 for simultaneous changes of m ≥2 and hence these sub-sets are the edges of the hyper graph having 2m+2 vertices .

6. Algorithm

INPUT: Let G be a hyper graph of 2m+2 vertices / modules for m ≥2

OUTPUT: Find the planar graph $G^{//}$ from G.

The following steps are considered for different structures of hyper graph G with same number of vertices 2m+2 for m ≥2.

Step1: If the hyper graph is a circuit of 2m+2 edges connecting all 2m+2 vertices for m ≥2, then go to Step2 to Step7 otherwise go to Step8.

Step2: Delete an hyper edge from the hyper graph of Step1.

Step3:Add one new edge connecting any two vertices of 2m+2 vertices for m ≥2 of the hyper graph.

Step4: Continue the deleting process of all hyper edges and addition of edges connecting the vertices of hyper graph till the deletion of all hyper edges are finished.

Step5: Obtain the new graph $G^{/}$ after addition of edges connecting the vertices of the hyper graph.

Step6: Make the graph $G^{//}$ as non-complete, non-regular simple planar graph adding some new edges connecting the vertices of $G^{/}$.

Step7:The graph $G^{//}$ is the required graph obtained from the hyper graph G and then go to Step25.

Step8: If the hyper graph G is a tree having one vertex of degree 2m+1 for m ≥2 and all other vertices are of degree one ,then go to Step9 to 11 ,otherwise go to Step 12.

Step9: Delete one hyper edge and connect the vertices v_i and v_j of the hyper graph. [this edge will be the first edge of the required graph]

Step10: Continue the deleting process till the hyper edges are deleted and for every deletion of hyper edge connect the vertices v_i and v_j, v_j and v_k………v_l and last vertex of 2m+2 vertices for m≥2.

Step11: This step will give a path $G^{/}$ of length 2m for m≥2, and there after go to Step6 to Step7.

Step12:If the hyper graph is a tree but not binary tree then go to Step13 to 16 ,otherwise go to Step17.

Step13:Deleting all hyper edges and adding the edges connecting the vertices of the hyper graph making the new graph as a tree again.

Step14:The tree obtained from the Step13 is a spanning tree of the graph $G^{//}$

Step15: Apply the cyclic interchange process on the spanning trees and obtain the final graph $G^{//}$. GO to Step16.

Step16:The graph $G^{//}$ is the required graph and GO to Step7.

Step17: If the one element sub-sets form a hyper graph with one vertex of degree 2m+1 for m ≥2 and two vertices are of degree 2 and all other vertices are of degree one then go to Step18 to Step20, otherwise go to Step21.

Step18: Delete 2m+2 hyper edges for m ≥2 of the hyper graph of Step17 and construct a graph with 2m+2 edges having one cycle of length four and another cycle of length three and one pendent vertex.

Step19: Make the graph of Step18 as planar graph $G^{//}$.

Step20: GO to Step7.

Step21:If the two element disjoint sub-sets (edges) form a hyper graph with degree one ,then go to Step22 to Step23, otherwise go to Step24.

Step22:Join any two vertices of the hyper graph by two edges so that after joining it form a spanning tree of the graph $G^{//}$. Then go to Step16.

Step23: The graph $G^{//}$ is formed. Go to Step25.

Step24: If two element non-disjoint sub-sets (edges) form a hyper graph as a tree ,then go to Step13 to Step16, otherwise go to Step25.

Step25: Stop.

7. Conclusion

This paper mainly focuses the application of some sub-graph of the complete graphs and the procedure of obtaining the graphs from hyper graphs. The further research will include the rectangular floor plane from which a planar triangulated graph without complex triangle (proposition 5.3) is to be focused related to the present discussion.

REFERENCES

[1]. Harary, F, “Graph Theory”,Narosa publishing House.pp.123. 1995.

[2]. Kalita. B, “A new set of non-planar graphs”. Bull. Pure & applied Sciences, vol.24E (No-1) pp. 29-38/2005.

[3]. Kalita .B,“ Some investigations on graph theory”, (Ph.D thesis), Finance India,Vol.xix No-4, Dec/2005, pp 1430-1438,2005.

[4]. Ajtai. M.V.Chvatal, M.M. Neuborn and E.Szemeredi,“Crossing–free subgraph, Theory and practice of Combinatorics”, Ann. Disc.Math.12, pp9- 12, 1982.

[5]. Erdos. P and R.K.Guy, “ Crossing number problems”. Amer. Math.Momthly, 80, pp 52-58,1973.

[6]. Guy.R.K,“Crossing numbers of graphs”, In graph theory and appln Kalamazoo, 1972 (ed. Y .Alavietal) Lect. Notes, Math 303 Springer (1972), 111-124, 1972.

[7]. Kleitman.D.J, “ The crossing number of $K_{5.n}$”,J.Comb. Th. 9, 315-323, 1970.

[8]. Pach.J, and G.Toth,“Graphs drawn with few crossings per Edge”, Combinatorica, 17, 427-439, 1997.

[9]. Szekely.L.A,“Crossing numbers and hard Erdos problems in discrete geometry”,Combin. Prob. Comput.6, 353-358,1997.

[10]. Douglas.B,"Introduction to graph theory", Pearson education , pp 260 -269,2002.
[11]. Dutta.A, Kalita.B, and Baruah.H.K, " Crossing and thickness of special type of non-planar graph", submitted, 2007.
[12]. M.Sarrafzadeh,C.K.Wong,"An introduction to VLSI physical design", Mc graw Hill, 1996.
[13]. Yeap.K.H, and M. Sarrafzadeh. " Floor planning by graph dualization: 2-concave rectilinear modules", 1993.
[14] .Makedon.F,and S.Tragoudas, " Approximate solutions for graph and Hyper graph partitioning in Algorithmic Aspect of VLSI layout" (M. Sarrafzadeh and. T.Lee,eds), pp.133-166.world Scientific, Singapore,1993.
[15]. Imai.H and T.Asano. " Efficient algorithms for geometric graph search Problems", SIAM Journal on Computing 15(2) 478-494, 1986.
[16]. Ihler.E, D.Wagner and F.Wagner, " Modeling hyper graphs with same mincut properties", Technical report B92-22, Freie universitat,Berlin, October. 1992.
[17]. Kalita. B, " sub-graphs of complete graphs" , Proceedings of the 2006 international conference on foundations of computer science (FCS06), LAS VEGAS, U.S.A pp. 71-77, 2006.
[18]. Kalita. B, " Construction of Strongly regular graph from TEEP graphs" Bulletin of Pure and Applied Sciences, Vol.22E (No -1), P-113-117,2003.
[19]. Kalita.B,"Non-isomorphic Hamiltonian Sub-graphs of Complete graph and their Application", Proc 92ND Indian Science Congress/2005.
[20]. Jennifer et.al " Graphical Basis partitions, Graphs and Combinatorics", 14: 241-261, 1998