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Implementation of Differential Optical Flow Algorithms in Natural Rigid Video Motion

Norazlin Ibrahim, Slamet Riyadi, Noorazrin Zakaria, Mohd Marzuki Mustafa and Aini Hussain

***Abstract*— Computing the optical flow of a sequence of images still remains a challenge in low-level video processing. Till present, none of the existing techniques has sufficiently generated accurate and dense optical flow fields to robustly represent video motion. In this paper we implement the optical flow algorithms through different lengths of displacements that exist in video motion of natural objects. We investigate the outcome of the differential optical flow algorithms based on Lukas-Kanade, Horn-Schunck and Brox's warping techniques. Experiments on natural images show that the warping technique produces smoother and consistent pattern of optical flow compared to the outputs of Lukas-Kanade and Horn-Schunck. The behaviors of optical flow fields for each algorithm can be observed accordingly with respect to their displacements.**

***Index Terms*— Optical flow, Horn-Schunck algorithm, Lukas-Kanade algorithm, warping technique.**

I. Introduction

Estimating optical flow plays an important role in detecting movements of objects from a sequence of images. Baron et al. [1] regroup major optical flow techniques into 4 classes. They are differential techniques; region-based matching, energy based methods and phased based approaches. In this comparison the differential techniques perform with lesser average errors. In addition, these differential techniques compute faster thanks to their simple set of linear equations as described by Alireza and David [2]. In this paper, we implement differential approaches mainly the original Horn-Shunck[3] and Lukas Kanade[4] algorithms, in comparison with the Brox's warping technique[5]. Natural images are used without implementing any pre-processed.

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Norazlin Ibrahim is with the Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering, Universiti Kebangsaan Malaysia, Bangi 43000 Selangor, Malaysia. (e-mail: norazlin@vlsi.eng.ukm.my).

Slamet Riyadi is with the Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering, Universiti Kebangsaan Malaysia, Bangi 43000 Selangor, Malaysia. (e-mail: riyadi@vlsi.eng.ukm.my).

Noorazrin Zakaria is with the Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering, Universiti Kebangsaan Malaysia, Bangi 43000 Selangor, Malaysia. (e-mail: drazrin@vlsi.eng.ukm.my).

Mohd Marzuki Mustafa is with the Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering, Universiti Kebangsaan Malaysia, Bangi 43000 Selangor, Malaysia. (e-mail: marzuki@.vlsi.eng.ukm.my).

Aini Hussain is with the Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering, Universiti Kebangsaan Malaysia, Bangi 43000 Selangor, Malaysia. (e-mail: aini@.eng.ukm.my).

II. Previous Work

A. The optical flow constraint equation

Most of the optical flow methods assume that the image objects keep the same intensity value under motion for at least a short period of time, expressed as follow: $\forall(x,y)\in\Omega$, $\forall t\in[0,T]$,

$$I(x,y,t) = I(x+dx, y+dy, t+dt) \quad (1)$$

Thus, the optical flow constraint equation (OFCE) is obtained by using Taylor expansion on (1) and dropping its nonlinear terms. The (OFCE) is expressed as follow:

$$I_x u + I_y v + I_t = 0 \quad (2)$$

where (u,v) represent the optical flow vectors $\left(\frac{dx}{dt}, \frac{dy}{dt}\right)$ and (I_x, I_y, I_t) represent the derivatives of image intensities at coordinate (x,y,t).

B. Regularization techniques

This single equation (2) with two unknowns poses an aperture problem as describe by Tikhonov et al [6][7]. They regularize the ill-posed equation by incorporating prior information to the equation. Generally, smoothness assumptions on the solution are used in the optimization procedure. In computer vision, the (OFCE) is replaced by the following minimization problem:

$$\min_{u,v} \int_x\int_y E(u,v)dxdy = 0, \quad (3)$$

where $E = E_d + \alpha \cdot E_r$. Here $E_d(u,v) = (I_x + I_y + I_t)^2$ represents the data term and E_r represents a regularization term. The parameter α is a positive scalar to trade-off the influence of E_r over E_d. In practice, it requires interactive adjustment to find the best value of α. Depending on the choice of E_r, some previously used terms such as quadratic smoother by Horn and Shunck [3], oriented smoother by Lukas Kanade[4] and anisotropic smoother by Brox [5].

C. Horn-Schunck algorithm

Horn and Schunck [3] assumed as an additional constraint that the optical flow is varying smoothly with neighboring object points that have almost the same velocity. This corresponds to a standard choice of E_r to be the following isotropic regularizer:

$$E_r(u,v) = \tfrac{1}{2}\left(|\nabla u|^2 + |\nabla v|^2\right). \quad (4)$$

Hence, two elliptic PDEs are obtained from (3) as expressed here-after:

$$\begin{aligned} \alpha\ \Delta u - I_x\left(I_x u + I_y v + I_t\right) = 0 \\ \alpha\ \Delta v - I_y\left(I_x u + I_y v + I_t\right) = 0 \end{aligned} \tag{5}$$

This coupled system is symmetric in the two components of the velocity u and v. Horn-Schunck solve these two equations simultaneously by using block Gauss-Seidel relaxation in order to capture the coupling effect between them, expressed as

$$u^{<n+1>} = \bar{u}^{<n>} - I_x\left(\frac{I_x\bar{u} + I_y\bar{v} + I_t}{\alpha + I_x^2 + I_y^2}\right), \tag{6}$$

$$v^{<n+1>} = \bar{v}^{<n>} - I_y\left(\frac{I_x\bar{u} + I_y\bar{v} + I_t}{\alpha + I_x^2 + I_y^2}\right)$$

where $(\bar{u},\bar{v})$ represent an average of the neighboring points to (u,v). The images of optical flow are computed using the first-order differentials of $\left(I_x, I_y, I_t\right)$, which have been approximated with the neighboring points in successive-image quadrants:

$$I_x = \frac{(I(0)_{x+1,y} + I(1)_{x+1,y} + I(0)_{x+1,y+1} + I(1)_{x+1,y+1}) - (I(0)_{x,y} + I(1)_{x,y} + I(0)_{x,y+1} + I(1)_{x,y+1})}{8}$$

$$I_y = \frac{(I(0)_{x,y+1} + I(1)_{x,y+1} + I(0)_{x+1,y+1} + I(1)_{x+1,y+1}) - (I(0)_{x,y} + I(1)_{x,y} + I(0)_{x+1,y} + I(1)_{x+1,y})}{8} \tag{7}$$

$$I_t = \frac{(\mathrm{I}(1)_{x,y} + \mathrm{I}(1)_{x+1,y} + \mathrm{I}(1)_{x,y+1} + I(1)_{x+1,y+1}) - (\mathrm{I}(0)_{x,y} + \mathrm{I}(0)_{x+1,y} + \mathrm{I}(0)_{x,y+1} + I(0)_{x+1,y+1})}{8}$$

D. Lukas-Kanade algorithm

Lukas and Kanade [4] assumed as an additional constraint that the optical flow is varying smoothly with neighboring object points that have exactly the same velocity. The least squares estimator has been adapted in (3) to minimize the squared error, expressed as:

$$E_{LK}(u,v) = \sum_{\vec{x}} g(\vec{x})[I_x u + I_y v + I_t]^2, \tag{8}$$

where $g(\vec{x})$ is the Gaussian weighting function that determines the support of the centered estimator. Thus, two PDEs are obtained from (3) expressed as:

$$\frac{\partial E(u,v)}{\partial u} = \sum_{\vec{x}} g(\vec{x})[uI_x{}^2 + vI_x I_y + I_x I_t] = 0 \tag{9}$$

$$\frac{\partial E(u,v)}{\partial v} = \sum_{\vec{x}} g(\vec{x})[vI_y{}^2 + uI_x I_y + I_y I_t] = 0$$

Lukas-Kanade solve these equations using Least Mean Square estimation:

$$\hat{u} = M^{-1}b, \tag{10}$$

where $M = \begin{bmatrix} \sum gI_x{}^2 & \sum gI_x I_y \\ \sum gI_x I_y & \sum gI_y{}^2 \end{bmatrix}$, and $\vec{b} = -\begin{pmatrix} \sum gI_x I_t \\ \sum gI_y I_t \end{pmatrix}$.

The images of optical flow are computed using partial derivatives between pixels in the *x, y* and *t* directions:

$$\begin{aligned} I_x &= m_x * (I_1 + I_2) & m_x &= \tfrac{1}{4}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \\ I_y &= m_y * (I_1 + I_2)\ , & m_x &= \tfrac{1}{4}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \\ I_t &= m_t * (I_2 - I_1) & m_t &= \tfrac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{aligned} \tag{11}$$

E. The Brox's warping algorithm

Brox [5] assumed 3 additional constraints in (3). Firstly, the gradient consistency assumption allows small variations in grey-level. This leads to the equation below:

$$E_d = \int \psi(|\, I(x+w) - I(x)\,|^2 + \gamma\,|\,\nabla I(x+w) - \nabla I(x)\,|^2\ dx \tag{12}$$

Secondly, the piecewise smooth flow field assumption estimates the displacement of a point only locally. It can be expressed as:

$$E_r = \int \psi(|\,\nabla_3 u\,|^2 + |\,\nabla_3 v\,|^2)dx \tag{13}$$

Finally, a multi-scale approach is used in order to find global minimum by down-sampling the images. It is performed during the linearization of (12) and (13) using Euler-Lagrange equations, expressed as:

$$(\psi')_d^{k,l}\begin{pmatrix} I_x^k\left(I_z^k + I_x^k du^{k,l+1} + I_y^k dv^{k,l+1}\right) \\ + \gamma \cdot I_{xx}^k\left(I_{xz}^k + I_{xx}^k du^{k,l+1} + I_{xy}^k dv^{k,l+1}\right) \\ + \gamma \cdot I_{xy}^k\left(I_{yz}^k + I_{xy}^k du^{k,l+1} + I_{yy}^k dv^{k,l+1}\right) \end{pmatrix} - \tag{14}$$

$$\alpha \cdot div\left((\psi')_r^{k,l}\,\nabla_3\left(u^k + du^{k,l+1}\right)\right) = 0$$

Common numerical method such as Gauss-Siedel or SOR iterations can be used to solve the above equation; with the expressions of *I* that can be computed by means of bilinear interpolation as demonstrated in Horn-Schunck approach.

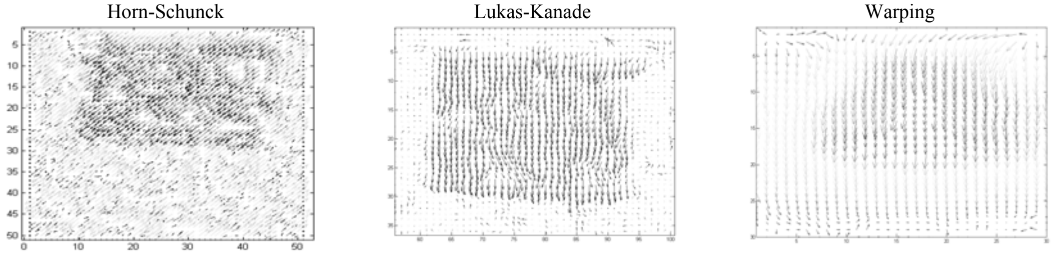
III. Experiments Results

For comparison purposes, we have implemented Horn-Schunck, Lukas-Kanade and warping algorithms using their best appearance adjustments. The testing sequences are obtained from a moving box with size 256x190 on a conveyer belt with various speeds of vertical displacement. We point out that all tested images were neither pre-smoothed nor hierarchal filtered before processing in order to observe the real outcome of each algorithm. The flow of the tested box is as shown in Figure 1using the computer specification of AMD Turion™64x2,1.181GHz,1.46GB of RAM. The estimated optical flow fields are illustrated in Figure 2.

(a) (b) (c)

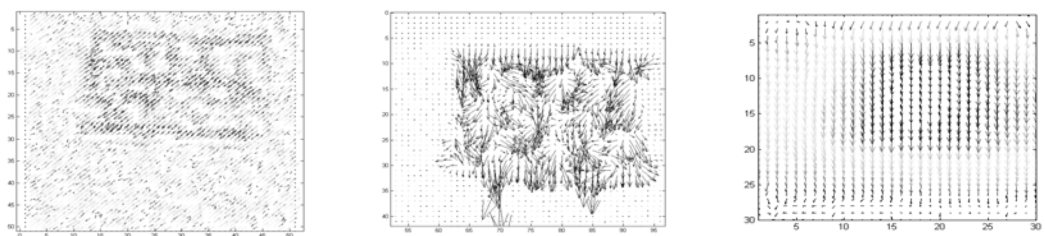
Fig. 1(a),(b) and (c) show the movement of the box.

Horn-Schunck | Lukas-Kanade | Warping



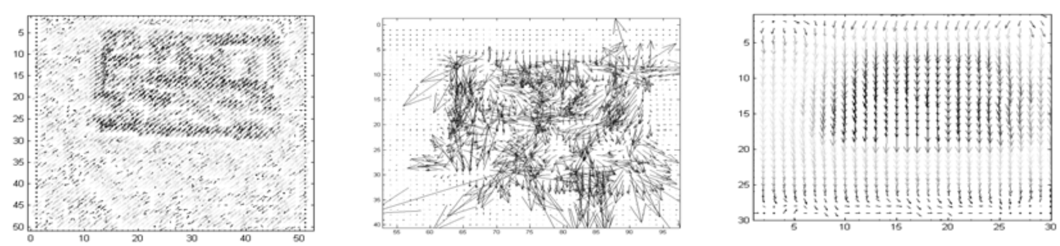
Total time taken = 1.936510 | Total time taken = 6.781360 | Total time taken = 2.224532

i)Displacement = 2 pixels



Total time taken = 2.001442 | Total time taken = 6.869587 | Total time taken = 2.167259

ii)Displacement = 6 pixels



iii)Displacement = 10 pixels

Fig. 2 The optical flow fields

The standard error of measurement (SEM) of optical flow fields is computed as equation (15).

$$SEM = \frac{\sqrt{u^2 + v^2}}{\sum (no\_arrow\_u + no\_arrow\_v)} \quad (15)$$

The SEM of optical flow fields against vertical object displacement is illustrated as below

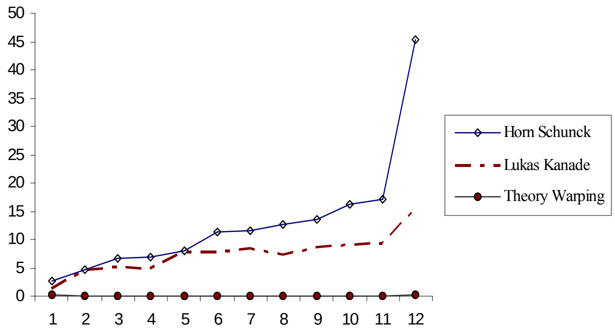


Fig. 3: The SEM of optical flow fields against vertical object displacement

IV. Discussion And Conclusion

We have compared differential optical flow fields from Horn-Schunck, Lukas-Kanade and Brox's warping techniques. The implementation on natural images of rigid object using various displacements has revealed the drawbacks of each approach regardless the improvement assumptions taken in the algorithms. The Horn-Schunck algorithm aims for better smoothing effect by providing denser fields compared to others. Within large range of object displacements, it provides consistent fields of optical flow. However they are very sensitive to errors derived from the variations of their neighboring points. On the other hand, the assumption of same neighboring velocities of optical flow in Lukas-Kanade only applies on small displacements. On larger object displacements, the fields deviate exponentially from their real displacements. Last but not least, the Brox's warping technique provides consistent density of fields with robust field orientations. However the local and global smoothing processes have rapidly increased the complexity of the algorithm. As a result it consumes greater execution time. Future work will be focused on reducing the time of execution so that the optical flow can be implemented in real-time process. Thus a novel, simpler but efficient regularization technique is inevitable.

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