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A Neural Network Model Based on Quadratic Programming to the Single-period and Multi-product Newsvendor Problem

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***Abstract**: The newsvendor problem now has been extended into complicated models to meet the commerce pattern's growing intricacy. For the requirement of outsourcing operation, supply chain management and cost-profit control, we need to develop practicable, potent and computer- operated approaches to resolve the newsvendor problems which the various costs and characteristic parameters of products are divers and the constraints of trade operation are multiplex. In this paper, we develop a quadratic programming approach (QPA) to settle the single-period, multi-product and multi-constraint newsvendor problem with regular distribution of market demand, and then we apply neural network method (NNM) to calculate the optimal order quantity of each potential available product. This model is quite practicable in actual application for that the QPA transforms the original newsvendor problem from integral pattern to quadratic function, which not only increases the maneuverability but also decreases the complexity of calculation greatly, and the NNM establishes a general model which is expansible and superior especially when the number of products is large.*

***Keywords**: cost-profit control, newsvendor problem, neural network, quadratic programming*

I. INTRODUCTION

The classical newsvendor problem (formerly the newsboy problem) is proposed by Hadley and Whitin [1] initially in 1963. Subsequently, they had extended their original model into the more complicated situations of multi-product with multi-constraint. Although they had applied many approaches such as Leibniz Rule, Lagrange Multiplier and Dynamic programming to solve these models, there were still some difficulties in resolving such problem effectively, particularly when the number of products considered is large.

After Hadley and Whitin's early work, many publications extended their original research by developing new solution methods and presenting new areas of application for these models. Gallego and Moon [2] and Khouja [3] had reviewed the extensions that were developed. And then, the related literatures continuously appear. Here we cite some of these articles that are relevant to the single-period and multi-product newsvendor problem. Erlebacher [4] developed both optimal and heuristic solutions for the single-period and multi-product newsvendor problem with the capacity constraints. He proved the optimality of the order quantities based on two special premises. The first was that the cost structure is the same for all considered products while the second was concerned with uniform probability density function for the demand distribution of all products. Then, he proceeded to developing heuristics for a few general probability distribution functions. Abdel-Malek and Montanari [5] developed exact, approximate, and iterative solution models to settle the capacitated newsvendor problem.

A quadratic programming solution model for the single-period and multi-product newsvendor problem with side constraints has been developed by Abdel-Malek and Areeratchakul [6] in 2006. The model accommodated lower bounds of products' demands that are larger than zero, and facilitated the performance of sensitivity analysis tasks.

While the newsvendor problem in our actual commerce is becoming more intricate, for that the number of products is gradually increasing and the constraints of available resources and some others are becoming more complicated, the optimization of multi-product procurement order quantities encounters so many difficulties in procedure of obtainment. We apply the quadratic programming approach to settle single-period and multi-product newsvendor model based on the demand forecasting of definite demand distribution such as uniform distribution, normal distribution and so on, which has transform the integral operation into the square operation that greatly reduces the complexity of calculation.

The quadratic programming problem is general existing in multi-subject. For its particular importance, many scholars have put forward a number of efficient

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algorithms. Goldfarb and Idnani [7] proposed a dual and original-dual algorithm to resolve the convex quadratic programming problem. This method started with the best optimal values in no constraints' case, and iterated to obtain the original problem's optimization values by operating a series of sub-issues of best objective functions which ones the objective values are increasing strictly. It has the superiority when the initial feasible point is not easy to find. Zhong and Tapia proposed an amended Newton method in 1992, which is ultra-linear convergence and the points of iteration are in the set of feasible solutions strictly. Nie and Magundho [8] put forward a simplex solution method to solve the strict convex quadratic programming problem. It's a robust method of searching the best objective point directly without using the condition of Kuhn-Tucker. According to the generalized multiplier method, He [9] transformed the convex quadratic programming problem of inequality constraints into ones which are only of non-negative constraints partly to resolve the large-scale sparse problems.

The calculation efficiency and practicability of quadratic programming functions can inadequately be satisfied by these traditional mathematics methods especially when the number of independent variables is large enough and the constraints are troublesome in practice. Since that the practicability and superiority of intelligent algorithm in handling the problem of optimizations have been recognized, here we apply the quadratic programming approach using neural network algorithm to settle the single-period and multi-product newsvendor problem.

In next section, the single-period and multi-product newsvendor model with constraints based on original work of Hadley's and Whitin is introduced, and then we present our developed quadratic programming approach. The neural network's establishment of obtained developed quadratic model is the subject of the third section. Finally, we conclude with a summary and conclusions of the paper.

II. The Newsvendor Model and Quadratic Programming Approach

This section is divided into two parts. In the first part we present one of the newsvendor models based on Hadley and Whitin's original work, and then in the second part, we show the development of our quadratic programming method.

A. Hadley and Whitin's newsvendor model

Johnson and Montgomery [10], Silver et al. [11] and Nahmias [12] have proposed different formulation for the newsvendor problem. These models are more or less mathematically equivalent, though the authors may slightly differ in their approaches to defining costs and profits. In this paper, we apply the one based on the Johnson and Montgomery's model.

Min$Z$

$$Z=\sum_{i=1}^{K}[C_iX_i + H_i\int_0^{X_i}(X_i-D_i)f_i(D_i)\,dD_i + O_i\int_{X_i}^{\infty}(D_i-X_i)\,f_i(D_i)\,dD_i]. \quad (1)$$

Subject to:

$$\sum_{i=1}^{K}(C_iX_i)\le B_g\ ;$$
$$\sum_{j=1}^{M}\sum_{i=1}^{K}(\beta_{j,i}X_i)\le R_j\ ,\ j=1,2,\cdots,M\ ;$$
$$X_i\ge L_i\ ,\ i=1,2,\cdots,K\ ;$$

where Z is the expected cost to be optimized, K is the number of products, i is the product index, C_i is the unit cost of product i, X_i is the amount to be ordered of product i, D_i is the random variable of product i's demand, $f_i(D_i)$ is the probability density function of demand for product i, H_i is the cost incurred per each product I's leftover at the end of the period, O_i is the out-stock cost per unit of product i, Bg is the firm's available budget, M is the number of constraints, $\beta_{j,i}$ is the coefficient of resource j of product i, R_j is the amount of available resource j and L_i is lower bound of order quantity of product i.

Eq. (1) consists of three major terms. The first term expresses the acquisition cost of the different products. The second one estimates the cost incurred when overstocking occurs for each of the considered products. The last shows the shortage cost for each of those products.

B. Quadratic programming approach

We transform the objective function shown in Eq. (1) into the generalized objective function with the simpler format.

$$Z=\sum_{i=1}^{K}[C_iX_i + H_i\int_0^{X_i}(X_i-D_i)f_i(D_i)\,dD_i + O_i\int_{X_i}^{\infty}(D_i-X_i)\,f_i(D_i)\,dD_i]$$
$$=\sum_{i=1}^{K}[\,C_iX_i + H_iX_i\int_0^{X_i}f_i(D_i)\,dD_i - H_i\int_0^{X_i}D_if_i(D_i)\,dD_i + O_i\int_{X_i}^{\infty}D_i\,f_i(D_i)dD_i - O_iX_i\int_{X_i}^{\infty}f_i(D_i)\,dD_i]$$
$$=\sum_{i=1}^{K}[\,C_iX_i + H_iX_iF(X_i) - H_i\int_0^{X_i}D_i\,f_i(D_i)\,dD_i + O_i(E(D_i) - \int_0^{X_i}f_i(D_i)\,dD_i) - O_iX_i[1-F(X_i)]]$$
$$=\sum_{i=1}^{K}[\,C_iX_i + H_iX_iF(X_i) - (H_i+O_i)\int_0^{X_i}D_if_i(D_i)\,dD_i + O_i(E(D_i) - O_iX_i[1-F(X_i)]]. \quad (2)$$

Implement integral by parts to $\int_0^{X_i}D_if_i(D_i)\,dD_i$, we Obtain [13] :

$$\int_0^{X_i}D_if_i(D_i)\,dD_i = X_iF(X_i) - \int_0^{X_i}F(D_i)\,dD_i. \quad (3)$$

Substitute (3) back into (2) and change the problem into maximization, we obtain Eq. (4).

Max Z

$Z=\sum_{i=1}^{K}[(O_i - C_i)X_i - (H_i+O_i)\int_0^{X_i} F(D_i)\, dD_i - O_i(E(D_i))].$ (4)

Then, we express the objective function given in Eq. (4) into the following quadratic format:

$$\text{Max } Z = \sum_{i=1}^{K}(A_i^{(*)}X_i^2 + B_i^{(*)}X_i + C_i^{(*)}). \quad (5)$$

Where $A_i^{(*)}$, $B_i^{(*)}$ and $C_i^{(*)}$ are constants to be determined for each product i according to its demand probability distribution function (*).

In the following, we consider three of the most commonly used probability density functions for the distribution of products' demand; the uniform, the exponential, and the normal.

Demand of uniform distribution

The integral of the cumulative density function (CDF) of the uniform distribution is show as follows:

$$\int_0^{X_i} F(D_i)\, dD_i = \frac{X_i^2 - 2a_iX_i + a_i}{2(b_i - a_i)},\ a_i \le X_i \le b_i.$$

The last term of the Eq. (4) requires the estimation of the expected value of the uniform：$(a_i + b_i)/2$.

Hence, substituting these values in Eq. (4), the constants of Eq. (5) are obtained as follows:

$$A_i^U = -\frac{(H_i + O_i)}{2(b_i - a_i)};$$
$$B_i^U = -C_i + \frac{(a_iH_i + b_iO_i)}{(b_i - a_i)};$$
$$C_i^U = -\frac{(a_i^2H_i + b_i^2O_i)}{2(b_i - a_i)}.$$

Demand of exponential distribution

Similar to the approach followed in the case of the uniform distribution, we integrate the CDF of the exponential distribution as follows:

$$\int_0^{X_i} F(D_i)\, dD_i = X_i + \lambda_i(e^{-\frac{X_i}{\lambda_i}} - 1)$$

We then substitute these values into Eq. (4) and hence the constants of Eq. (5) can be approximated using the first two terms of Taylor's expansion as follows:

$$A^{Exp} = -\frac{(H_i + O_i)}{2\lambda_i};$$
$$B^{Exp} = (O_i - C_i);$$
$$C^{Exp} = -O_i\lambda_i.$$

And the error is:

$$\varepsilon = \frac{(-2\lambda_i^2 e^{-\frac{X_i}{\lambda}} + X_i^2 - 2X_i\lambda_i + 2\lambda_i^2)(H_i + O_i)}{2\lambda_i((C_i + H_i)X_i + (O_i + H_i)\lambda_i e^{-\frac{X_i}{\lambda}} - H_i\lambda_i)}.$$

Demand of normal distribution

The exact value of the integral of the CDF of the normal distribution is difficult to obtain. To address this difficulty, we approximate the normal demand PDF to that of the uniform with its limits as follows:

$a_i^N = \max(\mu_i - \sqrt{3}\sigma_i, o)$; $b_i^N = \mu_i + \sqrt{3}\sigma_i$;

where μ_i and σ_i are the mean and the standard deviation of the normal distribution respectively.

Then, we can use the formulae in chapter (2.2.1) to determine the pertinent constants. It should be noted that the limits a_i^N and b_i^N cover more than 99% of the area under the normal curve. It should be also noted that the conducted numerical experiments show that this approximation yields less than 1% error in the optimum value of the objective function.

$$A_i^U = -\frac{(H_i + O_i)}{2(b_i^N - a_i^N)};$$
$$B_i^U = -C_i + \frac{(a_i^NH_i + b_i^NO_i)}{(b_i^N - a_i^N)};$$
$$C_i^U = -\frac{[(a_i^N)^2H_i + (b_i^N)^2O_i]}{2(b_i^N - a_i^N)}.$$

To sum up, we attain the table.1 of the objective function coefficients for the uniform, exponential, and normal distributions as follows:

Table 1 The objective function coefficients for the uniform, exponential and normal distributions

Distributions	$A_i^{(*)}$	$B_i^{(*)}$	$C_i^{(*)}$
Uniform	$-\frac{(H_i + O_i)}{2(b_i - a_i)}$	$-C_i + \frac{(a_iH_i + b_iO_i)}{(b_i - a_i)}$	$-\frac{(a_i^2H_i + b_i^2O_i)}{2(b_i - a_i)}$
Exponential	$-\frac{(H_i + O_i)}{2\lambda_i}$	$-(O_i - C_i)$	$-O_i\lambda_i$
Normal	$-\frac{(H_i + O_i)}{2(b_i^N - a_i^N)}$	$-C_i + \frac{(a_i^NH_i + b_i^NO_i)}{(b_i^N - a_i^N)}$	$-\frac{[(a_i^N)^2H_i + (b_i^N)^2O_i]}{2(b_i^N - a_i^N)}$

III. Neural network model of newsvendor's quadratic programming approach

We transform the equation (5) into the following pattern to establish our neural network model:

Max Z

$$Z=\sum_{i=1}^{K}(A_i^{(*)}X_i^2+B_i^{(*)}X_i+C_i^{(*)})$$
$$=(\sum_{i=1}^{K}A_i^{(*)}X_i^2)+(\sum_{i=1}^{K}B_i^{(*)}X_i)+(\sum_{i=1}^{K}C_i^{(*)})$$

Subject to:

$$\sum_{i=1}^{K}(C_iX_i)\le B_g\ ;$$
$$\sum_{j=1}^{M}\sum_{i=1}^{K}(\beta_{j,i}X_i)\le R_j\ ,\ j=1,2,\cdots,M\ ;$$
$$X_i\ge L_i\ ,\ i=1,2,\cdots,K\ .$$

It is equivalent to solve:

Min Z^*

$$Z^*=-(\sum_{i=1}^{K}A_i^{(*)}X_i^2)+(\sum_{i=1}^{K}B_i^{(*)}X_i)+(\sum_{i=1}^{K}C_i^{(*)})$$
$$=X^TAX+B^TX+C;$$

Subject to:

$$QX^T\le U.$$

Where:

$$X=[X_1\quad X_2\quad X_3\quad \ldots\quad X_K];$$

$$Q=\begin{bmatrix} C_1 & C_2 & \cdots & C_K \\ \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,K} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{M,1} & \beta_{M,2} & \cdots & \beta_{M,K} \\ -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{bmatrix};$$

$$A=\begin{bmatrix} -A_1^{(*)} & 0 & \cdots & 0 \\ 0 & -A_2^{(*)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -A_K^{(*)} \end{bmatrix};$$

$$B^T=\left[-B_1^{(*)}\quad -B_2^{(*)}\quad -B_3^{(*)}\quad \cdots\quad -B_K^{(*)}\right]^T;$$

$$C=-\sum_{i=1}^{K}C_i^{(*)}.$$

Then, in order to establish the neural network model, we define the function as follows:

$$Y=QX^T-U$$
$$=[Y_1\quad Y_2\quad Y_3\quad \cdots\quad Y_{M+K+1}]^T.$$

Where:

$$Y_\theta=Q_\theta X-U_\theta,\ \theta=1,2,3,\cdots,M+K+1.$$

Using the Augmented Lagrange Multiplier method, we define the energy function of above quadratic programming equations as follows:

$$E(X,\xi)=X^TAX+B^TX+\xi\sum_{\theta=1}^{M+K+1}Max\{Y_\theta,0\};$$

where ξ is denote the penalty parameter and $\xi>0$.

Using the discrete-time steepest descent method, we obtain the update equation:

$$\frac{\partial E(X,\xi)}{\partial X}=2AX^T+B^T+\xi\sum_{\theta=1}^{M+K+1}S_\theta\begin{bmatrix}Q_{\theta,1}\\ Q_{\theta,2}\\ \vdots \\ Q_{\theta,k}\end{bmatrix};$$

where $S_i=\begin{cases}0 & Y_\theta\le 0\\ 1 & Y_\theta\le 0\end{cases};$

$$X_i(n+1)=X_i(n)-\mu\{2AX^T+\mathrm{B_i}+\xi\sum_{\theta=1}^{G}S_\theta Q_{\theta,i}\}.$$

where μ is the learning parameter and G=M+K+1.

The neural network of abovementioned newsvendor quadratic programming model (NQPM) is showed as following figure 1:

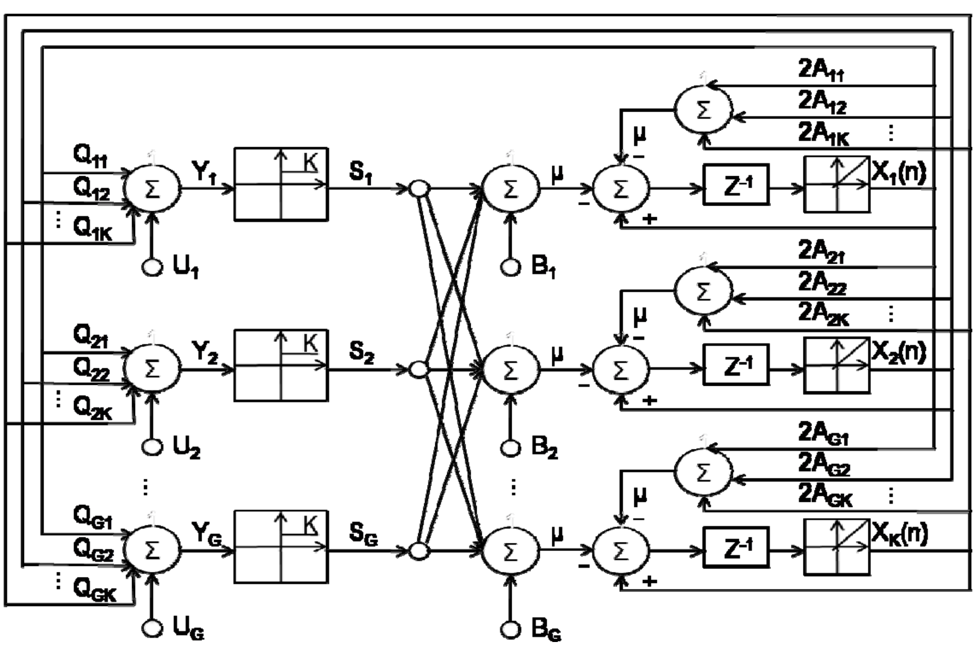

Figure 1 Neural network of NQPM

We take a representative case for example, which considers a newsvendor problem with five products and four resource constraints. The products characteristics are given in table 2, and constraints of these resources and their usage for each product are shown in table 3.

Table 2 Products characteristics and costs

Product	Distribution	Parameter	C_i	O_i	H_i
1	Exponential	(150)	7	3	2
2	Uniform	(75,125)	6.5	3	3
3	Normal	(130,20)	3	1	2
4	Exponential	(275)	8	3	4
5	Normal	(200,30)	7	4	2

Table 3 Numerical data for resources constraints

Constraint	Resource availability	Resource usage coefficient for product				
		1	2	3	4	5
1	400	2	3	4	2	2.5
2	400	2	3	1	3	1
3	400	3	1	2	2	4
4	400	2	2	3	1	3

The firm's available budget is 5000 and the lower bound of order quantity of each product is:

$$L_1=10\ ;\ L_2=10\ ;\ L_3=10\ ;\ L_4=10\ ;\ L_5=10.$$

Then we transform this representative newsvendor example into our developed quadratic programming model.

According to Table 1, we get the model and the constants obtained in quadratic transformation as follows in table 4:

Table 4 Required constants

Product	$A^{(*)}$	$B^{(*)}$	$C^{(*)}$
1	-0.0167	4	-450
2	-0.06	5.5	-637.5
3	-0.0144	4.5	-1044.87
4	-0.0127	5	-825
5	-0.0288	5.538	-1431.85

Using the neural network approach, we select the penalty parameter ξ=1 and the learning parameter μ=0.01. After 3000 iterations, the calculated values of this example as follows in table 5:

Table 5 Order quantity of each product

Product	X_i
1	10.0000
2	16.6501
3	30.2820
4	59.9959
5	35.5721
Total cost	3760.6711

IV. Summary and Conclusions

In this paper, we develop a neural network solution method which is based on quadratic programming model for the single-period, multi-product and multi- constraint newsvendor problem. The contribution of this work includes: (1) it addresses the complexities resulting from right skewed probability density function of products' demand by utilizing the familiar nonlinear quadratic programming approach to solve the multi-constraint model of the newsvendor problem, which offers "good" approximate solutions depending on the type of demand distribution functions, and (2) it offers the decision-maker a practicable method to settle the problem of calculation efficiency and feasibility by applying neural network model, especially when the number of products and constraints are quite large.

Illustrative examples are given to show the generality of the developed method in solving the problem under different demand probability distribution scenarios.

References

[1] Hadley, G., Whitin, T., "Analysis of Inventory Systems," Prentice-Hall, *Englewood Cliffs*, NJ. 1963.

[2] Gallego, G., Moon, I., "The distribution free newsboy problem: Review and extensions,"*Journal of the Operational Research Society* 44, 825 – 834. 1993.

[3] Khouja, M., "The single-period (news-vendor) problem: Literature review and suggestions for future research," *Omega International Journal of Management Science* 27, 537 – 553,1999.

[4] Erlebacher, S., "Optimal and heuristic solutions for the multi-item newsvendor problem with a single constraint," *Production and Operations Management* 9, 303 – 318. 2000.

[5] Abdel-Malek, L., Montanari, R., Morales, L.C., "Exact, approximate, and generic iterative models for the multi-product newsboy with budget constraint," *International Journal of Production Economics* 91, 189 – 198. 2003.

[6] Layek L. Abdel-Malek, Nathapol Areeratchakul, "A quadratic programming approach to the multi-product newsvendor problem with side constraints," *European Journal of Operational Research* 176, 1607 – 1619, 2006.

[7] Goldafrb D., A. Idnani， "Dual and primal-dual method for solving strictly convex quadratic programs",1987.

[8] Nie Y. Y., S. O. Magundho, "A quasi-simplex methods for solving strictly convex quadratic programs," *Mini-Micro System*, 22(1-6), 2001.

[9] He L. Q. "The new algorithm of convex quadratic programming problem with inequality constraints," *Journal of Beijing University of Technology*, 18(541-547) ,1998.

[10] Johnson, L.A., Montgomery, D.C., "Operations Research in Production Planning, Scheduling, and Inventory Control," *John Wiley and Sons*, Inc., NY. ,1974.

[11] Silver, E.A., Pyke, D.F., Peterson, R., "Inventory Management and Production Planning and Scheduling," *third. John Wiley and Sons*, NY. ,1998.

[12] Nahmias, S., "Production and Operations Analysis," f*ourthed. McGraw-Hill Higher Education*, NY. 2001.

[13] Layek L. Abdel-Malek, Nathapol Areeratchakul, "A quadratic programming approach to the multi-product newsvendor problem with side constraints." *European Journal of Operational Research* 176,1607 – 1619,2006.