

# Application of Neuro Fuzzy Reduced Order Observer in Magnetic Bearing Systems

M. A., Eltantawie, *Member, IAENG*

**Abstract—** Adaptive Neuro-Fuzzy Inference System (ANFIS) is used to design fuzzy reduced order observer for solving the problem of unbalance in magnetic bearing systems. Nonlinear model of magnetic bearing with unbalance is very complicated and coupling between various states cannot be neglected. With classical methods no one can eliminate disturbances without linearization of the system and neglecting some terms from the nonlinear model. In this paper ANFIS is used to acquire desired membership functions of the fuzzy rules for reduced order observer to achieve rejection of unbalance. Simulation results show that fuzzy observer can eliminate the effect of unbalance at any rotational speed.

**Index Terms—** ANFIS, gain scheduling, magnetic bearing, reduced order observer.

## I. INTRODUCTION

Magnetic bearings are being extensively used in industrial and manufacturing applications due to their unique properties, (attain high speed of rotation, operate in environments not amenable to the use of lubrication, and non-contacting nature leading to elimination of frictional losses. Magnetic bearing systems are unstable and fast, therefore they need well designed controllers to keep its precision position and to eliminate unbalance vibrations at any time. Rotor unbalance usually generated disturbance forces and moments, the forces due to the eccentricity between the center of rigid body and center of the shaft (static unbalance), but disturbance moments due to the inclination angle between axis of symmetry of rigid body and rotation axis (dynamic unbalance).

Several researches deal with unbalance in magnetic bearing system by different methods. Fumio [1], [4] deals with unbalance by two steps. The first step is to compensate the unbalance forces by generating electromagnetic forces that cancel it. The second step is to make the rotor rotates around its axis of inertia (automatic balancing). Another solution is filter based automatic balancing achieved by notch filter centered at the frequency of rotation inserted in the control loop to eliminate bearing stiffness at this frequency [5].

Unbalance compensation and automatic balance using Q-parameterization theory were done by [5], [6]-[9], they assumed that the unbalance is a sinusoidal noise in the measured signal and choose the controller Q-parameter to reject this noise. Also this problem was solved by using

gain scheduled  $H_\infty$  robust controller based on the Loop Shaping Design Procedure (LSDP) [10].

In this paper a combination of ANFIS technique and gain scheduling was proposed to derive fuzzy membership functions and fuzzy IF-THEN rules for fuzzy reduced order observer from data point to cancel disturbances at any rotational speed. Whence, the same observer was jointed to the nonlinear model.

## II. MATHEMATICAL MODEL

### A. Equations of Motion

Consider the vertical shaft magnetic bearing system shown in Figure 1, the fundamental equations of motion of the rotor [11] for four radial degrees of freedom are:

$$\begin{aligned} m\ddot{x} + m\omega^2 \sin \omega t &= f_{ax} + f_{bx} \\ m\ddot{y} + m\omega^2 \cos \omega t &= f_{ay} + f_{by} \\ I_x \ddot{\psi} - I_z \omega \dot{\theta} - \varepsilon \omega^2 (I_x - I_z) \sin \omega t &= af_{ax} - bf_{bx} \\ I_x \ddot{\theta} - I_z \omega \dot{\psi} - \varepsilon \omega^2 (I_x - I_z) \cos \omega t &= -af_{ay} + bf_{by} \end{aligned} \quad (1)$$

where

$a, b$  :Distances from center to bearings  $a$  &  $b$ .

$e$  :Eccentricity  $\mu m$ .

$I_i$  : Moment of inertia around  $i$  axis  $kg.m^2$ .

$m$ : Rotor mass  $kg$ .

$f_{ij}$ : Forces in  $x$  &  $y$  axis at bearings  $a$  &  $b$ .

### B. Electromagnet Equations

Forces  $f_{ax}, f_{bx}, f_{ay}$  and  $f_{by}$  can be expressed in terms of coil current  $i$  and the gap length  $S$  as follows:

$$f_B = K \left( \frac{i_B^2}{S_B^2} \right), \quad K = \frac{\mu_0 A g N^2}{4} \quad (2)$$

where

$A$ : Area of magnetic coil  $mm^2$ .

$i_B$ : coil current of  $j^{th}$  stator electromagnet  $Amp$ .

$N$ : Number of turns in each electromagnet coils.

$S$ : Steady state air gap length  $mm$ .

From linearization forces as a function of current and air gap

$$f_B = K_{SB} Z_B + K_{IB} i_B \quad (3)$$

where

$$i_B = (i_{ax} \ i_{bx} \ i_{ay} \ i_{by})$$

$$K_{SB} = 2diag(k_{sa}, k_{sb}, k_{sa}, k_{sb})$$

$$K_{IB} = 2diag(k_{ia}, k_{ib}, k_{ia}, k_{ib})$$

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M. A. Eltantawie is with the Higher Technological Institute –Six October City-Egypt, phone: 002-011-8869604; e-mail: manartantawie@maktoob.com.

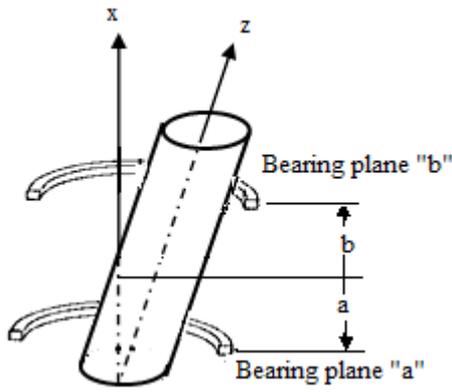


Fig.1. Rigid rotor with two radial bearings

The voltage across the  $j$  electromagnet coils can also expressed as follows:

$$u_j = Ri_j + L \frac{di_j}{dt} + k_i \dot{z}_B(j) \quad (4)$$

where

$R$ : electromagnetic coil resistance *ohm*.

$L$ : coil inductance *H*.

$z_B = [x_a \ x_b \ y_a \ y_b]^T$

Let equations (1-4) are written in the form:

$$M\ddot{Z} = G\dot{Z} + f + Ew(t) \quad (5)$$

where

$M = \text{diag.}(m, I_x, m, I_x)$

$G = \text{zeros matrix excepted } g_{24} = -g_{42} = \omega I_z$

$$E = \omega^2 \begin{bmatrix} me & 0 \\ \varepsilon(I_x - I_z) & 0 \\ 0 & me \\ 0 & \varepsilon(I_x - I_z) \end{bmatrix}, \quad w(t) = \begin{bmatrix} \sin \omega t \\ \cos \omega t \end{bmatrix} \quad (6)$$

Transform (5) to bearing coordinates

$$M_B \ddot{z}_B = G_B \dot{z}_B + f_B + E_B w(t)$$

$$M_B = T_B^T M T_B \quad (7)$$

$$G_B = T_B^T G T_B$$

$$E_B = T_B^T E$$

$T_B$  is the transformation matrix from principle axes to bearing axes. If we assume that the force displacement factor and force current factor are the same for bearings (a) and (b), then

$$\ddot{z}_B = M_B^{-1} G_B \dot{z}_B + M_B^{-1} K_{SB} z_B + M_B^{-1} K_{IB} i_B + E_B w(t) \quad (8)$$

The state vector is composed of displacements, their derivatives and electrical currents in coils. For this model the voltage is the input variable. The final state space model is

$$\begin{aligned} \dot{x}_B &= A_B x_B(t) + B_B u(t) + \Gamma_1 w(t) \\ y &= C x_B(t) \end{aligned} \quad (9)$$

$$x_B = \begin{bmatrix} z_B \\ \dot{z}_B \\ i_B \end{bmatrix}$$

$$A_B = \begin{bmatrix} 0_{4 \times 4} & I_{4 \times 4} & 0_{4 \times 4} \\ (M_B^{-1} K_{SB})_{4 \times 4} & (M_B^{-1} G_B)_{4 \times 4} & (M_B^{-1} K_{IB})_{4 \times 4} \\ 0_{4 \times 4} & \left(-\frac{K_i}{L}\right)_{4 \times 4} & \left(-\frac{K_i}{L}\right)_{4 \times 4} \end{bmatrix}$$

$$C = [I_{4 \times 4} \ 0_{4 \times 4} \ 0_{4 \times 4}], \quad \Gamma_1 = \begin{bmatrix} 0_{4 \times 2} \\ (M_B^{-1} E_B)_{4 \times 2} \\ 0_{4 \times 2} \end{bmatrix}$$

Assume that  $\omega = \text{zero}$  and there is no disturbance, we can divide the Multi-Input-Multi-Output (MIMO) system to four Single-Input-Single-Output (SISO) separate subsystems using decentralized method [12] and control each subsystem individually.

Discretization of each subsystem by using a zero order hold at a sampling time of  $t_s$  (sec) yields the following SISO discrete-time control system.

$$\begin{aligned} z(k+1) &= \Phi z(k) + \Gamma u(k) \\ Y(k) &= C z(k) \end{aligned} \quad (10)$$

Disturbance forces in whole system obey the following equations [13]:

$$\begin{aligned} \dot{x}_d &= F_d x_d \\ w &= H_d x_d \end{aligned} \quad (11)$$

where

$$F_d = \omega^2 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad H_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The discrete model is done by zero order hold with sampling time  $t_s$  sec, as

$$\begin{aligned} x_d(k+1) &= \Phi_d x_d(k) \\ w(k) &= H_d x_d(k) \end{aligned} \quad (12)$$

We augmented the system model with disturbance model, so that the state space equations become

$$\begin{aligned} \begin{bmatrix} x_B(k+1) \\ x_d(k+1) \end{bmatrix} &= \begin{bmatrix} \Phi & \Gamma_d \\ 0 & \Phi_d \end{bmatrix} \begin{bmatrix} x_B(k) \\ x_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} u(k) \\ Y(k) &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_w} \begin{bmatrix} x_B(k) \\ x_d(k) \end{bmatrix} \end{aligned} \quad (13)$$

where

$$\Gamma_d = \begin{bmatrix} 0 \\ M_B^{-1} E_B H_d \\ 0 \end{bmatrix}, \quad x_d(k) = \begin{bmatrix} \sin \omega t \\ \cos \omega t \end{bmatrix} \ \& \ \Phi = e^{-A_B t_s}$$

### III. CONTROLLER DESIGN

The first step to design the controller is to know the feedback gain  $K$  for system in equation (10), which satisfies the control law

$$u = -Kz \quad (14)$$

In our study, pole placement method is used to obtain the gain vector  $K$  for each subsystem. The second step is disturbance rejection where, we estimate disturbance signal by the reduced order observer for each subsystem (Fig. 2) and then we use that estimate in control law to force the error to be zero.

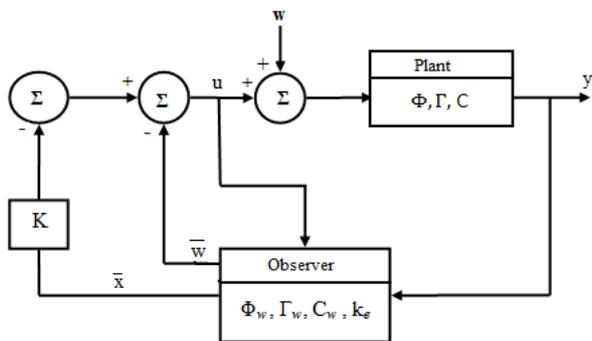


Fig.2. Block diagram disturbance rejection

#### IV. NEURO-FUZZY REDUCED ORDER OBSERVER

The interest in Neuro-Fuzzy systems has grown tremendously over the last few years. Fuzzy systems are designed to work with knowledge in the form of linguistic rules. But the translation of these linguistic rules into the framework of fuzzy set theory depends on the choice of certain parameters, for which no formal method is known. The optimization of these parameters can be carried out by neural networks, which are designed to learn from training data [14], [15].

We present new learning strategies to derive fuzzy reduced order observer rules from data. A regular first-order Sugeno fuzzy controller uses its inputs both in the premise part to determine firing strengths and in the consequent part to determine each rule's output; this is shown in Figure 3(a), where both the premise and consequent parts receive the same inputs.

For certain applications, it suffices to use only some of the inputs for the premise part and the others for the consequent part, as shown in Fig. 3(b). Obviously, Fig. 3(b) is a special case of Fig. 3(a), but is very useful in designing fuzzy observer based on the concept of gain scheduling.

Specifically, the inputs to a fuzzy observer contain two types of variables: scheduling and state. Scheduling variables are used in the premise part to determine what mode or characteristics the plant has. Once the plant mode or characteristics have been determined, the corresponding rule is fired with an output equal to the state variables multiplied by appropriate state feedback gains and reduced order observer gain. In Fig. 3(b), z and u are the scheduling variables but x and y are the state variables [16].

A Neuro-Fuzzy observer has the following fuzzy IF-THEN rules:

$$\begin{aligned} \text{IF } u \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ THEN } k_{e1n} &= F_{1n} \\ \text{IF } u \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ THEN } k_{e2n} &= F_{2n} \end{aligned} \quad (15)$$

where

$k_e$  is the estimator gain at any rotational speed.

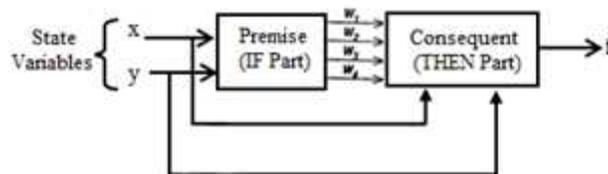


Fig. 3.a Sugeno fuzzy controller

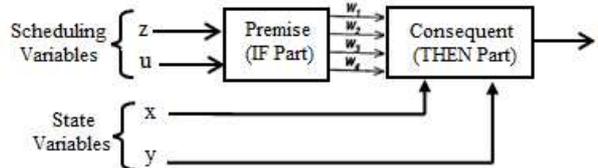


Fig. 3. (b) Fuzzy gain scheduling controller

#### V. DESIGN APPROACH

The design of Neuro-Fuzzy observer procedure is as follows:

- 1- Determination of a set of various values of rotational speed, for each speed determine the estimator gain and the value of estimated disturbance.
- 2- Use of ANFIS to construct fuzzy inference system whose Membership function parameters are tuned or adjusted using the back propagation algorithm and least square method for a set of rotational speed.
- 3- Combination of nonlinear plant with both feed-back controller and Neuro Fuzzy observer Fig. 4.

#### VI. RESULTS

Simulation with MATLAB program using data in Table I is done. The feedback gains are obtained via decentralized controller with pole placement method for each subsystem based on zero rotational speed. Then collect and rearrange the obtained feedback gains to the form given in gain matrix for the complete system.

$$\begin{bmatrix} K_{xa} & 0 & 0 & 0 \\ 0 & K_{xb} & 0 & 0 \\ 0 & 0 & K_{ya} & 0 \\ 0 & 0 & 0 & K_{yb} \end{bmatrix}_{12 \times 12} \quad (16)$$

where

$K_{xa}, K_{xb}, K_{ya}, K_{yb}$  are controllers of the  $x_a, x_b, y_a, y_b$  subsystems respectively.

The reduced order observer is designed separately for each subsystem to twenty-five different numbers of rotational ( $\omega$ ) speeds from  $2\pi 20$  to  $2\pi 250$  rad/sec. Values of static and dynamic unbalance are  $e=0.1$  mm,  $\epsilon=4 \times 10^{-4}$  rad.

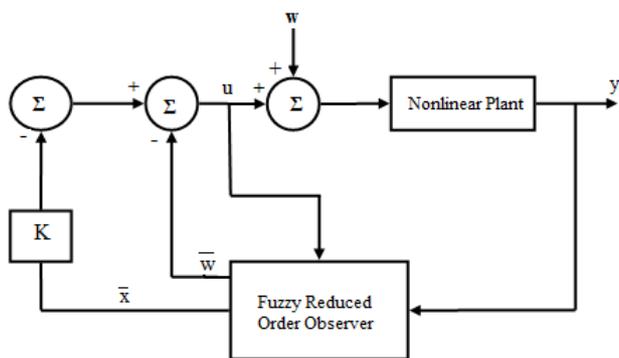


Fig.4. Block diagram for controlled magnetic bearing system

As similar as controller gains, we collect the estimator gains and estimated disturbances to whole system at every rotational speed.

TABLE I  
DATA for NUMERICAL SIMULATION

Symbol	Quantity	Value
$m$	Mass of rotor	14.46 kg
$I_z$	Mass moment of inertia in axial direction	0.0136 kg. $m^2$
$I_x$	Mass moment of inertia in radial direction	0.333 kg. $m^2$
$a$	Distance from center to bearing $a$	0.13 m
$b$	Distance from center to bearing $b$	0.13 m
$i_o$	Steady state current	0.38 Ampere
$S_o$	Steady state air gap	0.55 mm
$R$	Resistance of coil	10.7 ohm
$N$	Number of turns per coil	400 turns
$A$	Area of magnetic coil	1531 $mm^2$

The data collected are used as training data for ANFIS technique to get membership functions and IF-THEN rules. Fig. 5 shows the membership function of estimator gains.

Whence Neuro-Fuzzy reduced order observer is joined to nonlinear model. To check the performance of exact nonlinear system simulation, a MATLAB program using numerical Rung-Kutta fourth order method is obtained.

It is shown that the motion in z-direction is necessary for describe the motion of rotor for nonlinear model, so we design a controller similar to one of radial direction and joint it with controller in gain matrix (13).

Note that there is no disturbance in z-direction, and then we don't need to design an estimator.

Results show in Fig. (6-8) show that the Neuro-Fuzzy reduced order observer based controller gives excellent performance to the nonlinear model.

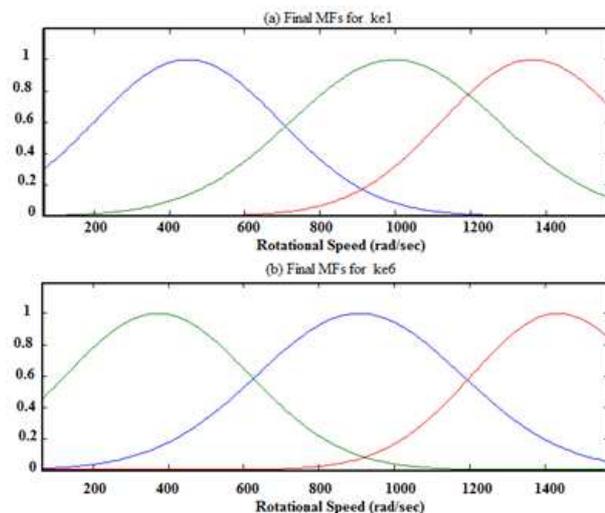


Fig.5. MFs for gain estimator

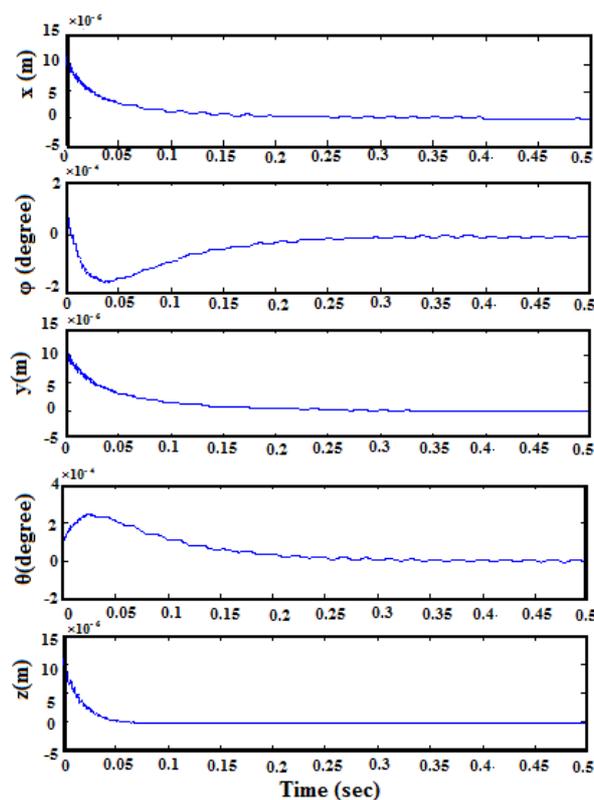


Fig.6. Response due to initial condition for  $\omega=2\pi60$  rad/sec

VII. CONCLUSION

In this paper a Neuro-Fuzzy reduced order observer is employed to eliminate sinusoidal disturbances in magnetic bearing system at all operating speeds.

A controller is designed to regulate the system and Neuro-Fuzzy observer to estimate the disturbance and negative feed it back to cancel the actual one.

A conventional reduced order observer is designed for twenty-five values of rotational speed ranging from  $\omega=2\pi 20$  to  $2\pi 250$  rad/sec. Then obtain estimator gain and estimated disturbance for each value to be the training data.

ANFIS technique used training data to obtain membership functions and IF-THEN rules for Neuro-Fuzzy observer.

The Neuro Fuzzy observer is combined with both controller and nonlinear plant of magnetic bearing. The results obtained show that we can reject the disturbance at any rotational speed in between each of the twenty-five considered intervals.

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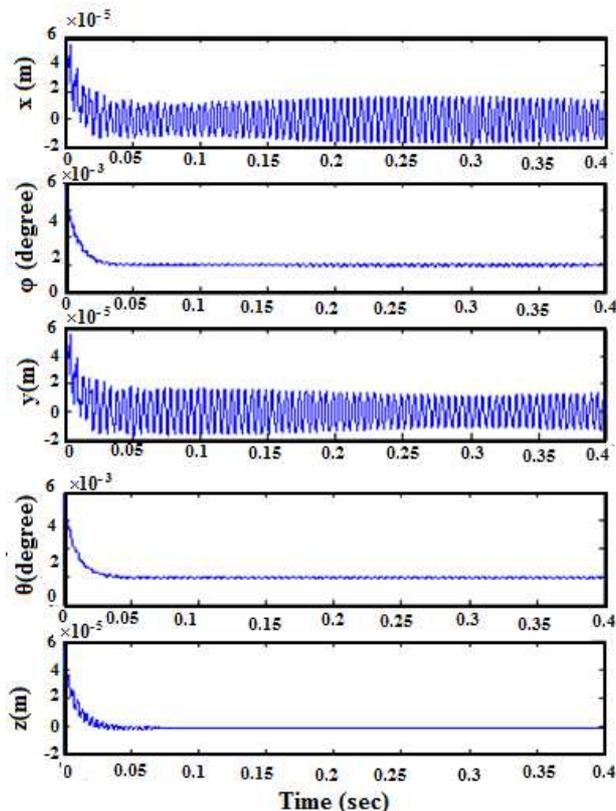


Fig.7. Response due to initial condition for  $\omega=2\pi 200$  rad/sec

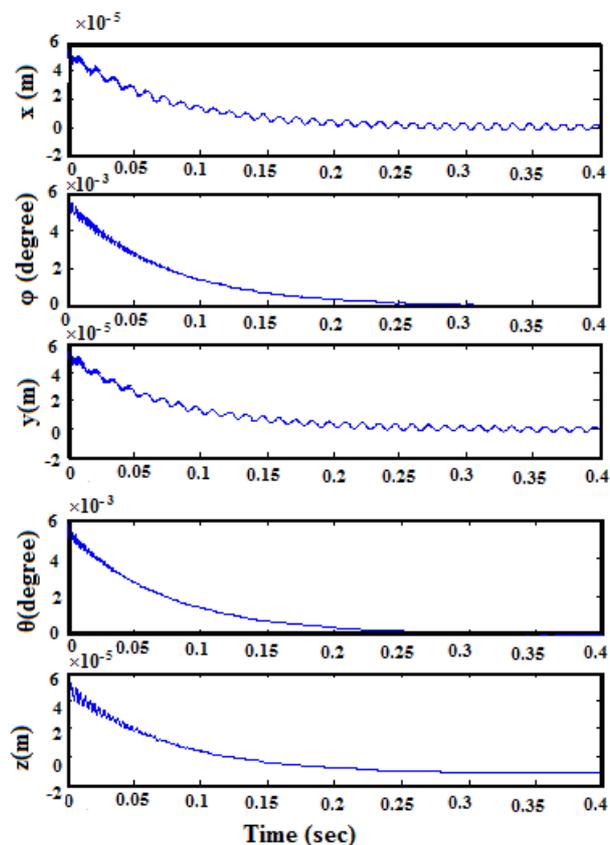


Fig.8. Response due to initial condition for  $\omega=2\pi 120$  rad/sec

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