Profit Maximizing Probabilistic Inventory Model under the Effect of Permissible Delay

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Abstract - Nothing is sure in this world except death, then how can surely; a businessman can determine demand for his items. In the classical EOQ models it has been considered that demand is deterministic but in actual practice it is not possible to have a fixed demand therefore it's necessary for us to consider stochastic demand. In this paper, maximum demand is dependent on average yearly demand and prescribed demand function, thus initial inventory level is taken to be maximum demand derived with the help of demand function and average demand. The concavity of the profit equation has been established and which shows that given model is helpful for finalizing the ordering policy for any supplier/retailer , also different pattern of the given demand function are also considered. In this paper, we have tried to increase the profit of organization by reducing the inventory cost. As shortages are not allowed therefore initial inventory level is equal to the maximum demand level formulated with the given demand function. In this study first time of effect of permissible delay in payments is considered with probabilistic demand. The major draw back of earlier models considering selling price and cost price equal has been removed in this model.

Keywords- C*oncavity, Permissible Delay Probabilistic Demand, Profit Maximization*

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1 INTRODUCTION

In a competitive market environment it is very difficult to forecast the demand exactly , for example you are making any type of auto parts if you don't have dependent demand for your item it will be very difficult for you to judge your demand from your past experience, sometimes demand is very low, then company may go for a low stock and has to face shortages if it rises immediately then it can create panic, most of companies do not like to have shortages, as it effects their goodwill, as well as sometimes they have to loose their customer.

Although many probabilistic models have been proposed using various demand patterns, most of the literature dealing with probabilistic inventory models assumes that the demand rate as the probability distribution of the demand rate rather than the exact value of the demand rate itself is known. Some of important ones are Datta and Pal (1987) who have considered the both deterministic and probabilistic version of power demand pattern with variable rate of deterioration, Hadley and Whitin (1963) have considered probabilistic demand in their book Some Inventory Models, Nita Shah (1993) has given a probabilistic time scheduling model for an exponentially decaying inventory when delays in payments are permissible. Naddor (1966) proposed probabilistic demand pattern in his book "Inventory Systems". Taha, H. (1997) considered inventory models with probabilistic demand pattern in his book "Operations Resesarch". Warren H. Hausman and L. Joseph Thomas (1972) proposed an inventory control model with probabilistic demand and periodic withdrawals.

The concept of trade credit is not new but in classical EOQ models this concept has been neglected. Goyal (1985) succinctly explain this concept in his model, this concept has given new

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dimension to EOQ models and made them more realistic, but it has a major drawback, he considered cost price as well as selling price of the items equal which is an absurd, how can any type of business have cost price equal to the selling price. Although most of the inventory models still considers that selling price equal to the cost price, but many new features has been added to this concept of permissible delay in payments, Other important researchers are Shah et al(1988), studied the Goyal model for shortages. Mandal and Phaujdar (1989) studied the same situation by including interest earned from the sales revenue on the stock remaining beyond the settlement period

In this model we considered the demand pattern proposed by Naddor (1966) in his book "Inventory Systems" with various realistic factors. The realistic factors considered are selling price is always greater than cost price, permissible delay in payments and even the optimality of profit equation has been checked. We have proved by optimality conditions that the profit maximization equations derived in this model helps to maximize profit. This model can be further extended for perishable items and even other factors like inflation and time value of money can be considered, this model can also be extended with fixed trade credit in which buyer will get the benefit of permissible delay only if he buys certain fixed number of items.

2 ASSUMPTIONS

To develop the mathematical model, the following assumptions are being made:

1) Probabilistic demand pattern is considered
2) Shortages are not allowed , hence initial level inventory is equal to the maximum level demand $x_{\max}(T)$ during the time period , so that there will be no shortages, hence $I_o = x_{\max}(T)$
3) The maximum and minimum value of probabilistic demand is known with certainty.
4) Time horizon is infinite
5) In this we have considered the effect of permissible and buyer pays only on the completion of cycle period.

The following notations are applied in the model below

- I_o Initial inventory level
- p Selling price per unit
- i Inventory carrying cost per unit per year
- c Cost price per unit
- I_e The interest earned per Rupee per year
- I_p The interest paid per Rupee per year
- A The ordering cost per order
- Z The total annual profit
- f(x) The probability density function of demand for cycle period
- r Average demand rate
- T Cycle period

3 MATHEMATICAL MODEL

The average level of inventory in any cycle period, T during which there is a demand x is $I_o - x$, where $\bar{x}(T)$ be its mean and let the average rate of demand $r = \frac{\bar{x}(T)}{t}$ is known and constant. Hence the expected average amount in inventory is

$$I = \int_{\min}^{x_{\max}} \left(I_0 - \frac{x}{2} \right) f(x)\,dx = I_0 - \frac{\bar{x}(T)}{2} = I_0 - \frac{rT}{2} \tag{1}$$

The total annual profit consists of the following

(a) Sales revenue = $p\bar{x}(T)$ (2)

(b) Cost of placing orders = $\frac{A}{T}$ (3)

(c) Cost of purchasing = $\frac{cI_0}{T}$ (4)

(d) Cost of carrying inventory

$$= i\left(I_0 - \frac{rT}{2} \right) \tag{5}$$

For permissible delay , there are two distinct cases in this inventory of inventory system:

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1) Payment at or before the total depletion of inventory , $T > M$
and
2) After depletion payment, $T \le M$

Case I- $T > M$

This situation indicates that the permissible payment time expires on or before the inventory depleted completely to zero. As a result, the total cost is comprised of the sum of ordering cost, carrying cost, and the interest payable minus the interest earned. Here the buyer sells rM units in total by the end of the permissible delay M, and has crM to pay the supplier. The items in stock are charged at interest rate I_p by the supplier starting at time M. Therefore, the interest payable per cycle for the inventory not sold after due date M is given by

$$cIp\int_M^T I \, dt/T == c\frac{I_p}{T}\left[I_0 - \frac{rT}{2}\right](T-M) \qquad (6)$$

But, during the period of permissible delay, the buyer sells the product and the revenue from the sales can be used to earn interest , therefore the interest earned during the positive inventory is given by

$$pI_e\int_0^M \bar{x}(T)t \, dt/T = pI_e\frac{r}{2}M^2 \qquad (7)$$

Therefore , the total annual profit Z is given by

$$Z = p\bar{x}(T) - \frac{A}{T} - \frac{cI_0}{T} - i\left(I_0 - \frac{rT}{2}\right) - p\frac{I_p}{T}\left[I_0 - \frac{rT}{2}\right](T-M) - pI_e\frac{r}{2}M^2 \qquad (8)$$

Case II In this case customer sells as customer sells all the items before expiration of permissible delay; hence no interest is paid only interest is earned on the given inventory. hence, the interest earned per year is

$$pI_e\left[\int_0^T \bar{x}tdt + \bar{x}T(M-T)\right] \qquad (9)$$

$$= pI_e\bar{x}\left(M - T/2\right)$$

Hence , total profit is given by

$$Z = p\bar{x}(T) - \frac{A}{T} - \frac{cI_0}{T} - i\left(I_0 - \frac{rT}{2}\right) - pI_e\bar{x}\left(M - T/2\right) \qquad (10)$$

Now to decide the cycle period we have to find the value of I_o or $x_{\max}(T)$

suppose

$$x_{\max} = \bar{x}(T)G(T) = rtG(T) \qquad (11)$$

Where G (T) is some function relating the maximum demand during any period T to the average demand during that period. Obviously,

$$G(T) \ge 1 \qquad (12)$$

We can consider different values of G(T) ,

Case I- G(T) =K, when ratio of maximum demand and average demand to be a constant, substituting these values in equation (8) and (10), we get the required profit maximization equation,

Case II G(T)=1+a/T, generally it is found that the ratio of maximum demand to average demand during T would generally depend on T. i.e., the larger the value of T , the smaller is the ratio, substituting these values in equation (8) and (10) , we get the required profit maximization equations

Differentiating equation (8) with respect to T, we obtain

$$\frac{\partial Z}{\partial T} = \frac{A}{T^2} + \frac{cI_0}{T^2} + \frac{ir}{2} - \frac{pI_pI_0M}{T^2} \qquad (13)$$

Again differentiating equation (13) with respect to T , we get

$$\frac{\partial^2 Z}{\partial T^2} = -\frac{2A}{T^3} - \frac{cI_0}{T^3} + \frac{2pI_pI_oM}{T^3} < 0 \qquad (14)$$

Which is obviously less than zero, as interest payable cannot be equal to the total cost price of the inventory, hence above equation is strictly concave function. Hence we can easily obtain the optimal value of T, which maximizes the total annual profit.

Differentiating equation (10) with respect to T , we have

$$\frac{\partial Z}{\partial T} = \frac{A}{T^2} + \frac{cI_0}{T^2} + \frac{r}{2} - \frac{pI_e}{2}\vec{x} \qquad (15)$$

Again differentiating with respect to T, we get

$$\frac{\partial^2 Z}{\partial T^2} = \frac{-2A}{T^3} - \frac{cI_0}{T^3} < 0 \qquad (16)$$

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Which is obviously less than zero as cost price and ordering cost both cannot be negative, hence the equation (10) is strictly concave, thus equation (10) provides the profit maximization equation.

4. CONCLUSIONS

By optimality test it has been verified that above model will help to increase the total profit of any organization. This model also helps inventory manager to deal with the items having probabilistic demand; concept of permissible delay in payments has been successfully applied first time to any probabilistic model. But still it's not complete and many of the latest of inventory concepts have to be incorporated in this model. But this can surely act as a catalyst for the development of more realistic and pertinent models.

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