Effect of Radiation and Magnetic Field on Mixed Convection at a Vertical Plate in a Porous Medium with Variable Fluid Properties and Varying Wall Temperature

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Abstract— In this paper the effects of radiation, magnetic field, variable viscosity and variable thermal conductivity on similarity solutions of mixed convection at a vertical flat plate embedded in a porous medium are studied numerically. Temperature of the plate as well as the free stream velocity are assumed to vary as power functions of x, where x is the distance measured vertically along the plate. The flow and heat transfer quantities are found to be functions of C, λ, γ_μ, γ_k, *RP* and *Rd* where *C* is magnetic field parameter, λ is power of index of the plate temperature, γ_μ is viscosity variation coefficient, γ_k is thermal conductivity variation coefficient, *RP*, mixed convection parameter is ratio of the *Rayleigh* number to the $Pe'clet$ number and *Rd* is radiation parameter. The cases of assisting and opposing flow are considered and in the opposing flow case dual solutions are found for certain values of the parameters. Ranges of values of the parameters for which there exist no solution or dual solutions or a unique solution are also obtained.

The influences of magnetic field, thermal radiation, variable viscosity, variable thermal conductivity and varying wall temperature on the velocity and temperature fields are studied and discussed with the help of graphs.

Key Words: Magnetic field, Mixed Convection, Radiation, Variable Fluid properties, varying wall temperature.

I.INTRODUCTION

Over the last four decades much insightful work has been done on mixed convection boundary-layer flows in porous media. The analogous problems have important applications in fields such as geothermal energy extraction, oil reservoir modeling, and the dispersion of chemical contaminants in different industrial processes in the environment. References [8] and [11] stand evident to the fact that convective flows in porous media are of vital importance to such processes.

As pointed out in reference [1], in mixed convection flows, similarity exists only if the free stream velocity and temperature of the plate vary as the same power functions of distance along the plate. Also, in mixed convection flows, there arise four cases (ref. [11]), of which two correspond to assisting flow and two to opposing flow. They are (i) hot plate-assisting flow (ii) hot plate- opposing flow (iii) cold plate-assisting flow and (iv) cold plate- opposing flow. Of these four cases, only two, namely (i), (iv) are taken into consideration in this study.

Reference [6] discussed the effect of variable viscosity on convective heat transfer in three different cases of natural convection, mixed convection and forced convection, taking fluid viscosity to vary inversely with temperature. However, the authors have confined their attention to the assisting flow case only. Reference [1] studied mixed convection boundary layer flow on a vertical surface in a porous medium, when both the temperature of the plate and the free stream velocity vary as the same power function of distance along the plate. Similarity solutions are found as functions of two parameters λ and ε where λ represents the power of index of the plate temperature and ε represents the mixed convection parameter which is the ratio of the *Rayleigh* number to the $Pe'clet$ number. Both assisting flow and opposing flow were discussed. Ranges of values of ε for different values of λ were presented for which either a unique solution, dual solutions or no solution exist. The effects of λ and ε on the flow and heat transfer characteristics were discussed.

The effect of radiation on free convection flow of fluid with variable viscosity from a porous plate is discussed in reference [2]. The fluid considered in that paper is an optically dense viscous incompressible fluid of linearly varying temperature dependent viscosity. Reference [9] discussed coupled heat and mass transfer in Darcy-Forchheimer Mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation.

Reference [4] discussed mixed convection boundary layer flow over a vertical surface for the Darcy model when viscosity varies inversely as a linear function of temperature. Results of both assisting flow and opposing flow were discussed as functions of the mixed convection parameter ε and variable viscosity parameter θ_e. In the opposing flow case, the existence of dual solutions and boundary layer separation were noticed. Mixed convection boundary layer flow on a vertical surface in a saturated porous medium is studied in reference [7]. In that paper the flow of a uniform stream past an impermeable vertical surface embedded in a saturated porous medium and which is supplying heat to the porous medium at a constant rate is considered.

The effect of magnetic field and varying plate temperature on free convection past a vertical plate in porous medium has been discussed in ref. [10]. Magneto hydrodynamic mixed convection flow in an annular region filled with a fluid saturated porous medium has been analyzed in ref. [3]. A transverse magnetic field which acts radially is created by a stationary electric current that flows through a cylindrical

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shaped electrical cable present in the annular region. The effect of non uniform magnetic field on the flow and heat transfer of the Darcy model is discussed. Magneto hydrodynamic free convection in a horizontal cavity filled with a fluid saturated porous medium with internal heat generation has been studied in ref. [5]. Assuming that the magnetic field is inclined at angle γ with the horizontal plane, the flow and heat transfer are discussed as functions of inclination angle γ, Hartmann number *Ha, Rayleigh* number *Ra* and aspect ratio *a.*

For fluids all the properties change with temperature and, in particular, changes in viscosity and thermal conductivity are quite significant for many fluids. Hence, in this study we assume both viscosity and thermal conductivity to vary with temperature and make a numerical study of the effect of variable viscosity, variable thermal conductivity and radiation on mixed convection flow at a vertical plate in a porous medium. The plate temperature and the free stream velocity are assumed to vary as power functions of distance (x) along the plate and a magnetic field is assumed to act normal to the plate. The fluid considered here is assumed to be gray. In the opposing flow case, dual solutions are obtained for certain values of the mixed convection parameter *RP.* Both assisting flow and opposing flow are discussed. Ranges of values of *RP* are obtained for which either a unique solution, dual solutions or no solution exist for the problem. Significant differences are noticed between the flow and heat transfer quantities related to the two solutions of the dual solution case.

II. FORMULATION

Let a flat plate be embedded vertically in a porous medium saturated with a viscous incompressible electrically conducting, gray, emitting, absorbing and non scattering fluid. The porous medium is assumed to be homogeneous and is in thermal equilibrium with the surrounding fluid. Let a magnetic field of uniform strength be applied in a direction normal to the plate. Let *X*-axis be taken vertically along the plate and *Y*-axis perpendicular to the plate. The temperature of the plate (T_0) is assumed to vary as a power function of distance along the plate, as $T_0 = T_\infty + A x^\lambda$ where T_∞ is temperature of the ambient fluid, *A* is a constant and λ is a real number. Fluid viscosity (μ) and effective thermal conductivity (k_m) are assumed to vary with temperature as $\mu = \mu_f s_\mu(t)$, $k_m = k_f s_k(t)$, where μ_f, k_f are viscosity and thermal conductivity evaluated at the film temperature. We take $s_\mu(t), s_k(t)$ as

$$s_\mu(t) = 1 + \left(\frac{d\mu}{dt}\right)_f (T - T_f), \quad s_k(t) = 1 + \left(\frac{dk}{dt}\right)_f (T - T_f)$$

where $T_f \left(= \frac{T_0 + T_\infty}{2} \right)$ is the film temperature.

A viscosity variation coefficient γ_μ and a thermal conductivity variation coefficient γ_k are introduced as

$$\gamma_\mu = \frac{1}{\mu_f}\left(\frac{d\mu}{dT}\right)_f (T_0 - T_\infty), \; \gamma_k = \frac{1}{k_f}\left(\frac{dk}{dt}\right)_f (T_0 - T_\infty).$$

Density of the fluid ρ is assumed to be a function of temperature only in the body force term and other fluid properties are assumed to be constant. The ambient fluid is assumed to flow vertically upwards with a velocity U_∞ parallel to the vertical plate.

The equations governing the mixed convection problem for the Darcy model are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial (p - p_\infty)}{\partial x} \pm (\rho - \rho_\infty) g + \frac{\mu}{K}(u - U_\infty) + \sigma B_0^2 (u - U_\infty) = 0 \tag{2}$$

$$\frac{\partial p}{\partial y} + \frac{\mu}{K} v = 0 \tag{3}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k_m \frac{\partial T}{\partial y} \right) - \frac{\partial q_r}{\partial y} \tag{4}$$

where u, v are fluid velocity components, T is fluid temperature, K is Permeability, k_m is effective thermal conductivity of the fluid saturated porous medium, B_0 is the magnetic flux, σ is the electric conductivity and q_r is the radiative heat flux. It is also assumed that there is radiation only from the fluid. Further it is assumed that thermal radiation is present in the form of a unidirectional flux, transverse to the vertical plate. The Rosseland approximation is used in the energy equation to describe the thermal radiative heat transfer.

It may be noted that by the use of the Rosseland approximation, the applicability of the present analysis is limited to optically thick fluids only.

The appropriate boundary conditions are

$$\left.\begin{array}{l} at\ y = 0, \quad T_0 = T_\infty + A x^\lambda, \ v = 0, \\ as\ y \to \infty, T \to T_\infty, \ u \to U_\infty \end{array}\right\} \tag{5}$$

Taking the free stream velocity as $U_\infty = b x^\lambda$ where b is a constant, introducing $Pe'clet$ number (Pe_x), non dimensional functions f, θ; a similarity variable η and radiative flux q_r through the relations

$$Pe_x = \frac{U_\infty x}{\alpha_m}, \; f(\eta) = \frac{\psi}{\alpha_m (Pe_x)^{\frac{1}{2}}}, \; \theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty}$$
$$\eta = \frac{y}{x}(Pe_x)^{\frac{1}{2}}, \; q_r = -\frac{16 \sigma_s T_\infty^3}{3 k_c}\left(\frac{\partial T}{\partial y}\right) \tag{6}$$

(where σ_s is the Stefan-Boltzmann's constant and k_e is the mean absorption coefficient) & eliminating fluid pressure from (2) and (3), the governing equations are obtained as

$$\left[1 + C\gamma_\mu\left(\theta - \frac{1}{2}\right)\right] f'' + C\,\gamma_\mu\, f'\,\theta' = C\,(RP)\,\theta' \tag{7}$$

$$\left[1 + \gamma_k\left(\theta - \frac{1}{2}\right) + \frac{4(Rd)}{3}\right]\theta'' - 2\lambda f'\theta + (\lambda + 1) f\theta' + \gamma_k \theta'^2 = 0 \tag{8}$$

Here $C = \frac{K^*}{K^* + M^2}$ is the magnetic field parameter,

$$M^2 = \frac{B_0^2 L^2 \sigma}{\mu_f} \;\&\; K^* = \frac{L^2}{K}, \;\; Rd = \frac{4\sigma_s T_\infty^3}{k\,k_c}$$

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is the radiation parameter, and $RP = \frac{Ra_x}{Pe_x}$ where $Ra_x = \frac{\rho_\infty K g \beta (T_0 - T_\infty) x}{\mu_f \alpha}$ is the Rayleigh number.

The boundary conditions (5) become

$$\left.\begin{array}{l} at \quad \eta = 0, \quad \theta = 1, \quad f = 0, \\ as\ \eta \to \infty, \theta \to 0, f' \to 1 \end{array}\right\} \qquad (9)$$

Equation (7) can be integrated once using the condition on f' at $\eta = \infty$ to get

$$f' = \frac{\left(1 - \frac{C\gamma_\mu}{2}\right) + C\,(RP)\,\theta}{\left[1 + C\gamma_\mu\left(\theta - \frac{1}{2}\right)\right]} \qquad (10)$$

Evaluating this at $\eta = 0$ we get the slip velocity $f'(0)$.

III.a PARAMETERS OF THE PROBLEM AND THEIR EFFECT ON THE FLOW AND HEAT TRANSFER

The flow and heat transfer depend on the parameters C, $\lambda, \gamma_\mu, \gamma_k$, RP and Rd where C is magnetic field parameter, λ is power of index of the plate temperature, γ_μ is viscosity variation coefficient, γ_k is thermal conductivity variation coefficient, RP, mixed convection parameter is ratio of the *Rayleigh* number to the *Pe'clet* number and Rd is radiation parameter. Positive and negative values of A correspond to $T_0 > T_\infty$, $T_0 < T_\infty$ and in turn to assisting flow and opposing flow respectively. The parameters γ_μ, γ_k take positive as well as negative values, the limiting values being '-2' and +2. Irrespective of the values of T_0 and T_∞, zero values of γ_μ and γ_k correspond to constant viscosity and constant thermal conductivity. In this paper, solutions are found for the values -1, 0, and 1 of γ_μ and γ_k.

The mixed convection parameter RP takes positive values for assisting flow and negative values for opposing flow. When RP is zero, the results correspond to the forced convection case. Enhanced flow can correspond to an increase in the positive value of RP, as an increase in its value can be due to an increase in the temperature difference ($T_0 - T_\infty$). Calculations are done for a wide range of positive and negative values of RP.

To determine certain important values for λ, the total heat convected in the flow, $Q(x)$ at any down stream location x is considered.

$$Q(x) = \int_0^\infty \rho\, C_p \beta (T - T_\infty)\, u\, dy \quad .$$

This can be seen to be proportional to $x^{\frac{3\lambda+1}{2}}$. For uniform heat flux surface, $Q(x)$ should vary linearly with x and so $\lambda = 1/3$. For an adiabatic surface, $Q(x)$ should be independent of x and so $\lambda = -1/3$. Zero value of λ corresponds to the isothermal case. In this study solutions are found for the values $-0.3,\ 0,\ 0.3$ and 0.5 of λ. When A is positive, an increase in the value of λ can correspond to an increase in the temperature of the plate, and, in a broader sense, it can result in enhanced flow.

When there is no magnetic field, the parameter C takes the value unity and for increasing intensity of the magnetic field, the parameter takes values smaller than unity. In the present study, solutions are found for the values 0.5 and 1 of C. Reduced flow can be expected for smaller values of C or for increased intensity of the magnetic field as the Lorentz force (due to the magnetic field) obstructs the flow.

When transfer of heat energy through radiation is neglected, the parameter Rd takes zero value and for increasing intensity of thermal radiation, the parameter takes larger values. Solutions are found for the values 0, 0.5, 10 of the parameter Rd. Thermal radiation causes thickening of the thermal boundary layer and hence increasing values of the parameter Rd can increase thermal boundary layer thickness.

Effects of simultaneous variation of the values of the parameters on the flow and heat transfer are presented in the discussion.

III.b. NUMERICAL SOLUTION

The equations for f and θ, i.e., equations 8 & 10 are integrated numerically subject to appropriate boundary conditions by Runge-Kutta-Gill method together with a shooting technique. The accuracy of the method is tested by comparing appropriate results of the present analysis with available results. Our results for C =1, Rd = 0, γ_μ =0, $\gamma_k = 0$ and appropriate values of λ (i.e., no magnetic field, no radiation, constant viscosity, constant thermal conductivity) are in very good agreement with those in ref. [1]. Also our results for C =1, Rd =0, γ_μ =0, $\gamma_k = 0$ and λ =0 (i.e., no magnetic field, no radiation, constant viscosity, constant thermal conductivity and isothermal plate) agree very well with those of ref. [4] and ref. [7].

IV. DISCUSSION OF THE RESULTS

In the following, more attention is paid to the discussion of solutions of the opposing flow case. Because of our choice, when $T_0 < T_\infty$ the opposing flow case arises in which RP takes negative values. When $T_0 < T_\infty$, for fluids like Methyl Chloride, $\gamma_\mu < 0, \gamma_k > 0$ while for fluids like Dichlorofluro Methane, $\gamma_\mu > 0, \gamma_k < 0$. For a given value of the parameter RP, in the dual solution case, the solution corresponding to a relatively larger value of $f''(0)$ is referred to as the upper solution and the one corresponding to a smaller value of $f''(0)$ as the lower solution. We know that the local drag coefficient is directly proportional to the skin friction $f''(0)$ and local Nusselt number is directly proportional to the heat transfer coefficient or the wall heat transfer rate '$-\theta'(0)$'.

ISOTHERMAL CASE ($\lambda = 0$)

Qualitatively distinct behaviours of the flow and heat transfer Characteristics of the isothermal case are presented in figures 1 -11.

Variations in Skin friction with negative values of the mixed convection parameter RP are shown in figures 1,2.

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Corresponding variations in heat transfer coefficient with negative values of *RP* are shown in figures 3,4. From figures 1 and 2, when viscosity and thermal conductivity are taken to be variable the local drag coefficient can be observed to take larger values in the presence of magnetic field (C =0.5) than in its absence (C =1). The coefficient of drag is also observed to be larger in the absence of radiation than in its presence. The range of values of *RP* over which solutions exist can be seen to be considerably larger in the presence of magnetic field than in its absence. The range can also be seen to be larger in the absence of radiation than in its presence. The drag coefficient also assumes larger numerical values in the presence of magnetic field, and also in the absence of radiation. The range of values of *RP* as well as the coefficient of drag can be seen to be larger when $\gamma_\mu < 0$ and $\gamma_k > 0$ (for fluids like methyl chloride) than when $\gamma_\mu > 0$ and $\gamma_k < 0$ (for fluids like dichloro fluro methane). Local drag coefficient can be seen to assume larger values as *RP* takes increasing negative values up to a certain stage (i.e., as *RP* changes from 0 to -0.1, -0.1 to -0.2 and so on), and beyond that again take diminishing values till *RP* assumes a critical value. No solution exists beyond this critical value of *RP*. As *RP* increases from this value, up to a certain negative value, the drag coefficient further diminishes. Thus, a single solution exists for certain values of *RP*, dual solutions exist for another set of values of *RP* and no solution exists beyond the critical value of *RP*.

From figures 3 and 4 it may be observed that, unlike skin friction, the heat transfer coefficient takes diminishing values with increasing negative values of *RP* up to a critical value of *RP*, and diminishes further with increasing values of *RP* up to a certain negative value. This is because, in the opposing flow case the buoyancy forces work against the fluid flow, and hence the retardation in the heat transfer process. Further, in the presence of magnetic field as well as in the absence of radiation, heat transfer coefficient decreases for larger numerical values of *RP*. The heat transfer coefficient can also be observed to take larger numerical values for $\gamma_\mu = 1,\ \gamma_k = -1$ than for $\gamma_\mu = -1,\ \gamma_k = 1$. Thus, the behavior of heat transfer coefficient with γ_μ, γ_k is also different from that of skin friction with these parameters.

Variations in shear stress for the dual solutions are shown in figures 5, 7. Changes in the shear stress of the lower solution with the parameter *RP* are significant than the changes in the upper solution. Boundary layer thickness of the lower solution is considerably larger than that of the upper solution.

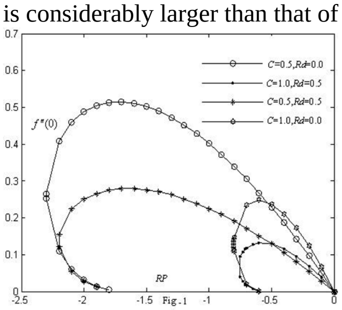

Fig - 1 Variations in $f''(0)$ for $\gamma_\mu = 1,\ \gamma_k = -1\ \&\ \lambda = 0$

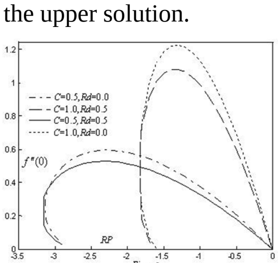

Fig - 2 Variations in $f''(0)$ for $\gamma_\mu = -1, \gamma_k = 1\ \&\ \lambda = 0$

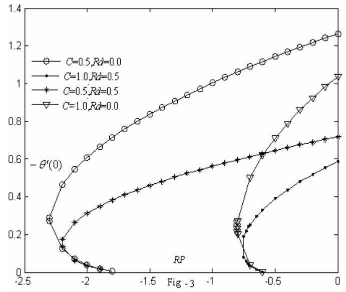

Fig - 3 Variations in '$-\theta'(0)$' for $\gamma_\mu = 1,\ \gamma_k = -1\ \&\ \lambda = 0$

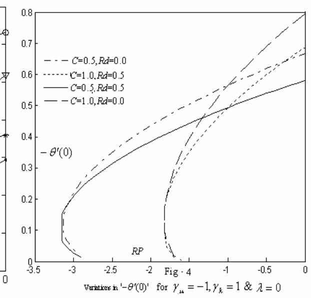

Fig - 4 Variations in '$-\theta'(0)$' for $\gamma_\mu = -1, \gamma_k = 1\ \&\ \lambda = 0$

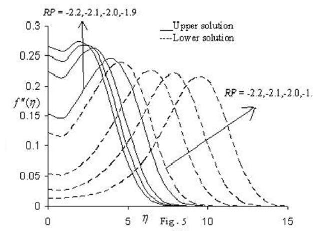

Fig - 5 Variations $f''(\eta)$ for $\gamma_\mu = 1,\ \gamma_k = -1, C = 0.5,\ Rd = 0.5\ \&\ \lambda = 0$

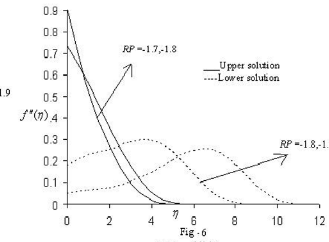

Fig - 6 Variations $f''(\eta)$ for $\gamma_\mu = -1, \gamma_k = 1, C = 1\ \&\ Rd = 0.5\ \&\ \lambda = 0$

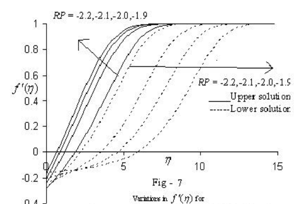

Fig - 7 Variations in $f'(\eta)$ for $\gamma_\mu = 1,\ \gamma_k = -1, C = 0.5,\ Rd = 0.5\ \&\ \lambda = 0$

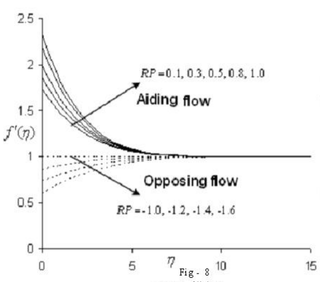

Fig - 8 Variations in $f'(\eta)$ for $\gamma_\mu = -1,\ \gamma_k = 1, C = 0.5,\ Rd = 10\ \&\ \lambda = 0$

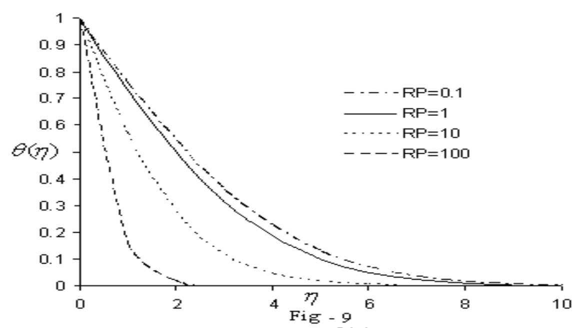

Fig - 9 Variations in $\theta(\eta)$ for $\gamma_\mu = -1, \gamma_k = 1, C = 0.5, Rd = 10\ \&\ \lambda = 0$

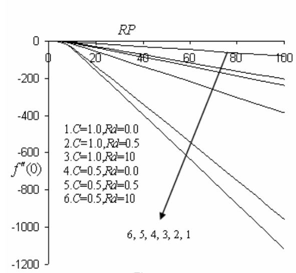

Fig - 10 Variations in $f''(0)$ for $\gamma_\mu = -1, \gamma_k = 1\ \&\ \lambda = 0$

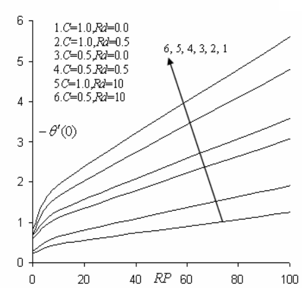

Fig - 11 Variations in '$-\theta'(0)$' for $\gamma_\mu = -1, \gamma_k = 1\ \&\ \lambda = 0$

Near the plate, the upper solution takes relatively larger numerical values than the lower solution, this behaviour being quite significant in the absence of magnetic field. The values of the shear stress are also larger in the absence of radiation than in its presence. Boundary layer thickness can also be

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observed to be more in the presence of radiation than in its absence.

Variations in fluid velocity are shown in figures 7,8 and variations in fluid temperature are shown in fig. 9. It may be noted that, variations in shear stress with different parameters are reflected in the fluid velocity. Variations in lower solution with the parameters are significant than the variations in the upper solution. Boundary layer thickness is more for lower solution than for upper solution. Boundary layer thickness is more in the presence of radiation than in its absence. From fig.9, it may be noted that in the variable fluid property case, in the presence of magnetic field, temperature assumes relatively larger values in the boundary layer. This may be due to the resistance offered by the Lorentz force to the flow, and as a result increase in the temperature. Also it may be observed that increase in thermal radiation parameter (*Rd*) produces significant increase in the thickness of the thermal boundary layer of the fluid and so the temperature profile $\theta(\eta)$ increases and tends to zero at the edge of the boundary layer. This is due to the fact that the presence of thermal radiation causes thickening of the thermal boundary layer. Thermal boundary layer thickness can be seen to increase with diminishing values of *RP*.

Variations in skin friction with positive values of the mixed convection parameter *RP* are shown in fig.10. One important point worth noting is that, for positive values of *RP* skin friction is negative for all values of the other parameters, indicating boundary layer separation for positive values of *RP*. Absolute values of skin friction can be observed to be larger in the absence of magnetic field than in its presence and also in the absence of radiation than in its presence.

Variations in the heat transfer coefficient, '$-\theta'(0)$' with positive values of *RP* are shown in fig.11. We may note that heat transfer coefficient assumes positive values for all values of the parameters. Heat transfer coefficient assumes larger values in the absence of magnetic field (*C*=1) and in the absence of radiation (*Rd*=0). It diminishes with diminishing values of *C* as well as with increasing values of *Rd* (i.e., with increasing intensity of the magnetic field and with increasing effect of radiation).

VARYING WALL TEMPERATURE CASE $(\lambda \neq 0)$

Some qualitatively interesting behaviours of the flow and heat transfer characteristics of this case are presented in figures 12-20.

Variations in skin friction with negative values of the mixed convection parameter *RP* are presented in fig.12 for a negative value of λ ($\lambda = -0.3$) and in fig.13 for a positive values of λ ($\lambda = 0.5$). For negative values of λ, we may notice that there arise two cases either a single solution exists or no solution exists depending on the value of *RP*. As *RP* takes increasing negative values, $f''(0)$ is seen to increase, indicating that the drag coefficient increases. In some cases, $f''(0)$ diminishes as *RP* takes further increasing negative values. Changes in the range of values of *RP* over which solution exist can also be observed with changes in magnetic field, radiation, the working fluid and the temperatures of the fluid and the ambient. In the presence of the magnetic field as well as the presence of radiation, skin friction can be seen to take diminishing values. For fluids for which $\gamma_\mu > 0, \gamma_k < 0$, in the presence of magnetic field and in the absence of radiation, $f''(0)$ assumes larger numerical values than for fluids for which $\gamma_\mu < 0, \gamma_k > 0$. However, in the presence of both magnetic field and radiation, $f''(0)$ assumes smaller numerical values for fluids for which $\gamma_\mu > 0, \gamma_k < 0$ than for fluids for which $\gamma_\mu < 0, \gamma_k > 0$. From fig.13, we may notice that for $\lambda = 0.5$ and in the presence of radiation, $f''(0)$ assumes both positive and negative values. Solutions exist over a wider range of values of the mixed convection parameter for $\lambda = 0.5$ than for $\lambda = -0.3$. Unlike for $\lambda = -0.3$, in this case, dual solutions also exist. In the presence of magnetic field the range of existence of solutions will be larger than the one in the absence of magnetic field. In the presence of magnetic field, the range also increases with increase in the radiation parameter. The drag coefficient assumes maximum value in the absence of magnetic field and in the presence of radiation for fluids for which $\gamma_\mu < 0, \gamma_k > 0$. Either in the presence or absence of magnetic field, for fluids for which, $\gamma_\mu < 0, \gamma_k > 0$ skin friction is more in the variable fluid property case than in the constant fluid property case(compare curves 7, 6 or curves 3, 2).But for fluids for which $\gamma_\mu > 0, \gamma_k < 0$ opposite is the behaviour of skin friction (compare curves 5, 6 or curves 1, 2).

Variations in wall heat transfer rate '$-\theta'(0)$' with negative values of *RP* are presented in fig.14 for a negative value of λ ($\lambda = -0.3$) and in fig.15 for a positive value of λ ($\lambda = 0.5$). For negative values of λ, as *RP* takes increasing negative values, the wall heat transfer rate diminishes up to a certain stage. For positive values of λ, similar is the behaviour except that the values corresponding to the lower solution diminish as *RP* takes increasing values. From fig.14, for negative values of λ, we may note that, in general, the effect of the magnetic field is to increase the wall heat transfer rate and the effect of radiation is to diminish it. However this nature varies from one working fluid to another.

Variations in shear stress pertaining to the dual solutions in the constant fluid property case are shown in fig. 16 for smaller negative values of *RP* and in fig.17 for larger negative values of *RP*. On a comparison of figures 16, 17 we may note that there is significant difference in the behaviour of shear stress for smaller and relatively larger numerical values of *RP*. For smaller numerical values of *RP*, the upper solution starts with a positive value at the plate and the lower solution starts with a negative value at the plate. For relatively larger values of *RP*, boundary layer thickness of upper solution is seen to increase while that of the lower solution decreases as compared to boundary layer thickness for smaller values of *RP* (compare figures 16 and 17). Thermal boundary layer thickness of the lower solution is more than that of the upper solution and it increases with diminishing numerical values of *RP*.

Velocity profiles pertaining to the dual solutions of the constant fluid property case are presented in fig.18 for smaller absolute values of *RP* and in fig.19 for relatively larger absolute values of *RP*. The behaviours of the shear stress presented in the figures 16 and 17 are reflected in the velocity profiles of figures 18 and 19. Temperature profiles in one of the variable fluid property cases are presented in fig.20. One important

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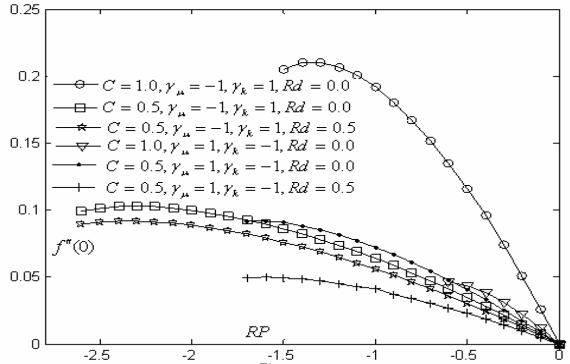

Fig -12
Variations in $f''(0)$ for $\lambda = -0.3$

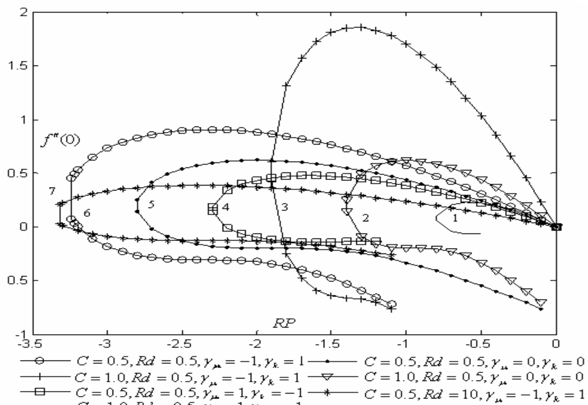

Fig - 13
Variations in $f''(0)$ for $\lambda = 0.5$

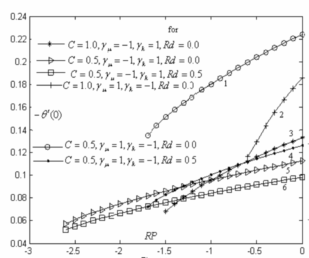

Fig - 14
Variations in $-\theta'(0)$ for $\lambda = -0.3$

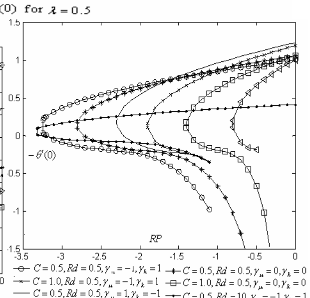

Fig - 15
Variations in $-\theta'(0)$ for $\lambda = 0.5$

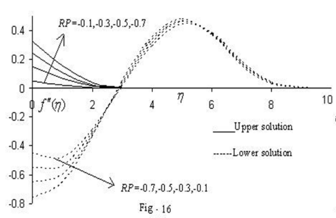

Fig - 16
Variations in $f''(\eta)$, Dual solutions for
$\gamma_\mu = 0, \gamma_k = 0, C = 0.5, \lambda = 0.5 \& Rd = 0.5$

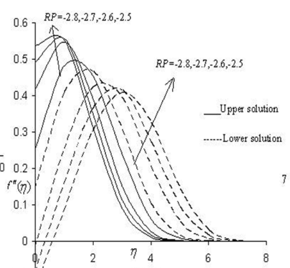

Fig - 17
Variations in $f''(\eta)$, Dual solutions for
$\gamma_\mu = 0, \gamma_k = 0, C = 0.5, \lambda = 0.5 \& Rd = 0.5$

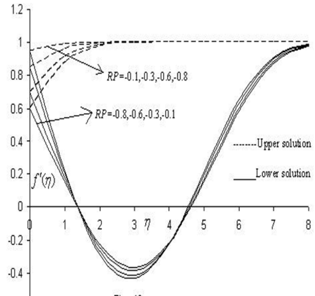

Fig - 18
Variations in $f'(\eta)$, dual solutions for
$\lambda = 0.5, \gamma_\mu = 0, \gamma_k = 0, C = 0.5 \& Rd = 0.5$

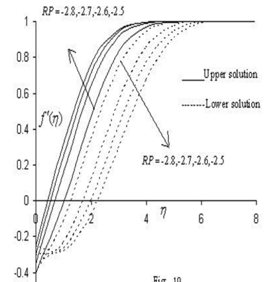

Fig - 19
Variations in $f'(\eta)$, dual solutions for
$\lambda = 0.5, \gamma_\mu = 0, \gamma_k = 0, C = 0.5 \& Rd = 0.5$

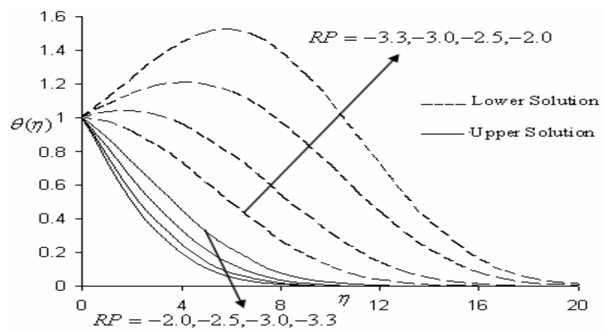

Fig - 20
Variations in $\theta(\eta)$ for
$\gamma_\mu = -1, \gamma_k = 1, \lambda = 0.5, C = 0.5 \& Rd = 0.5$

observation is that the temperature of the fluid can exceed that of the plate temperature in the lower solution case. Variations in the lower solutions are significant with changing values of *RP*. Thermal boundary layer thickness of the lower solution is more than that of the upper solution and it increases with diminishing numerical values of *RP*.

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