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Multiple Periodic Preventive Maintenance for Used Equipment under Lease

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Abstract—Upgrading action and preventive maintenance are alternatives to reduce the used equipment failures rate which have more disruptions than the new. The optimal maintenance policy will effective to the most failure decrease. This research is to determine the optimal PM actions that minimize the total maintenance cost for leased equipment when we consider the penalties for equipment failures and repairs time over limit. The formulated models are combined from the advantages of sequential PM and periodic PM policies. Assumption of failures form is applied by Nonhomogeneous Poisson process (NHPP) and failures distribution is appraised Weibull. The optimal solution is achieved from the optimal upgrade level and two timeframes approach which is obtained the optimal of time intervals to carry out PM and the optimal level of PM actions.

***Index Terms*— Periodic preventive maintenance, Reliability, Lease, Used equipment.**

I. Introduction

Equipments under leasing is the one strategy of business management and the most important of leasing business is a reliability. The used equipment should be focused because of failure rate is higher than the new one and less reliability. The upgrading action and suitable PM are necessary to decrease failure rate, moreover they can reduce the total maintenance cost. Multi periodic preventive maintenance policy for the new equipment is combining the advantages of sequential and periodic PM policy under lease condition which is more flexible than implementation [17]. This policy is apply from sequential PM policy for leased equipment which a unit is preventively maintained at unequal time intervals. Usually, the time intervals become shorter and shorter as time passes, considering that most units need more frequent maintenance with increased ages [5] and periodic PM policy for leased equipment, unlike the sequential PM policy, which a unit is preventively maintained at fixed time intervals of jT, $j=1,2,...,k$ over the lease period. This implies that the implementation of periodic PM policy is more convenience than the sequential PM policy because the time intervals between successive PM actions are constant [20]. The lease period is divided in to two stages which is the first and the second lease period. Each period have equal time intervals for PM actions, but the frequency of each PM actions will be different in the first and the second lease period e.g., the first lease period, the PM actions are carried out at periodic times of jT, $j=1,2,...,k_1$ and in the second lease period, the PM actions are carried out at periodic times $j'T/2$, $j'=1,2,...,k_2$. Frequency of PM actions in the second period is higher than the first because of increasing failure rate correspondingly. The separated lease timing will be considered after the equipments have been used A years and upgrading will be carried out before lease. Any intervenient failures over the lease period, we will assume to be rectified through minimal repairs.

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II. Model Formulation

We use the following notation:

$F(t)$ = failure distribution function

$f(t)$ = failure density function associated with $F(t)$

$r(t)$ = failure rate [hazard] function associated with $F(t)$

$\lambda_0(t)$ = failure intensity function with no PM [= $r(t)$]

$\lambda(t)$ = failure intensity function with PM actions

$\Lambda_0(t)$ = cumulative failure intensity function with no PM $\left[=\int_0^t \lambda_0(x)dx\right]$

$\Lambda(t)$ = cumulative failure intensity function with PM actions $\left[=\int_0^t \lambda(x)dx\right]$

$N(t)$ = number of failures over $[0,t]$

Y = time to repair

$G(y)$ = repair-time distribution function

$g(y)$ = repair-time density function [= $dG(y)/dy$]

L = lease period

L_1 = 1st lease period

L_2 = 2nd lease period

T = period of time instant to carry out PM

k_1 = number of PM actions over the 1st lease period

k_2 = number of PM actions over the 2nd lease period

t_j = time instant for j^{th} PM action over the 1st lease period

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$t_{j'}$ = time instant for j'^{th} PM action over the 2nd lease period

δ_j = reduction in intensity function due to j^{th} PM action over the 1st lease period

$\delta_{j'}$ = reduction in intensity function due to j'^{th} PM action over the 2nd lease period

A = age of equipment before lease

x = upgrade level before lease (percentage of A)

C_u = upgrade cost

$C_p(\delta)$ = cost of PM action resulting in a reduction δ_j in intensity function over the 1st lease period

$C_p(\delta_{j'})$ = cost of PM action resulting in a reduction $\delta_{j'}$ in intensity function over the 2nd lease period

TC_p = total cost of PM actions

C_f = average cost of CM action to rectify failure

TC_f = total cost of CM actions

τ = repair time limit [parameter of lease contract]

C_t = penalty cost per unit time if repair not completed within τ [Penalty-1]

C_n = penalty cost per failure if $N(t) > 0$ [Penalty-2]

ϕ_1 = total cost due to Penalty-1

ϕ_2 = total cost due to Penalty-2

A. *Lease Contact*

For lease period L, after A years, that compose of $L_1 + L_2$, when L_2 have a frequency of PM action more than L_1, that benefit to reliability increasing. The equipment is leased for a period L with two types of penalty resulting from failures.

Penalty-1: The lessor incurs a penalty if the time to repair failure exceeds τ. Let Y denote the time to repair, then there is no penalty if $Y \le \tau$ and a penalty $(Y-\tau)C_t$ if $Y > \tau$ [21].

Penalty-2: The lessor incurs a penalty cost C_n for each failure that occurs over the lease period [21].

B. *Modeling Failures and PM Actions*

Equipment failures are rectified through minimal repairs and the repair times are small relative to the mean time between failures. We assume that equipment had been used before new lease. Furthermore, at the primal lease no PM needed at early phase of equipment because of low failure. As a results, it is not worth to PM, however the lessor has provided PM when ever equipment failed. The first failures timing is a distribution function $F(t)$ and failures rate $r(t)$ is an increasing function of time.

For the first period, the lessor carries out periodic PM with a period of T and $T/2$ over the second lease period to compensate increasing of failure rate. The time instants of PM actions are given by $t_j = jT$, $j = 1,2,\dots,k_1$ for the first lease period and $t_{j'} = L_1 + j'T/2$, $j' = 1,2,\dots,k_2$ for the second lease period. Each PM action results in a reduction in the intensity function. The reductions resulting from the j^{th} PM in the first period is given by δ_j and $\delta_{j'}$ for j^{th} PM in the second period [17].

The first period, the failures over the lease period for used equipment, no PM action before lease that occurs according to the intensity function given by

$$\lambda(t) = \lambda_0(A + t_j - x) - \sum_{i=1}^{k_1} \delta_i \quad \text{for } t_j \le t \le t_{j+1} \tag{1}$$

Where $x = 0$ this refer to no upgrade case and $t_0 = A - x$. $\delta_{i=0} = \delta_0 = \lambda(A-x)$ and δ_j is constrained as follows

$$0 \le \delta_j \le \lambda_0(A + t_j - x) - \sum_{i=0}^{j-1} \delta_i \quad \text{for } 1 \le j \le k_1 \tag{2}$$

The second period,

$$\lambda(t) = \lambda_0(A + t_{j'} - x) - \sum_{i=1}^{k_1} \delta_i - \sum_{i'=1}^{k_2} \delta_{i'} \tag{3}$$

for $t_{j'} \le t \le t_{j'+1}$ where $t_{0'} = A + L_1 - x$ and $\delta_{j'} = 0$

$\delta_{j'}$ is constrained as follows

$$0 \le \delta_{j'} \le \lambda_0(A + t_{j'} - x) - \sum_{i=0}^{j-1} \delta_i - \sum_{i'=0}^{j'-1} \delta_{i'} \tag{4}$$

for $;1 \le j' \le k_2$

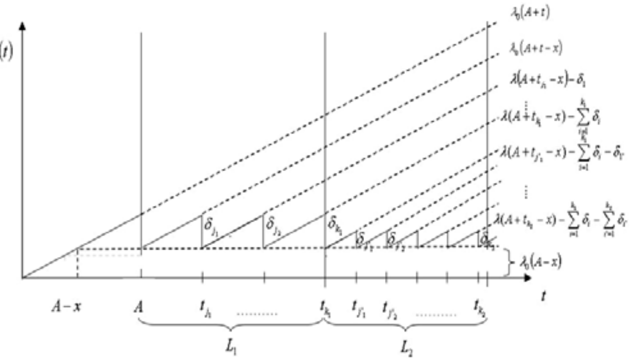

Fig.1 Plot of failure intensity function of both period of L_1 and L_2

C. *Cost of Lessor*

(i) Cost of CM Actions

Let $N(t)$ is a number of failures over $[0,t]$ and C_f is a mean cost of repair. The cost of repairing failures is given by

$$TC_f = C_f N(L) \tag{5}$$

(ii) Cost of PM Actions

The cost of PM action depends on the reduction result in the intensity PM cost function. We model this through a fixed cost and the variable cost and is given by

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$$C_p(\delta_j) = a + b\delta_j \ , \quad j = 1,2,...,k_1 \tag{6}$$

for the first lease period and

$$C_p(\delta_{j'}) = a + b\delta_{j'} \ , \quad j' = 1,2,...,k_2 \tag{7}$$

For the second period

with $a > 0$ and $b \ge 0$. Hence, the total cost of PM actions is given by

$$TC_p(\delta_j, \delta_{j'}) = \sum_{j=1}^{k_1}(a + b\delta_j) + \sum_{j'=1}^{k_2}(a + b\delta_{j'}) \tag{8}$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

(iii) Upgrade Costs

The upgrade cost $C_u(x)$ is an increasing function of x and given by

$$C_u(x) = \omega x / (1 - e^{-\varphi(A-x)}) \tag{9}$$

Where $\omega > 0$ and $\varphi > 0$ when ω is a scale parameter and φ is a shape parameter of $C_u(x)$

(ix) Penalty Costs

Penalty-1 results from failure to complete a repair within a specified time τ . Let Y (a random variable from a distribution $G(y)$) denote the time to rectify the i^{th} failure, $1 \le i \le N(L)$. Then, the total Penalty-1 cost incurred is given by

$$\phi_1(N(L), Y_i, \tau) = C_t \left\{ \sum_{i=1}^{N(L)} \max[0, Y_i - \tau \right\} \tag{10}$$

Penalty-2 results whenever there is any failure over the lease period, the lessor incurs the penalty-2 costs and this is given by

$$\phi_2(N(L)) = C_n \{\max[0, N(L)\} \tag{11}$$

III. Model Analysis

A. Expected number of failurwes

The equipment failures with no PM actions occur according to a Non homogeneous Poisson process (NHPP) with intensity function $\lambda_0(t) = r(t)$ where $r(t)$ is failure rate [hazard] function associated with the distribution $F(t)$ and we define [3]

$$\Lambda(L) = \int_0^L \lambda(t)dt \tag{12}$$

The expected number of failures over the lease period, with no PM action, is given by

$$E[N(L)] = \Lambda_0(A + L - x) - \Lambda_0(A - x) \tag{13}$$

The expected number of failures over the lease period, with PM action, is given by

$$E[N(L)] = \Lambda_0(A + L - x) - \Lambda_0(A - x) - \sum_{j=1}^{k_1}\delta_j(L - t_j) - \sum_{j'=1}^{k_2}\delta_{j'}(L - t_{j'}) \tag{14}$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

B. Expected Cost

(i) Expected CM cost

From (5) and (14), the total expected cost of CM actions is given by

$$E(TC_f) = C_f E[N(L)] \tag{15}$$

the used equipment consideration in (15) result in

$$E(TC_f) = C_f[\Lambda_0(A + L - x) - \Lambda_0(A - x)] - \sum_{j=1}^{k_1}\delta_j(A + L - x - t_j) - \sum_{j'=1}^{k_2}\delta_{j'}(A + L - x - t_{j'}) \tag{16}$$

(ii) Expected Penalty-1 cost

From (10) and (14) the total expected Penalty-1 cost is given by

$$E[\phi_1(N(L), Y_i, \tau)] = C_t E[N(L)] \left\{ \int_\tau^\infty (y - \tau) g(y) dy \right\} \tag{17}$$

Using integrating by parts on (16) results in

$$E[\phi_1(N(L), Y_i, \tau)] = C_t E[N(L)] \left\{ \int_\tau^\infty (1 - G(y)) dy \right\} \tag{18}$$

(iii) Expected Penalty-2 cost

From (11) and (14) the total expected Penalty-2 cost is given by

$$E[\phi_2(N(L))] = C_n E[N(L)] \tag{19}$$

Combining all of costs, expected CM cost, total PM cost, upgrade cost, expected Penalty-1 cost and expected Penalty-2 cost, yields the total expected cost to the lessor given by

$$\begin{aligned} J(T, \underline{\delta}) = {} & C_f N(L) + \sum_{j=1}^{k_1}(a + b\delta_j) + \sum_{j'=1}^{k_2}(a + b\delta_{j'}) \\ & + \omega x / (1 - e^{-\varphi(A-x)}) + C_t N(L) \left\{ \int_\tau^\infty (1 - G(y)) dy \right\} \\ & + C_n N(L) \end{aligned} \tag{20}$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

We can define

$$C' = C_f + C_t \int_\tau^\infty (1 - G(y)) dy + C_n$$

Then, (20) can be rewritten as

$$\begin{aligned} J(T, \underline{\delta}) = {} & C' \begin{bmatrix} \Lambda_0(A + L - x) - \Lambda_0(A - x) \\ - \sum_{j=1}^{k_1}\delta_j(A + L - x - t_j) - \sum_{j'=1}^{k_2}\delta_{j'}(A + L - x - t_{j'}) \end{bmatrix} \\ & + \omega x / (1 - e^{-\varphi(A-x)}) + \sum_{j=1}^{k_1}(a + b\delta_j) + \sum_{j'=1}^{k_2}(a + b\delta_{j'}) \end{aligned} \tag{21}$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

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C. Optimization

The optimal parameters of the PM policy are parameter values that yield a minimum for $J(T,\underline{\delta})$. We obtain the optimal values using a two process. In State one we apply differential calculus method to obtain $\underline{\delta}^*(T)$. In State 2 we obtain T^* by using one-dimensional minimization method with the iterative procedure.

(i) State 1

Fix k_1, k_2 and obtain $t_j, t_{j'}$ from $t_j = jT$, $j = 1,2,...,k_1$ and $t_{j'} = L_1 + j'T/2$, $j' = 1,2,...,k_2$.

As a result, $J(T,\underline{\delta})$ is only a function of $\underline{\delta}$, and from (2) and (4), the $\underline{\delta}$ is constrained. Determine the extreme point of $J(T,\underline{\delta})$ by determining the first partial derivatives of $J(T,\underline{\delta})$ corresponding to $\underline{\delta}$ as below

$$\partial J(T,\underline{\delta})/\partial\delta_1 = ... = \partial J(T,\underline{\delta})/\partial\delta_j = \partial J(T,\underline{\delta})/\partial\delta_{1'} ... = \partial(J,\underline{\delta})/\partial\delta_{jj'} = 0 \quad (22)$$

then we have the constraints of t_j and $t_{j'}$ as follow

$$0 < t_1 < t_2 < ... < t_{k_1} < L - b/C' \text{ and} \quad (23)$$

$$0 < t_{1'} < t_{2'} < ... < t_{k_2} < L - b/C' \quad (24)$$

where $0 < t_j < t_{j'} < L - b/C'$ (25)

As a result, $J(T,\underline{\delta})$ is a linear function of $\underline{\delta}$ and constrained as indicated in (2),(4),(23)-(25). Therefore, the optimal values are the end points of the constraint intervals. This yields

$$\delta_j^* = \begin{cases} \lambda_0(A + t_j - x) - \sum_{i=0}^{j-1}\delta_i & \text{if } t_j < L - b/C' \text{ for } 1 \le j \le k_1 \\ 0 & \quad\; t_j \ge L - b/C' \quad\;\; j > k_1 \end{cases} \quad (26)$$

where $t_0 = A - x$ and $\delta_0 = \lambda(A - x)$

and

$$\delta_{j'}^* = \begin{cases} \lambda_0(A + t_{j'} - x) - \sum_{i'=0}^{k_1}\delta_{i'} - \sum_{i'=0}^{j'-1}\delta_{i'} & \text{if } t_{j'} < L - b/C' \\ 0 & \quad\; t_{j'} \ge L - b/C' \end{cases} \text{ for } \begin{matrix} 1 \le j' \le k_2 \\ j' > k_2 \end{matrix} \quad (27)$$

where $t_{0'} = A + L_1 - x$ and $\delta_{0'} = 0$

This implies that the optimal PM action at $t_j = jT$, $j = 1,2,...,k_1$ or $t_{j'} = L_1 + j'T/2$, $j' = 1,2,...,k_2$ is to reduce failure intensity by the maximum amount when $t_j < L - b/C'$ or $t_{j'} < L - b/C'$ and not to carry out any PM when $t_j \ge L - b/C'$ or $t_{j'} \ge L - b/C'$.

(ii) State 2

We obtain T^*, the optimal T, by minimizing $J(T,\underline{\delta}^*)$ using $\underline{\delta}^*(T)$ obtained from Stage 1. One can obtain T^* by using one-dimensional minimization method with the iterative procedure according to the algorithm given by.

Step 1:

Given $k_1 = 1$.

Step 2:

Evaluate side constraints of T from $L_1/k_1 + 1 < T \le L_1/k_1$.

Step 3:

Find T over the interval $L_1/k_1 + 1 < T \le L_1/k_1$ with one dimensional method and step size $\to 0$ and then compute k_2 from $k_2 = 2L_2/T = 2(L - L_1)/T$

Step 4:

Compute t_j and $t_{j'}$ from $t_j = jT$, $j = 1,2,...,k_1$ and $t_{j'} = L_1 + j'T/2$, $j' = 1,2,...,k_2$.

Step 5:

Evaluate δ_j^* and $\delta_{j'}^*$ by placing t_j and $t_{j'}$ in (26) and (27) respectively.

Step 6:

Evaluate $J(T,\underline{\delta}^*)$ from (21)

Step 7:

Set new $k_1 \leftarrow k_1 + 1$, and repeat Step 1 onwards until $k_1 = k_{1\max}$ where $k_{1\max} = C'\Lambda_0(L_1)/a$, then go to Step 8.

Step 8:

Search for T^* which yields the smallest values for $J(T,\underline{\delta}^*)$. Using this, the optimal PM actions are given by $\underline{\delta}^* = \underline{\delta}^*(T^*)$ and the minimum expected cost to the lessor given by $J(T^*, \underline{\delta}^*(T^*))$.

IV. NUMERICAL EXAMPLE

We assume that the failure distribution for the equipment is given by the two-parameter Weibull distribution [20]. As a result,

$$\lambda_0 = (\beta/\alpha)(t/\alpha)^{\beta-1} \quad (28)$$

with scale parameter $\alpha > 0$ and shape parameter $\beta > 1$ (implying an increasing failure rate). According to [3] we can assume $\alpha = 1$ because of the scale parameter α has no influence to the model if they are two or three parameter Weibul model.

The repair time, Y, is a random variable with distribution function $G(y)$. We assume that $G(y)$ is also a two-parameter Weibull distribution function given by

$$G(y) = 1 - \exp\left[-(y/\varphi)^m\right]; 0 \le y \le \infty \quad (29)$$

with the scale parameter $\varphi < 0$ and the shape parameter $m < 0$ (implying a decreasing repair rate). We consider the following nominal values for the model parameters

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$L = 5$ (years), $L_1 = 2$ (years), $C_f = 100\$$,
$C_t = 300\$$, $C_n = 200\$$, $C_u(\omega) = 10$,
$C_u(\varphi) = 0.01$, $a = 100\$$, $b = 50\$$,
$\tau = 2$ (days), $\beta = 3, \alpha = 1$ $m = 0.5$, $\varphi = 0.5$

A. The Optimal Parameters for the PM Policy

As a result, $T^* = 0.2755$ years that means time interval of PM action is 3.31 months for the first period and 1.66 months for the second period and we have , $k_1^* = 7$, $k_2^* = 20$, $k = 27$ that means under leasing time, PM actions time is 27 and the optimal of upgrade level is 87% that give the minimum total maintenance cost \$19,882.20 . The optimal parameters of multi periodic PM policy with upgrade cost are given in Table.1

Table.1 The optimal parameters for the PM policy with upgrade cost $(A = 5)$

A=5					
x%	k_1	k_2	k	T	J (\$)
20	11	31	42	0.1802	119,984.78
40	10	29	39	0.1923	73,846.82
60	9	26	35	0.2135	40,692.76
80	8	23	31	0.2417	21,740.93
87	7	20	27	0.2755	19,882.20
90	7	20	27	0.2755	20,696.27

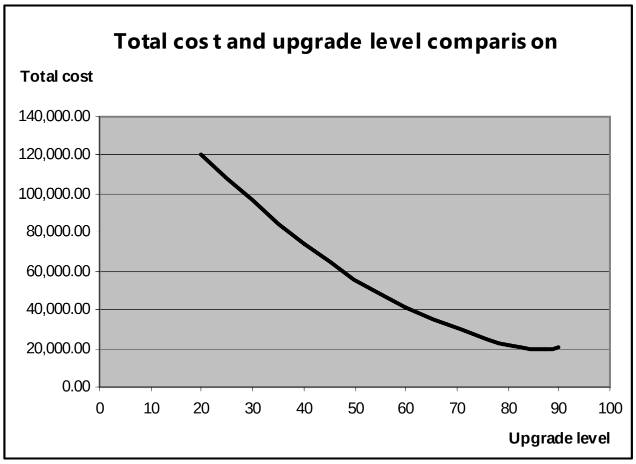

Fig.1 Total cost (J) and upgrade level $(x\%)$ comparison

As Fig.1 we can notice that the highest upgrade level have not given the minimum maintenance cost but the upgrade level will directly variable with age of the equipment as the result in Table.2

Table.2 The optimal parameters for the PM policy with upgrade cost (compare A with the several age)

Upgrade						
A	x%	k_1	k_2	k	T	J
1	63	7	20	27	0.2755	12,314.79
2	76	7	20	27	0.2755	14,670.29
3	81	7	20	27	0.2755	16,611.01
4	85	7	20	27	0.2755	18,322.05
5	87	7	20	27	0.2755	19,882.20
7	89	8	23	31	0.2417	22,683.59
9	91	8	23	31	0.2417	25,214.98

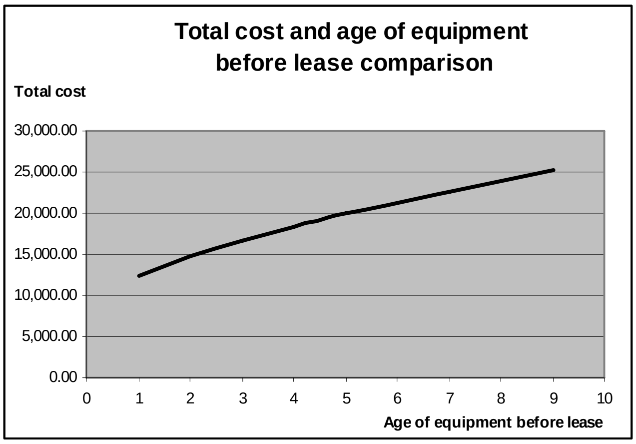

Fig.2 Total cost (J) and age of equipment before lease (A) comparison

We can compare the further age of used equipment between using multi PM policy with upgrade and multi PM policy without upgrade. The results are shown in the Table.3, upgrading action have effected to decrease total maintenance cost. Moreover the older used equipment, we have been relieved too much maintenance cost with multi PM policy with upgrade as shown in the Table.3

Table.3 The optimal parameters for the PM policy with upgrade cost and without upgrade cost

Upgrade				No upgrade		
A	x%	k	J	k	J	ΔJ (\$)
1	63	27	12,314.79	31	17,720.90	5,406.11
2	76	27	14,670.29	35	39,177.71	24,507.42
3	81	27	16,611.01	39	73,170.10	56,559.09
4	85	27	18,322.05	42	119,729.75	101,407.70
5	87	27	19,882.20	50	178,880.69	158,998.49
7	89	31	22,683.59	54	334,986.12	312,302.53
9	91	31	25,214.98	58	541,584.13	516,369.15

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V. Conclusion

The proposed of this paper is to research the multiple periodic preventive maintenance policy which for used equipments under lease. Upgrade is the action which decrease failure rate and total maintenance cost. Available multi periodic PM is more advantage than periodic PM cause of correspond with degeneration of the used equipment while sequential PM is more delicate to adopt than multi periodic PM which divide time to two intervals. Results of this paper can support leaser to carries out the used equipment for lease with the minimum cost and invoke profits as long as the total minimum cost still higher than invest the new one.

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