An Analysis of Inverse Heat Conduction Problem on Irregular Shape Fins

J. C. Li, David T. W. Lin, C. C. Wang, C. Y. Yang

***Abstract*—This study is proposed an inverse method to estimate the unknown heat flux boundary conditions on the irregular shape fins. An inverse algorithm based on the sequential method and the concept of future time combined with the finite element method is used to determine the 2-D heat conduction problems. The estimated results are considered with the different future time, the sensor location, the sensor's number and the measured errors. The estimated results are agreed with the exact solution, even in the irregular shape fin. The results of the study show that the proposed method is an accurate, stable, and efficient method for solving the complex heat conduction problems.**

***Keywords:* irregular shape fins, sequential method, future time, finite element method**

I. Introduction

The direct heat conduction problems are concerned with the value of heat flux on unknown boundary in interior region when the initial and boundary conditions, thermal physical properties and heat generation are acquired easily. On the contrary, the inverse heat conduction problem involves the determination of the surface boundary conditions, energy generation and thermo-properties from the knowledge of the temperature measurements are obtained within the physical model. In our study, it is not difficult to use sensor to measure boundary condition and utilize a simple transformation of matrix with no iteration process for solving inverse heat problem of fins. Even for irregular shape fins, the proposed method in this paper also can combine with some numerical manner to estimate the unknown boundary condition accurately.

Several methods have been used to solve the inverse heat conduction problem (IHCP), these include the analytic methods or the numerical approaches, such as polynomial method [1], graphical method [2], finite difference method and finite element method [3,4], Laplace transform method [5], direct sensitivity coefficient method [6], conjugate gradient method [7], and regularization method [8].

During the past several decades, more researches have been investigated the thermal phenomenon in fin problems [9-12]. Al-Sanea and Mujahid [9] used a finite volume method to analyze the time dependent boundary of fins. Yang [10] derived the analytic solution for convective fins under a periodic heat transfer condition. In Chung and Eslinger's [11] paper, it is used a finite element method to solve a convective and radiative fin. Yang [12] proposed a numerical method to estimate the periodic boundary conditions on the non-Fourier fin problem.

Besides, some studies have been considered the effect of variable shape fins in heat transfer problem. Bejan and Almogbel [13] reports the geometric optimization of T-shape fin in their paper. Lorenzini and Moretti [14] analyze the optimal problem in thermal system characterized by Y-shaped fins through a numerical method which is based on CFD software. Other paper [15] is also showed the same software to investigate the heat transfer problem of fins. Razelos and Satyaprakash [16] presented an analysis of trapezoidal profile convective pin fins.

To overcome the instability of the inverse problem, different methods have been developed. Several studies investigated the inverse heat transfer phenomena related to this topic [17-20]. However, the above approaches have some limits in the application, for example, an iterative process, an essential pre-select function, used in the nonlinear domain, or only specialized in the simple shape. From the above, this paper proposes a sequential inverse algorithm [21] combined with finite element method to solve the heat problems in order to release the above limitation.

This paper is intended as an investigation of the fin with irregular shape. Then, the proposed method is a numerical method that can estimate unknown heat flux boundary condition sequentially without the sensitivity analysis. In the process of the derivation, a finite element method combined with the concept of future time [22] is used to derive the result of simulation. This matter will be proved numerically in the section of discussions. Then, the boundary condition is determined along with the temporal coordinate step by step. It can prove that the heat flux boundary can estimate feasible in this paper. Furthermore, the results of this study show that the estimation of heat flux boundary in irregular shape fins by inverse method is effective.

II. System Description

2.1 Linear least-squares error method

Linear least-squares error method [23] utilizes the inverse matrix method to deal with inverse problem. First, the

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inverse problem is discreted to construct a linear inverse model. Then, linear least-squares error method is used to solve linear the inverse model directly. According to the rearrangement of the matrix of the direct problem, the form of the unknown condition can be shown identically, and the iteration and initial guess are unnecessary. It only employs a simple matrix algorithm to solve the optimal solution at one time. The unique of the solution can be proofed easily.

2.2 Problem statement

In general, the inverse method is used in the analysis of simple shape. Nevertheless, the proposed method in this paper based on the linear least-squares error method with finite element method will solve the stiffness matrix of irregular shape in the spatial discrete process, and construct a general solving process. Consider a two-dimensional body with irregular shape which is subjected to the following three boundary types: (1) the specified temperature $T = T_b$ on Γ_T, (2) the specified heat flux $q_n = q_0$ on Γ_q, and (3) the specified convection $q_n = h(T - T_f)$ on Γ_c. The interior of the body is denoted as V, and the boundary is denoted as $\Gamma = \Gamma_T \cup \Gamma_q \cup \Gamma_c$. The transient heat conduction problem is listed as below:

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q \,, (x,y) \in V \tag{1.1}$$

$$T(x,y,t) = T_b(x,y,t)\,, (x,y) \in \Gamma_T \tag{1.2}$$

$$q_n = q(x,y,t)\,, (x,y) \in \Gamma_q \tag{1.3}$$

$$q_c = h(T(x,y,t) - T_\infty)\,, (x,y) \in \Gamma_c \tag{1.4}$$

$$T(x,y,0) = T_0(x,y)\,, (x,y) \in \{\Gamma_T \cup \Gamma_q \cup \Gamma_c\} \cup V \tag{1.5}$$

where T represents the temperature field $T(x_i, y_i, t)$, k is the thermal conductivity, ρC is the heat capacity per unit volume and Q is the heat source generation.

The inverse problem is to estimate the unknown boundary conditions when the temperature field is measured at the known boundary. Hsu et al. [24] mentioned that it is difficult to have an algorithm with the ability to estimate the heat flux and the surface temperature through the same technique in the multi-dimensional inverse heat conduction problem. The first step of process uses a matrix to express equation (1.1) and uses a vector to express boundary conditions (1.2) - (1.5) separately. In the second place, it is more stable for estimated solution by add the concept of future time. Finally, we can make a process of solving inverse problem.

The proposed method uses a finite-element method and with linear triangular element to discretize the spatial coordinate and time domain. By the conventional finite-element procedure with n_p grids at $t = t_j$ [25], equations (1.1) - (1.5) can be converted to the following discrete form:

$$[B]\frac{\{T_m\} - \{T_{m-1}\}}{\Delta t} = \{R_{m-1+\lambda}\} - [A]\left(\lambda\{T_m\} + (1-\lambda)\{T_{m-1}\}\right) \tag{2}$$

where $[A]$ is the space stiffness matrix of the problem with n_p dimensions

$[B]$ is the transient matrix of the problem with n_p dimensions

$\{R_m\}$ is the boundary condition vector with n_p components

$\{T_m\}$ is the temperature vector with n_p components

The time index is m, λ is the time step weight. The equation (2) is called Explicit Scheme when $\lambda = 0$. And then, it is called Implicit Scheme and Crank-Nicolson Scheme as λ is equal to 1 and 1/2 respectively. In general, the error of discrete time is increased from $O(\Delta t)$ to $O(\Delta t^2)$ when $\lambda = 1/2$. Therefore, all of calculation in this study is set $\lambda = 1/2$.

When $t = t_m$, the temperature distribution $\{T_m\}$ can be derived from equation (2) as follows:

$$\begin{aligned}\{T_m\} &= \left(\frac{[\mathrm{B}]}{\Delta t} + \lambda[A]\right)^{-1}\left(\frac{[\mathrm{B}]}{\Delta t} - (1-\lambda)[A]\right)\{T_{m-1}\} \\ &\quad + \left(\frac{[\mathrm{B}]}{\Delta t} + \lambda[A]\right)^{-1}\{R_{\overline{m}}\} \\ &= [C]\{T_{m-1}\} + [D]\{R_{\overline{m}}\}\end{aligned} \tag{3.1}$$

where the time index is $\overline{m} = m - 1 + \lambda$

$$[D] = ([B]/\Delta t + \lambda[A])^{-1} \tag{3.2}$$

$$[C] = [D]([B]/\Delta t - (1-\lambda)[A]) \tag{3.3}$$

Similarly, the temperature distribution at $t = t_{m+1}$ and $t = t_{m+2}$ can be represented as follows:

$$\begin{aligned}\{T_{m+1}\} &= [C]\{T_m\} + [D]\{R_{\overline{m}+1}\} \\ &= [C]^2\{T_{m-1}\} + [C][D]\{R_{\overline{m}}\} + [D]\{R_{\overline{m}+1}\}\end{aligned} \tag{4.1}$$

$$\begin{aligned}\{T_{m+2}\} &= [C]\{T_{m+1}\} + [D]\{R_{\overline{m}+2}\} \\ &= [C]^3\{T_{m-1}\} + [C]^2[D]\{R_{\overline{m}}\} + [C][D]\{R_{\overline{m}+1}\} + [D]\{R_{\overline{m}+2}\}\end{aligned} \tag{4.2}$$

Therefore, we have the temperature vector at ($m+n$)-temporal grid

$$\begin{aligned}\{T_{m+n}\} &= [C]^{n+1}\{T_{m-1}\} + \sum_{l=0}^{n}[C]^l[D]\{R_{\overline{m}+n-l}\} \\ &= [C]^{n+1}\{T_{m-1}\} + \\ &\quad \sum_{l=0}^{n}[C]^l[D]\left(\{R^0_{\overline{m}+n-l}\} + \{R^T_{\overline{m}+n-l}\} + \{R^q_{\overline{m}+n-l}\} + \{R^c_{\overline{m}+n-l}\}\right)\end{aligned} \tag{5}$$

where n is an integer and n =0, 1, 2,..., r − 1

$\{R\} = \{R^0\} + \{R^T\} + \{R^q\} + \{R^c\}$

$\{R^0\}$ is the vector of known boundary condition.

$\{R^T\}$ is the vector of unknown temperature boundary.

$\{R^q\}$ is the vector of unknown heat flux boundary.

$\{R^c\}$ is the vector of unknown convection boundary.

After the measured temperature T^i_j (measured at $t = t_j$ and $x = x_i$) is substituted into vector $[\Theta]$, the components of vector $[\Phi]$ can be found through a linear least-squares error method [23]. Therefore, the result is:

$$[\Phi_m] = ([\Omega]^T[\Omega])^{-1}[\Omega]^T[\Theta] \tag{6}$$

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Therefore, the unknown boundary condition $\{R_m^T\}$, $\{R_m^q\}$, and $\{R_m^c\}$ can be solved at successive time steps along with the temporal coordinate. In other words, equation (6) provides a sequential algorithm that can be used to estimate the boundary conditions through increasing the value of m by one for each time step because the estimated conditions depend on the measured temperature, the known boundary condition $\{R_m^0\}$, and the previous state $\{T_{m-1}\}$.

III. Numerical Results and discussion

This study presents a numerical method to analyze the inverse heat conduction problem of fins with irregular shape. In this research, we use the temperature sensors on the wall surface of the heat sinks to predict the unknown heat flux boundary of the fins as shown in fig.1 by the inverse method. The accuracy results of this study are discussed with the different future time, the sensor location, the sensor's number and the measured errors. The inverse problem is defined from equations (1) to (5) to estimate the variation of heat flux at the unknown boundary. The geometry and coordinate system diagram is shown in fig.1. The accuracy of the estimated heat flux distribution on the time-varying of the sinusoid heat flux boundary conditions are investigated and demonstrate the effect of the proposed method.

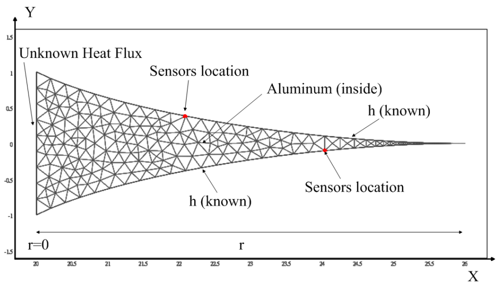


Fig. 1 Geometry and coordinate system.

The simulated temperature is generated from the exact temperature in each problem and it is presumed to have measured errors. In other words, the random errors of measurement are added to the exact temperature. It can be shown in the following equation:

$$\theta_{mea} = \theta_{exact}(1+\sigma\omega) \tag{7}$$

where θ_{exact} and θ_{mea} in equation (7) is the exact temperature and measured temperature respectively. σ is the standard deviation of the measurement error, ω is a random variable of normal distribution with zero mean and unit standard deviation. The value of ω is calculated by the IMSL subroutine DRNNOR [26] and chosen over the range $\pm\sigma\omega$, which represent the 99% confidence bound for the measured temperature.

We consider a function of heat flux distribution which is assumed for unknown boundary to illustrate the advantage of the proposed method and defined the heat transfer coefficient of fins as equation (8).

Known boundary

$$h(r) = 0.01*\exp((r - 20)/6) \tag{8}$$

In addition, the unknown heat flux distributions of fins are assumed to be a function to examine the accuracy of the estimated results. These heat flux distributions of fins are listed as below.

Case: unknown heat flux boundary

$$T(t) = 100 + 10*(t/150)^2 * \sin((t/30)^{2.4} * \pi) \tag{9}$$

The case is assumed the unknown boundary as the time-varying sinusoid to generate different heat flux distribution. Furthermore, the result of the estimated heat flux is discussed with the different future time, the sensor location, the sensor's number and the measured errors. The material of the fin of heat sink is assumed as aluminum ($k = 237\ W/mK$, $\rho = 2700\ kg/m^3$, $c = 900\ J/kg\cdot K$). It is initially at an uniform temperature, $T_0 = 373K$. The temporal domain is from 0 to 100 seconds with 0.05 seconds increment (i.e. time steps, τ , is 2000) for each problem.

To investigate the deviation of the estimated results from the error-free solution, the absolute average errors for the estimated solutions are defined as follows:

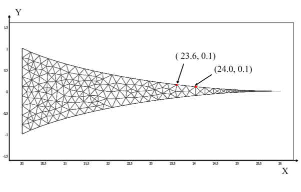
$$\varepsilon = \frac{1}{n_t}\sum_{j=1}^{n_t}\left| f - f_0 \right| \tag{10}$$

where f is the estimated result with measurement errors and f_0 is the estimated result without measurement errors. n_t is the number of the temporal steps. It is clear that a smaller value of ε indicates a better estimation and vice versa.

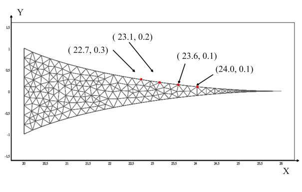
The transient temperature is estimated when the temperature measurement is available in the different measured locations. The measured locations are listed as below and shown in fig.2 (a)-2 (c).

2 sensors: (23.597, 0.14951), (24.0309, 0.1025)
4 sensors: (22.7323, 0.26903), (23.164, 0.20494),
(23.597, 0.14951), (24.0309, 0.1025)
6 sensors: (21.8735, 0.4258), (22.302, 0.34235),
(22.7323, 0.26903), (23.164, 0.20494),
(23.597, 0.14951), (24.0309, 0.1025)

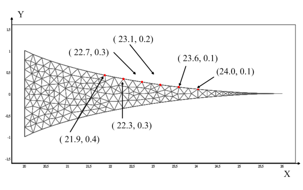
(11)



(a) 2 sensors



(b) 4 sensors



(c) 6 sensors

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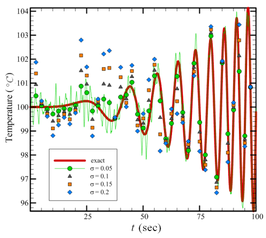
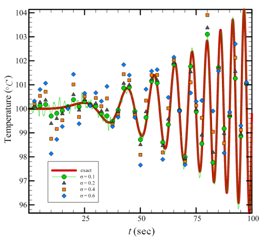
Fig. 2 The schematic diagrams of sensor's location (a) 2 sensors (b) 4 sensors (c) 6 sensors.

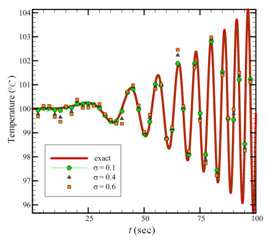
In the fig.3 (a)-3 (c), we will begin our discussion by consider the effect of measured errors between estimated results and exact results in case temperature distribution. From fig.3 (a), the value of measured errors is changed from 0.2 to 0.05, the absolute average errors becomes smaller than before and reduces 74% error. This message means the measured errors have big ability to affect and change the estimated results. The more clearly, we can follow the fig.3 (b) and fig.3 (c) to realize that the sensor number is also an important parameter. In the fig.3 (c), absolute average errors are 0.02568 when the measured errors are equal to 0.6 and it compares better with $\sigma = 0.05$ in fig.3 (a). This means the variation of sensor number is affect directly the estimated result very much. We will devote table 1 to the discussion of the sensor number on the fins surface that must be taken to prevent the promotion of absolute average errors in case temperature profile. From table 1, it is clear that the measured error reduces from 0.2 to 0.05 and the sensor number raise from 2 to 4, the absolute average errors decrease 92% quickly. For the reasons mentioned above, it can be concluded that the sensor number is a key factor for the inverse result.



(a) 2 sensors



(b) 4 sensors



(c) 6 sensors

Fig. 3 Comparison of exact result and inverse results (σ =0.05, 0.1, 0.15, 0.2, 0.4 and 0.6) for Case temperature profile (a) 2 sensors (b) 4 sensors (c) 6 sensors. ($r = 30$)

Table 1: The absolute average errors with different σ and sensor number in Case.

case	Measured error			
	$\sigma = 0.2$	$\sigma = 0.15$	$\sigma = 0.1$	$\sigma = 0.05$
2 sensors	0.1123	0.0843	0.0563	0.0283
4 sensors	0.0266	0.0203	0.0141	0.0081
6 sensors	0.0124	0.0109	0.0094	0.0080

Fig.4 shows that the absolute average errors have a sequence variation between $r = 15$ and $r = 24$. We can observe this profile to obtain the influence and the sensitivity of future time for absolute average errors. In the fig.4, the estimated value curve is more close to the exact temperature curve when the future time changes from 15 to 24. Therefore, we will now examine the ability of future time more clearly. Absolute average errors in this profile reduce from 0.02914 to 0.00818 and the final result decreases 71.9% error with the exact one. The table 2 presents that the variation of absolute average errors relate to the future time, sensor number and measured error. In this table, we can find the measured error decreases, the future time and sensor number increase, the error will reduce observably.

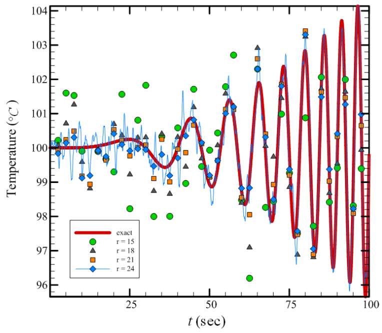


Fig. 4 Comparison of exact result and inverse results (r =15, 18, 21 and 24) for Case temperature profile. ($\sigma = 0.4$)

Table 2: The absolute average errors with the variation of sensor number, the measured errors and the future time r in Case.

case	Future time			
	$r = 15$	$r = 17$	$r = 19$	$r = 21$
4 sensors $\sigma = 0.2$	0. 5277	0. 2874	0. 1703	0. 1080
$\sigma = 0.1$	0. 2639	0. 1438	0. 0852	0. 0541
6 sensors $\sigma = 0.2$	0. 0582	0. 0386	0. 0274	0. 0203
$\sigma = 0.1$	0. 0291	0. 0193	0. 0138	0. 0104

It compares with the difference type of future time which can cause the absolute average errors to decrease plentifully in fig.5. The result in this profile shows the linear combination of future time is better than the other and it also decreases the absolute average errors effectively. The reason is the linear combination of future time can close to the exact solution approximately but the constant type isn't enough to match. According to the fig.5, we can understand that the type of future time is also an important parameter.

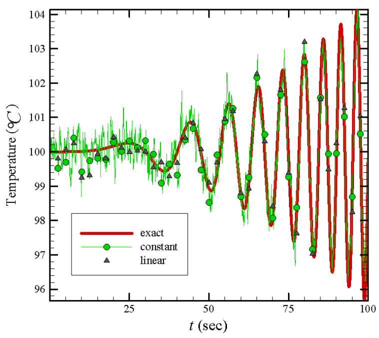


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Fig. 5 Comparison of exact result and inverse results with the type of future time (constant and linear) for Case temperature profile. ($\sigma = 0.4$, r =25, 6 sensors)

In this investigation, an inverse algorithm of the proposed method based on the sequential method, the concept of future time and the finite element method is used to analyze the time-varying boundary condition of the irregular shape and it can verify that the heat flux boundary condition is estimated possibility in this research. The estimated results of this study are compared with the measured errors, the sensor location, sensor's number and future time. From what has been discussed above, we can conclude that the proposed method can handle the estimation of unknown boundary condition in irregular shape by inverse method effectively.

IV. Conclusion

This investigation shows that an inverse method based upon the sequential method and the finite element method estimates the time-dependent heat flux distribution in the heat conduction problem of fins with irregular shape. When this study considered the measurement error, the sequential method is needed to combine with future time to stabilize the estimation solutions. In the process of the derivation, the future time can increase the accuracy of the solutions apparently. A sequential method is proposed in this paper that can handle the determination of unknown boundary conditions for the inverse heat conduction problem efficiently and obtain a higher exact result. From fig.4, we can make a conclusion that the estimated heat flux and the exact one are became closely as the future time from 15 to 24. In order to reduce errors, this paper attempts to add the sensor number and change the type of future time separately. It has been shown that the accuracy of the estimated results can be raised by the measurement error is restrained or by the more temperature sensors.

From this paper, we combine the traditional inverse algorithm and the finite element method to determine the excellent results under the irregular shape condition. It should be pointed out that the proposed method is an accurate, stable and efficient method to determine the unknown heat flux boundary conditions on fins in inverse heat conduction problem of irregular shape successfully. Necessarily, two dimension and three dimension inverse problems can also be applied by the proposed method in the same way.

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