Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol III,
IMECS 2010, March 17 - 19, 2010, Hong Kong

Competitive Facilities Location Problems with Fuzzy Random Demand

Takeshi Uno*, Hideki Katagiri†, and Kosuke Kato‡

Abstract— **This paper proposes a new location problem of competitive facilities, e.g. shops and stores, with uncertainty and vagueness including demands for the facilities in a plain. By representing the demands for facilities as fuzzy random variables, the location problem can be formulated as a fuzzy random programming problem. For solving the fuzzy random programming problem, first the α-level sets for fuzzy numbers are used for transforming it to a stochastic programming problem, and secondly, by using their expectations and variances, it can be reformulated to a deterministic programming problem. After showing that one of their optimal solutions can be found by solving 0-1 programming problems, their solution method is proposed by improving the tabu search algorithm with strategic oscillation. The efficiency of the solution method is shown by applying it to numerical examples of the facility location problems.**

Keywords: facility location, competitiveness, fuzzy random variables, tabu search, strategic oscillation.

1 Introduction

Competitive facility location problem (CFLP) is one of optimal location problems for commercial facilities, e.g. shops and stores, and the objective of a decision maker (DM) for the CFLP is mainly to obtain as many demands for her/his facilities as possible. Mathematical studies on CFLPs are originated by Hotelling [9]. He considered the CFLP under the conditions that (i) customers are uniformly distributed on a line segment, (ii) each of DMs can locate and move her/his own facility at any times, and (iii) all customers only use the nearest facility. CFLPs on the plain were studied by Okabe and Suzuki [17], etc. As an extension of Hotelling's CFLP, Wendell and McKelvey [26] assumed that there exist customers on a finite number of points, called demand points (DPs), and they considered the CFLP on a network whose nodes are DPs. Based upon the CFLP proposed by Wendell and McKelvey, Hakimi [7] considered CFLPs under the conditions that the DM locates her/his facilities on a network that other competitive facilities were already located. Drezner [4] extended Hakimi's CFLPs to the CFLP on the plain that there are DPs and competitive facilities. As extension of their CFLPs, CFLPs with quality or size of facilities are considered by Fernández et al. [5], Uno et al. [22, 23], Bruno and Improta [3], and Zhang and Rushton [27], and CFLPs based on maximal covering are considered by Plastria and Vanhaverbeke [18].

In the above studies of CFLPs, the demands of customers for facilities are represented as definite values. Wagnera et al. [24] considered facility location models with random demands in a noncompetitive environment. For the details of location models with random demands, the reader can refer to the study of Berman and Krass [2]. Uno et al. [23] considered CFLPs with random demands. The above CFLPs represent some uncertainty including demands for facilities as randomness. However, the demands generally include not only uncertainty but also vagueness, which is often represented as fuzziness. Moreno Pérez et al. [15] considered a facility location problem with fuzziness in a noncompetitive environment. For variables with both uncertainty and vagueness, Kwakernaak [14] proposed the fuzzy random variable representing both fuzziness and randomness. For the details of fuzzy random variable, the reader can refer to the book of Kruse and Meyer [13]. Wang and Qiao [19, 25] considered fuzzy random programming and its distribution problem. For the recent studies of fuzzy random programming problems, Katagiri et al. [11] considered multiobjective fuzzy random linear programming, and Ammar [1] considered fuzzy random multiobjective quadratic programming.

In this paper, we proposes a new CFLP with uncertainty and vagueness by extending the CFLP proposed by Uno et al. [23]. By representing the demands for facilities as fuzzy random variables, the CFLP can be formulated as a fuzzy random programming problem. For solving the fuzzy random programming problem, first the α-level sets for fuzzy numbers are used for transforming it to a stochastic programming problem, and secondly, by using their expectations and variances, it can be reformulated to a deterministic programming problem.

*The manuscript was submitted at December 31, 2009. Graduate School of Human and Natural Environment Sciences, the University of Tokushima, 1-1, Minamijosanjima-cho, Tokushima-shi, Tokushima, 770-8502 Japan (Email: uno@ias.tokushima-u.ac.jp)

†Graduate School of Engineering, Hiroshima University, 1-4-1, Kagamiyama, Higashihiroshima-shi, Hiroshima, 739-8527 Japan (Email: katagiri-h@hiroshima-u.ac.jp)

‡Faculty of Applied Information Science, Hiroshima Institute of Technology, 2-1-1, Miyake, Saeki-ku, Hiroshima, 731-5193, Japan (Email: k.katoh.me@it-hiroshima.ac.jp)

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The formulated CFLP cannot be solved directly because of its nonconvexity and nonlinearity. Then, we first consider an integer programming problem to obtain one of optimal solutions to the CFLP. Since the reformulated problem is NP-hard, we need to propose an efficient solution method for the problem. For discrete optimization problems, the tabu search algorithm, proposed by Grover [6], is one of the efficient solution algorithms; for the details of the tabu search algorithm, the reader can refer to the book of Reeves [20]. Hanafi and Freville [8] proposed an efficient tabu search approach for the 0-1 multidimensional knapsack problem, which is designed based on a strategic oscillation. For CFLPs with random demand, Uno et al. [23] proposed an efficient solution method improving their tabu search approach. Their method utilizes the characteristic that the DM can improve her/his objective function value if she/he locates her/his facilities so that the facilities obtain demands from as many DPs as possible. Unfortunately, the problem does not have the above characteristic because increasing the number of DPs whose demands are obtained by the locating facilities may make the variance of obtaining demands worse. Then, we propose a new tabu search algorithm with strategic oscillation for solving the CFLP. We apply it to numerical examples of the CFLPs with fuzzy random demands, and show its efficiency by comparing to other solution algorithms.

The remaining structure of this article is organized as follows. The next section devotes to introducing the definition of fuzzy random variables. In Section 3, we formulate the CFLP with fuzzy random demand as a fuzzy random programming problem, and by using the α-level sets, expectation and variance, we reformulated the problem to a deterministic programming problem. Since it is difficult to solve the formulated problem directly, we show that one of its optimal solutions can be found by solving a 0-1 programming problem in Section 4. In Section 5, we propose its solution method based upon tabu search algorithms with strategic oscillation. We show the efficiency of the solution method by applying to numerical examples of the CFLPs with fuzzy random demands in Section 6. Finally, in Section 7, concluding comments and future extensions are summarized.

2 Fuzzy random variable

Let $\tilde{A}$ be fuzzy number and $\mu_{\tilde{A}} : \mathbf{R} \to [0,1]$ be membership function of $\tilde{A}$, where $\mathbf{R}$ is the set of real numbers. For $\alpha \in (0,1]$, the α-level set of $\tilde{A}$ is represented as the following equation:

$$\tilde{A}_\alpha \equiv \{x |\ \mu_{\tilde{A}}(x) \geq \alpha\} \tag{1}$$

In this paper, we use the following definition of fuzzy random variable, suggested by Kruse and Meyer [13]:

Definition 1 *Let (Ω, B, P) be a probability space, $\mathcal{F}(\mathbf{R})$ the set of fuzzy numbers with compact supports, and X a measurable mapping $\Omega \to \mathcal{F}(\mathbf{R})$. Then X is a fuzzy random variable if and only if given ω set membership, variant Ω, $X_\alpha(\omega)$ is a random interval for any $\alpha \in (0,1]$.*

3 Formulation of CFLP with fuzzy random demands

In the proposed CFLPs, we assume that all customers only exist on DPs in plain $\mathbf{R}^2$. For convenience sake, by aggregating all customers on the same DP, we regard one DP as one customer.

There are n DPs in $\mathbf{R}^2$, and let $D = \{1, \ldots, n\}$ be the set of indices of the DPs. Let m be the number of new facilities that the DM locates, and k be the number of competitive facilities which have been already located in $\mathbf{R}^2$. The sets of indices of the new facilities and the competitive facilities are denoted by $F = \{1, \ldots, m\}$ and $F_C = \{m+1, \ldots, m+k\}$, respectively.

Let $\boldsymbol{u}_i \in \mathbf{R}^2$ be the site of DP $i \in D$, and $\boldsymbol{x}_j \in \mathbf{R}^2$ and $q_j > 0$ be the site and quality of facility $j \in F \cup F_C$, respectively. Then, attractive power of facility j for DP i is represented as the following function introduced by Huff [10]:

$$a_i(\boldsymbol{x}_j, q_j) \equiv \begin{cases} \dfrac{q_j}{||\boldsymbol{u}_i - \boldsymbol{x}_j||^2}, & \text{if } ||\boldsymbol{u}_i - \boldsymbol{x}_j|| > \varepsilon, \\ \dfrac{q_j}{\varepsilon^2}, & \text{if } ||\boldsymbol{u}_i - \boldsymbol{x}_j|| \leq \varepsilon, \end{cases} \tag{2}$$

where $\varepsilon > 0$ is an upper limit of the distance that customers can move without any trouble. It is assumed that all customers only use one facility with the largest attractive power, and if the two or more attractive powers are the same, they use the facility in reverse numerical order of the indices of facilities; that is, in the order of competitive facilities and new facilities.

Let $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)$ be the location of the new facilities. Then we use the following 0-1 variable for representing whether DP i uses new facility $j \in F$:

$$\varphi_i^j(\boldsymbol{x}) = \begin{cases} 1, & \text{if DP } i \text{ uses the new facility } j, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Let $\tilde{\bar{W}}_i$ be the $L-R$ fuzzy random variable meaning the buying power (BP) of DP i. New facility $j \in F$ can obtain the BP $\tilde{\bar{W}}_i$ if $\varphi_i^j(\boldsymbol{x}) = 1$. The objective of the DM is maximizing the sum of BP that all the new facilities obtain. Then, the CFLP with random demand is formulated as the following fuzzy random programming problem:

$$\left.\begin{array}{ll} \text{maximize} & \tilde{\bar{f}}(\boldsymbol{x}) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{\bar{W}}_i \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{R}^{2m}. \end{array}\right\} \tag{4}$$

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

For solving (4), we first transform (4) to the following stochastic programming problem by using the α-level sets of $\tilde{\bar{f}}(\boldsymbol{x})$ with a given value $\alpha \in (0,1]$. For given $\alpha \in [0,1]$, we assume that the DM can decide the variable in each of α-level sets for maximizing the objective function value of (4). Then, (4) can be transformed as follows:

$$\left.\begin{array}{ll} \text{maximize} & \bar{f}_\alpha(\boldsymbol{x}) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} \bar{W}_i \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \bar{W}_i \in (\tilde{\bar{W}}_i)_\alpha,\ \forall i \in D, \\ & \boldsymbol{x} \in \mathbf{R}^{2m}. \end{array}\right\} \quad (5)$$

Next, by using the expectations and variances of random numbers, we reformulate (5) to a deterministic programming problem. For random variable $\bar{W}$, let $E[\bar{W}]$ and $Var[\bar{W}]$ be the expectation and variance of $\bar{W}$. For given positive parameter σ, let

$$W_i^\alpha \equiv \sup_{\bar{W} \in (\tilde{\bar{W}}_i)_\alpha} \{E[\bar{W}] - \sigma \cdot Var[\bar{W}]\}. \quad (6)$$

Then, (5) can be reformulated as follows:

$$\left.\begin{array}{ll} \text{maximize} & f_\alpha(\boldsymbol{x}) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} W_i^\alpha \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \mathbf{R}^{2m} \end{array}\right\} \quad (7)$$

Problem (7) is a nonconvex nonlinear programming problem. Besides, (7) has the characteristic that for distinct $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ ($\neq \boldsymbol{x}^1$), $f_\alpha(\boldsymbol{x}^1) = f_\alpha(\boldsymbol{x}^2)$ if the order of attractive power and dominating relation for $\boldsymbol{x}^1$ is the very same as those for $\boldsymbol{x}^2$. This means that (7) cannot be solved by directly applying general analytic solution methods with differential of the objective function, Kuhn-Tucker conditions, etc., since $f_\alpha(\boldsymbol{x})$ is a stepformed function. Moreover, the above characteristic also causes an adverse effect for applying heuristic solution methods for nonlinear programming problems, because we cannot utilize any information of the neighborhood of current solutions for finding their improving directions. Uno et al. [22] showed that their CFLPs, which is easier to solve than (7), cannot be also solved by directly applying heuristic solution methods for nonlinear programming problems, e.g. genetic algorithm for numerical optimization for constrained problem (GENOCOP) [12]. In the next section, we show that one of the optimal solutions of (7) can be found by solving a 0-1 programming problem.

4 Reformulation to 0-1 programming problem

In the location model in the previous section, if the new facilities are located, the values of (3) for all facilities and DPs are given, and then objective function value of (7) can be computed. On the other hand, we propose the following solution method:

1. Decide the set of DPs that the DM wants to obtain their BPs preferentially by giving the values of (3) for all facilities and DPs, and
2. Find the location of the new facilities such that the value of (3) for each facility and DP is equal to or more than the given value.

For DP $i \in D$, the largest attractive power among all competitive facilities is denoted by a_i^C. From (2), the set of DPs that new facility $j \in F$ cannot obtain their BPs wherever it is located can be represented as follows:

$$D_j^\triangle = \{i \in D \mid \sqrt{q_j / a_i^C} \leq \varepsilon\}. \quad (8)$$

Then, the set of DPs that there is at least one location of new facility j which can obtain their BPs is denoted by $D_j = D \backslash D_j^\triangle$. For new facility j, let $\bar{D}_j \subseteq D_j$ be the set of DPs that the DM wants to obtain their BPs by locating it preferentially. Let

$$l_{ij} = \begin{cases} 1, & \text{if } i \in \bar{D}_j, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Then, $\bar{D}_j$ can be represented as 0-1 vector $\boldsymbol{l}_j = (l_{1j}, \ldots, l_{nj})^T$. For new facility j and vector $\boldsymbol{l}_j$ given by the DM, we consider the following problem with an auxiliary variable $r_j \geq 0$:

$$\left.\begin{array}{ll} \text{minimize} & r_j^2 \\ \text{subject to} & ||\boldsymbol{x}_j - \boldsymbol{u}_i||^2 \leq \dfrac{q_j}{a_i^C} \cdot r_j, \\ & \qquad\qquad \forall i \in \{\bar{i} | l_{\bar{i}j} = 1\}, \\ & \boldsymbol{x}_j \in \mathbf{R}^2,\ r_j \geq 0. \end{array}\right\} \quad (10)$$

Let $(\boldsymbol{x}_j^{\boldsymbol{l}_j}, r_j^{\boldsymbol{l}_j})$ be an optimal solution of (10). Then, the following theorem plays an important role to find an optimal location.

Theorem 2 *For new facility $j \in F$, let $\bar{D}_j$ be the set of DPs given by the DM and $\boldsymbol{l}_j$ be the 0-1 vector corresponding to $\bar{D}_j$. Then, if $r_j^{\boldsymbol{l}_j} < 1$, the new facility j can obtain all DPs in $\bar{D}_j$ by locating it at $\boldsymbol{x}_j^{\boldsymbol{l}_j}$.*

PROOF: *For the constraint of (10) and $r_j^{\boldsymbol{l}_j} < 1$, $||\boldsymbol{x}_j^{\boldsymbol{l}_j} - \boldsymbol{u}_i||^2 < q_j / a_i^C$ is satisfied for all DPs in $\bar{D}_j$. Then, $a_i^C < q_j / ||\boldsymbol{x}_j^{\boldsymbol{l}_j} - \boldsymbol{u}_i||^2$ is satisfied. From (2), this equation means that the attractive power of new facility j is more than that of competitive facilities if the site of facility j is $\boldsymbol{x}_l^{\boldsymbol{l}_j}$.* □

Since (10) is a convex programming problem, (10) can be solved by using the solution algorithms for convex programming problems, such as sequential quadratic programming (SQP) method; for the details of the SQP method, the reader can refer to the book of Nocedal and Wright [16].

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Theorem 3 *Let* $L = (\boldsymbol{l}_1, \ldots, \boldsymbol{l}_m) \in \{0,1\}^{nm}$ *and* $\boldsymbol{x}^L = (\boldsymbol{x}_1^{\boldsymbol{l}_1}, \ldots, \boldsymbol{x}_m^{\boldsymbol{l}_m})$. *Then, there exists* L *such that* $\boldsymbol{x}^L$ *is an optimal solution of (7).*

PROOF: *Let* $\boldsymbol{x}^*$ *be an optimal solution of (7). We define the 0-1 matrix* $\bar{L} = (\bar{\boldsymbol{l}}_1, \ldots, \bar{\boldsymbol{l}}_m) \in \{0,1\}^{nm}$, *each of whose element for* $i \in D$ *and* $j \in F$ *is that* $\bar{l}_{ij} = \varphi_i^j(\boldsymbol{x}^*)$. *Then, from Theorem 2,* $\boldsymbol{x}^{\bar{L}}$ *is also an optimal solution of (7) because* $\varphi_i^j(\boldsymbol{x}^{\bar{L}}) = \varphi_i^j(\boldsymbol{x}^*)$ *for all* i, j *and* $r_j^{\bar{\boldsymbol{l}}_j} < 1$ *for all* j. *This means that* $\bar{L}$ *is one of the matrices satisfying the condition of the theorem.* $\square$

Let $r_{\max}^L \equiv \max\{r_1^{\boldsymbol{l}_1}, \ldots, r_m^{\boldsymbol{l}_m}\}$. From Theorem 3, finding an optimal solution of (7) can be formulated as the following 0-1 programming problem:

$$\left.\begin{array}{ll} \text{maximize} & f_\alpha(\boldsymbol{x}^L) \\ \text{subject to} & r_{\max}^L < 1, \\ & L \in \{0,1\}^{nm} \end{array}\right\} \quad (11)$$

Problem (11) is NP-hard because finding an optimal solution of (11) strictly needs to solve (10) at 2^{nm} times. In the next section, we propose an efficient solution method for (11).

5 Tabu search algorithm with strategic oscillation

Problem (11) is a 0-1 programming problem, one of whose approximate solution methods is the tabu search algorithm with strategic oscillation, proposed by Hanafi and Freville [8]. Uno et al. [23] proposed an efficient solution algorithm based upon the tabu search algorithm with strategic oscillation for the 0-1 programming problem reformulated from the CFLP with random demands. In this section, we propose a new efficient solution algorithm based upon the tabu search algorithm with strategic oscillation for (11).

The tabu search is one of the local search methods. In our solution method, we define moves from a current solution, denoted by L^{now}, as the increase or decrease of its one element. The neighborhood of a current solution of (11) is represented as a set of all solutions that can transfer by only one move from the solution. In the tabu search including our solution method, the next searching solution from L^{now}, denoted by L^{next}, is basically chosen to the best solution for given criteria, e.g. objective function value, in the neighborhood of L^{now}. However, if we use such a search without modification, a circulation of certain chosen moves occurs on the way of search, and then it can only find one local optimal solution. For preventing such a circulation, if a move is chosen in the search, the tabu constraint for its opposite move is activated for given tenure, called the tabu tenure and denoted by T_1. Then the activated moves are forbidden to choose in T_1 tenure, called tabu, even if the moves make the objective function value of (11) best in all solutions in a neighborhood. Such tabu moves are memorized in the tabu list for the search.

Although the tabu search method has advantage for searching in local areas, the feasible set of (11) is generally wide and there are many local optimal solutions in the feasible set. Then, we introduce the strategic oscillation to the tabu search for searching various local optimal solutions efficiently. Because of (10), there is a connection between increasing the value of $r_{\max}^{L^{\text{now}}}$ and increasing the number of obtaining DPs. This means that (11) has the tendency that while the expectation for L^{now} improves by increasing $r_{\max}^{L^{\text{now}}}$, the variance for L^{now} improves by decreasing $r_{\max}^{L^{\text{now}}}$. Then, we can find various types of solutions of (11) by using the strategic oscillation: the iterations of increasing and decreasing of $r_{\max}^{L^{\text{now}}}$. Therefore, our proposing solution method is described as follows.

Tabu search algorithm with strategic oscillation for CFLPs

Step 0: Generate the initial searching solution L^{now}, and initialize the tabu list and other variables. If $r_{\max}^{L^{\text{now}}} < 1$, then go to Step 4.

Step 1: Move L^{now} to L^{next} by decreasing an element of L^{now} with the purpose of decreasing $r_{\max}^{L^{\text{next}}}$ as much as possible. This step is repeated until it is satisfied $r_{\max}^{L^{\text{next}}} < 1$.

Step 2: Move L^{now} to L^{next} with the purpose of improving the objective function value of (11). This step is repeated until there is no non-tabu and improved moves from L^{now} in its feasible set.

Step 3: Move L^{now} to L^{next} with the purpose of improving the objective function value of (11). This step is repeated at a given certain tenure, denoted by $T_2 > 0$.

Step 4: Move L^{now} to L^{next} by decreasing an element of L^{now} with the purpose of decreasing $r_{\max}^{L^{\text{next}}}$ as much as possible. This step is repeated until $r_{\max}^{L^{\text{next}}}$ is less than a certain value, denoted by r_{low}.

Step 5: Do the same operations as Step 2.

Step 6: Do the same operations as Step 3.

Step 7: Move L^{now} to L^{next} with the purpose of improving the objective function value of (11) without considering the feasibility of (11). This step is repeated until $r_{\max}^{L^{\text{next}}}$ is more than a certain value, denoted by r_{upp}.

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Step 8: If the given terminal conditions are satisfied, then terminate the algorithm. Otherwise, return to Step 1.

Note that the tabu search algorithm with strategic oscillation for CFLPs with random demands, proposed by Uno et al. [23], is utilized the characteristic that the DM can improve her/his objective function value if she/he locates her/his facilities so that the facilities obtain demands from as many DPs as possible. Unfortunately, the problem does not have the above characteristic because increasing the number of DPs whose demands are obtained by the locating facilities may make the variance of obtaining demands worse. Then, we improve the algorithm to the CFLPs with fuzzy random demands by adding Step 2 and changing Step 5 to search moves of not only increase but also decrease.

6 Numerical experiments

In this section, we show the efficiency of the solution algorithm in the previous section by applying to three examples of the CFLPs. In these examples, the numbers of DPs are $n = 30, 40, 50$. The sites of DPs $\boldsymbol{u}_1, \dots, \boldsymbol{u}_n$ are given in $[0, 100] \times [0, 100]$ randomly, and their BPs $\tilde{\bar{W}}_1, \dots, \tilde{\bar{W}}_n$ are represented as fuzzy random variables; each of which have three scenarios whose probabilities are 0.5, 0.3, and 0.2, and BPs of each DP for each scenario are given from the set of fuzzy numbers whose membership functions are given by

$$\mu_{\tilde{W}}(x) = \begin{cases} 1 - (c - x)/\beta \vee 0, & x \leq c, \\ 1 - (x - c)/\gamma \vee 0, & x \geq c, \end{cases} \tag{12}$$

where $\vee$ means that $a \vee b = \max(a, b)$, and for each DP and scenario, c, β, and γ are randomly given in $[5, 10]$, $[1, 4]$, and $[1, 6]$, respectively. We give five competitive facilities, that is $k = 5$, and for each competitive facility $j \in F_C$, its site $\boldsymbol{x}_j$ are given on DPs chosen randomly, and its quality q_j is randomly given in $\{1, \dots, 5\}$. In this plain, the decision maker locates one facility, that is $m = 1$, whose quality is that $q_1 = 3$. We give the α-level set with $\alpha = 0.7$. For (6), we give $\sigma = 3$.

Next, we give parameters about our solution method; for the meanings of parameters of tabu searches, the reader can refer to the book of Reeves [20]. We set the tabu tenure $T_1 = n/2 - 10$. The terminal condition in Step 7 is that the iteration of the tabu search algorithm is false until 10 times. At Step 3, let $T_2 = 10$. At Steps 4 and 7, let $r_{\text{low}} = 0.3$ and $r_{\text{upp}} = 3$.

For showing the efficiency of our solution method, we compare its computational results to that of the genetic algorithm; for details of the genetic algorithms, the readers can refer to the studies of Sakawa et al. [21]. We set generation gap $G = 0.9$, population size $N_{GA} = 150$, and terminal generation $T_{GA} = 2000$. Probabilities of crossover, mutation, and inversion are $p_C = 0.9$, $p_M = 0.01$, and $p_I = 0.03$, respectively.

We apply the tabu search and the genetic algorithm to three examples of the CFLPs, where each of these algorithms is implemented 10 times for each example by using the PC (Intel Celeron(R) CPU 2.93 GHz, 992 MB RAM). The computational results of solving the CFLPs are shown in Tables 1-2. From Tables 1-2, the tabu search can obtain better solutions for (11) than those of the genetic algorithm with shorter computational times. This means that our solution method is efficient for the CFLPs with fuzzy random demands.

Table 1: Computational results by the tabu search algorithm with the strategic oscillation for (11)

n	30	40	50
Best	49.34	57.30	68.56
Mean	49.34	57.15	68.56
Worst	49.34	56.57	68.56
CPU times (sec)	43.23	62.92	98.27

Table 2: Computational results by the genetic algorithm for (11)

n	30	40	50
Best	49.34	57.30	68.56
Mean	49.27	56.79	63.50
Worst	48.63	56.57	56.67
CPU times (sec)	336.1	395.9	410.7

7 Conclusions and future researches

In this paper, we have proposed a new CFLP on the plain with fuzzy random demands. We have formulated the CFLP as a fuzzy random programming problem, and reformulated to a deterministic programming problem for finding an optimal solution of the problem. Because the problem is difficult to solve directly, we have shown that the problem can be reformulated as a 0-1 programming problem. Since the 0-1 programming problem is NP-hard, we have proposed an efficient solution method based upon the tabu search algorithm with strategic oscillation by utilizing characteristics of the CFLPs. The efficiency of the solution method is shown by applying to several examples of the CFLPs.

We considered the uncertainty and vagueness including demands in this paper. For the CFLPs, we think that the distance between customer and facility also include some uncertainty and vagueness; for the reasons that customers may go to the facilities by utilizing various means of transportation, that customers may come by facilities on the way to another destination, etc. Then, we can consider other CFLPs with uncertainty and vagueness, which are interesting future studies.

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

References

[1] Ammar, E.E., "On fuzzy random multiobjective quadratic programming," *European Journal of Operational Research*, V193, N2, pp. 329-340, 3/09.

[2] Berman, O., Krass K., "Facility location with stochastic demands and congestion," *Z. Drezner and H.W. Hamacher, Editors, Facility Location: Application and Theory*, Springer, Berlin, 2001.

[3] Bruno, G., Improta, G., "Using gravity models for the evaluation of new university site locations: A case study," *Computers & Operations Research*, V35, N2, pp. 436-444, 2/08.

[4] Drezner, Z., "Competitive location strategies for two facilities," *Regional Science and Urban Economics*, V12, pp. 485-493, 11/82.

[5] Fernández, J., Pelegrín, B., Plastria, F., Tóth, B., "Solving a Huff-like competitive location and design model for profit maximization in the plane," *European Journal of Operational Research*, V179, N3, pp. 1274-1287, 6/07.

[6] Glover, F., "Future paths for integer programming and links to artificial intelligence," *Computers & Operations Research*, V5, pp. 533-549, 5/86.

[7] Hakimi, S.L., "On locating new facilities in a competitive environment," *European Journal of Operational Research*, V12, pp. 29-35, 1/83.

[8] Hanafi S., Freville, A., "An efficient tabu search approach for the 0-1 multidimensional knapsack problem," *European Journal of Operational Research*, V106, pp. 659-675, 4/98.

[9] Hotelling, H., "Stability in competition," *The Economic Journal*, V30, pp. 41-57, 3/29.

[10] Huff, D.L., "Defining and estimating a trading area," *Journal of Marketing*, V28, pp. 34-38, 7/64.

[11] Hideki, K., Sakawa, M., Kato, K., Nishizaki, I., "Interactive multiobjective fuzzy random linear programming: Maximization of possibility and probability," *European Journal of Operational Research*, V188, N2, pp. 530-539, 7/08.

[12] Koziel S., Michalewicz Z., "Evolutionary Algorithms, Homomorphous Mappings, and Constrained Parameter Optimization," *Evolutionary Computation*, V7, N1, pp. 19-44, 4/99.

[13] Kruse R., Meyer, K.D., *Statistics with vague data*, D. Riedel Publishing Company, Dordrecht, 1987.

[14] Kwakernaak, H., "Fuzzy random variables-I. definitions and theorems," *Information Sciences*, V15, pp. 1-29, 7/78.

[15] Moreno Pérez, J.A., Marcos Moreno Vega, L., Verdegay, J.L., "Fuzzy location problems on networks," *Fuzzy Sets and Systems*, V142, N3, pp. 393-405, 3/04.

[16] Nocedal, J., Wright, S., *Numerical Optimization*, Springer-Verlag, New York, 1999.

[17] Okabe, A., Suzuki, A., "Stability of competition for a large number of firms on a bounded two-dimensional space," *Environment and Planning A*, V19, N8, pp. 1067-1082, 1/87.

[18] Plastria, F., Vanhaverbeke, L., "Discrete models for competitive location with foresight," *Computers & Operations Research*, V35, N3, pp. 683-700, 2/08.

[19] Qiao, Z., Wang, G., "On solutions and distribution problems of the linear programming with fuzzy random variable coefficients," *Fuzzy Sets and Systems*, V58, pp. 120-155, 9/93.

[20] Reeves, C.R. ed., *Modern Heuristic Techniques for Combinatorial Problems*, Blackwell Scientific Press, Oxford, 1993.

[21] Sakawa, M., Kato, K., Ushiro, S., "An interactive fuzzy satisficing method for multiobjective 0-1 programming problems involving positive and negative coefficients through genetic algorithms with double strings," *Proceedings of the 8th International Fuzzy Systems Association World Congress*, V1, pp. 430-434, 8/99.

[22] Uno, T., Katagiri, H., "A location of competitive facilities in the plane including cooperative facilities," *Proceedings of IEEE SMC Hiroshima Chapter 3rd International Workshop on Computational Intelligence & Applications*, pp.P3-1-P3-6, 12/07.

[23] Uno, T., Katagiri, H., Kato, K., "Competitive facility location with random demands," *IAENG Transactions on Engineering Technologies*, V3, pp. 83-93, America Institute of Physics, 2009.

[24] Wagnera, M.R., Bhaduryb, J., Penga, S., "Risk management in uncapacitated facility location models with random demands," *Computers & Operations Research*, V36, I4, pp. 1002-1011, 4/09.

[25] Wang, G.Y., Qiao, Z., "Linear programming with fuzzy random variable coefficients," *Fuzzy Sets and Systems*, V57, pp. 295-311, 8/93.

[26] Wendell, R.E., McKelvey, R.D., "New perspective in competitive location theory," *European Journal of Operational Research*, V6, pp. 174-182, 2/81.

[27] Zhang, L., Rushton, G., "Optimizing the size and locations of facilities in competitive multi-site service systems," *Computers & Operations Research*, V35, N2, pp. 327-338, 2/08.