Budget Constraint in Reforestation Meant for Minimizing Sediment Load at a Watershed Outlet

Pablo Vanegas, Dirk Cattrysse, Jos Van Orshoven

Abstract—**Policy and decision makers dealing with environmental conservation and land use planning often require identifying potential sites for contributing to minimize sediment flow reaching riverbeds. This could be one of the environmental objectives of reforestation initiatives. An Integer Programming (IP) formulation for selecting a predefined number of locations to minimize sediment load at a watershed outlet has been previously developed in [1]. This paper tests that formulation under the assumption that the area to be reforested is not known in advance, a budget constraint is included instead. Therefore the extension of the reforested area is subject to this budget constraint, which makes the problem more complex. Several experiments are performed for two watersheds in South Dakota in the USA. The results show the sediment load at the watershed outlet as well as the erosion levels, slopes and distances to the riverbeds of the locations selected to be reforested.**

Keywords: Budget, Site Location, Reforestation, Integer Programming, Exact Methods

I. Introduction

Regions meant for reforestation can have sediment flow minimization among their environmental objectives. With decision criteria represented by raster maps, sets of cells to be reforested can be identified in order to reduce sediment flow. Since flow is nonlinear in nature, [1] applied a piecewise linear convex function in an IP approach to model flow delivery from a cell to one of its neighbors. This function needs two breakpoints in order to define three segments, in turn each segment is associated with a specific flow delivery factor. Breakpoints, flow production, and flow delivery factors in a cell change when it is reforested. These local changes affect also the state of neighboring cells (spatial interaction).

This paper analyzes the effect of including a budget constraint in the Integer Programming formulation proposed by [1]. The formulation in [1] is modeled as a *general Network Flow (NF) problem* [2] satisfying arc capacities and mass balance constraints to all cells. Instead of specifying the number of cells to be reforested, the formulation in this paper includes a budget constraint which makes the problem more complex.

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Next section summarizes work performed for modeling flow transport. Whereas section III details the data representation and the proposed IP model, section IV shows the results obtained for 2 small watersheds in South Dakota in the USA. Finally, section V draws the conclusions.

II. Literature Review

The IP formulation proposed in the course of this paper attempts to model *interactions* between cells in order to minimize sediment flow reaching a predefined outlet. *Spatial Interaction* (SI) designate the existence of causal relations in space or the existence of spatial diffusion processes. In the problem at hand, SI implies that reforestation of a cell modifies also the state of one or more neighboring cells.

A. Spatial Interaction for Simulating Sediment Transport

During the last years, several simulation models have been developed in order to represent cells interaction. In this sense, particular attention was given to applications of Cellular Automata (CA) for simulating flow transport and channel dynamics [3], landscape evolution [4], hydrodynamics [5], lava flow [6], soil erosion by water [7], sediment discharge and alluvial fan development [8]. *Flow* simulations carried out within these applications are not dealing with the explicit location of high quality sites for minimizing cumulative sediment flow, i.e. optimality requirements are not present in these approaches. In addition, flow direction is a variable within the decision rules of these CA applications.

The traditional way of measuring flow direction takes the steepest-descent route (Single Flow Direction -SFD-) between cells. However, drawbacks of SFD algorithms have been discussed by several authors ([9], [10], [11]). Although SFD algorithms may be appropriate for convergent parts of the landscape, it is not for divergent hill slopes [9]. Research has also been conducted to compare the effect of *Single (SFD) and Multiple Flow Direction (MFD)* algorithms. TOPMODEL model efficiency and simulated flow paths were affected only slightly when the topographic index distribution was computed with the SFD instead of the MFD algorithm [12]. Any difference essentially disappeared when the model was calibrated by adjusting subsurface hydraulic parameters.

The IP formulation proposed by [1] makes use of a SFD map to model flow delivery from one cell to another until the wa-

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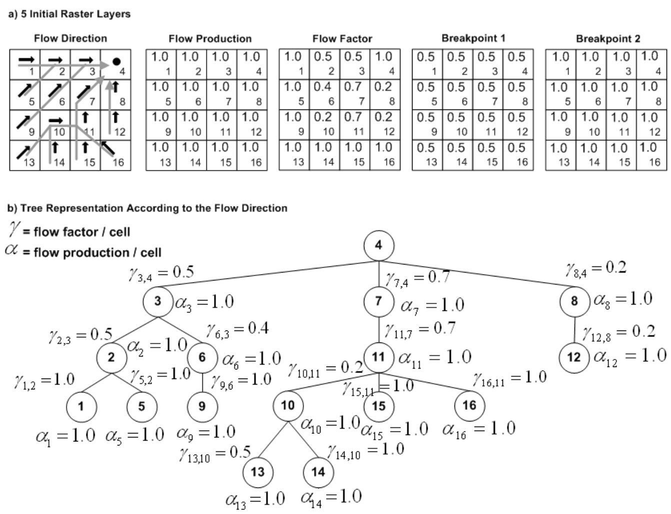

Fig. 1. Data and Problem Representation

tershed outlet is reached. The SFD map allows constructing a tree where every node corresponds with a cell location and the root corresponds with the outlet cell. Under the tree structure every node delivers its sediment to the parent. Moreover, the formulation in [1] requires that the number of nodes to be reforested is predefined. Nevertheless, there exist applications constrained by a fixed amount of resources available for reforestation of an area of unknown extension. The area is a variable in this sort of problems.

III. Materials and Methods

Locating optimal cells requires the specification of an outlet cell where total flow converges. Figure 1a shows a schematic representation of the five initial maps applied to minimize flow. The underlying tree structure shown in figure 1b is constructed from the *Flow Direction* map. Each node in the tree corresponds with a cell location, therefore *cell values* in each one of the raster maps can be referenced from the corresponding node; e.g. nodes of the tree presented in figure 1b are associated with values of flow factor between node i and node j ($\gamma_{i,j}$) and flow production in node i (α_i).

A. Notation

The following notation is introduced to facilitate the explanation of the IP formulation for minimizing sediment flow.

1) *Flow Production* (α_j) is a general term associated to levels of any kind of locally produced flow in node j. In the problem at hand flow production refers to sediment or erosion ($T \cdot ha^{-1} \cdot yr^{-1}$) when node j is not reforested.
2) *Flow Factor* ($\gamma_{j,k}$) stands for a factor of transporting sediment from node j to the parent node k. In problems where geographic relief plays an important role, as in the problem at hand, slope can act as a multiplier for flow delivery. $\gamma_{j,k}$ is applied when node j is not reforested.
3) *Breakpoint* $\#1$ ($\sigma_{1,j}$) is the first breakpoint of a piecewise linear convex function modeling the non-linear nature of sediment flow when node j is not reforested. Breakpoints are required to define linear segments within the flow delivery functions. In this paper, *breakpoint* $\#1$ is considered as the sediment retention capacity in node j, i.e. if total flow is less than or equal *breakpoint* $\#1$ it will not be delivered to the parent node.
4) *Breakpoint* $\#2$ ($\sigma_{2,j}$) is the second breakpoint of the sediment flow delivery function in node j.
5) *New Flow Production* (β_j), *New Flow Factor*($\delta_{j,k}$), *New Breakpoint* $\#1$ ($\sigma_{3,j}$), *and New Breakpoint* $\#2$ ($\sigma_{4,j}$) are the new values for Flow Production, Flow Factor, Breakpoint $\#1$, and Breakpoint $\#2$ for reforested nodes; i.e. new parameters of the piecewise linear convex function.
6) *Reforestation Cost* (ν_j) is the cost associated to the reforestation of node j. Distance to roads is used as cost in the present study.
7) *Streams* is a binary grid where cells upholding a value 1 represent streams or riverbeds.

B. Integer Programming Formulation

An IP model, based on the general *Network Flow (NF) formulation with piecewise linear convex functions* [2], is proposed to identify cells to be reforested in order to minimize sediment flow at a watershed outlet. This formulation requires that each individual link between a pair of nodes (j, k) in the original tree representation (fig 1) is substituted by a set of five links: $y_{j,k,1}$, $y_{j,k,2}$, $y_{j,k,3}$, $y_{j,k,4}$, $y_{j,k,5}$. Each link corresponds with a segment for delivering flow from node j to node k.

When node j is *not reforested*, flow is delivered to its parent (node k) according to a piecewise linear convex function with two breakpoints ($\sigma_{j,1}, \sigma_{j,2}$,) and three segments ($y_{j,k,1}$, $y_{j,k,2}$, $y_{j,k,5}$). Effective Accumulation in node j (EA_j) corresponds to the sum of flow coming into node j from its children nodes plus the flow produced in the node itself (α_j). Output flow delivered to cell k depends on EA_j: 1) when it is less than or equal to the breakpoint $\#1$ ($\sigma_{j,1}$) flow in segment 1 ($y_{j,k,1}$) is completely retained, i.e. no flow is delivered to the parent node (k); 2) if EA_j is less than or equal to breakpoint $\#2$ ($\sigma_{j,2}$) and larger than breakpoint $\#1$ ($\sigma_{j,1}$), a fraction $\gamma_{j,k}$ of the flow in segment 2 ($y_{j,k,2} = EA_j - \sigma_{j,1}$) is delivered; 3) when EA_j is larger than breakpoint $\#2$ ($\sigma_{j,2}$), a fraction $\gamma_{j,k}$ of the flow in segment 2 ($y_{j,k,2} = \sigma_{j,2} - \sigma_{j,1}$) plus the entire flow in segment 5 ($y_{j,k,5} = \sigma_{j,2} - EA_j$) is delivered to the parent node (k).

The procedure in the former paragraph is also applied to determine the amount of flow to be delivered by node j when it is reforested. In this case segments $y_{j,k,3}$, $y_{j,k,4}$ and $y_{j,k,5}$ are used. The parameters of the original piecewise linear

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convex function are replaced by the parameters for reforested nodes: 1) $\sigma_{j,1}$ by $\sigma_{j,3}$, and $\sigma_{j,2}$ by $\sigma_{j,4}$ (breakpoints), 2) $y_{j,k,1}$ by $y_{j,k,3}$ and $y_{j,k,2}$ by $y_{j,k,4}$ (segments), 3) $\gamma_{j,k}$ by $\delta_{j,k}$ (flow factors), and 4) α_j by β_j (flow production).

Therefore, when flow is delivered along segments $y_{j,k,2}$ or $y_{j,k,4}$, those flows are multiplied by factors $\gamma_{j,k}$ and $\delta_{j,k}$ respectively. Since these factors are between 0 and 1, flow can be partially delivered. On the contrary, flow along segment $y_{j,k,5}$ is always fully transported from node j to node k.

The objective function (eq 1) of the Integer Programming (IP) formulation minimizes the amount of flow reaching the outlet (root node). Flow delivered to the root from its j children (eq 1), through segments $y_{j,root,2}$ and $y_{j,root,4}$, are multiplied by flow factors $\gamma_{j,root}$, and $\delta_{j,root}$ respectively.

minimize:

$$\sum_j (\gamma_{j,root} \cdot y_{j,root,2} + \delta_{j,root} \cdot y_{j,root,4} + y_{j,root,5}) \quad (1)$$

s.t:

$$EA_j = \sum_i (\gamma_{i,j} \cdot y_{i,j,2} + y_{i,j,5} + \delta_{i,j} \cdot y_{i,j,4}) + \alpha_j \cdot (1 - f_j) + (\beta_j \cdot f_j) \quad \forall j \quad (2)$$

$$\begin{array}{lll}
EA_j = y_{j,k,1} + y_{j,k,2} + y_{j,k,3} + y_{j,k,4} + y_{j,k,5} & \forall j,k & (3) \\
y_{j,k,1} \leq \sigma_{1,j} \cdot (1 - f_j) & \forall j,k & (4) \\
y_{j,k,2} \leq (\sigma_{2,j} - \sigma_{1,j}) \cdot (1 - f_j) & \forall j,k & (5) \\
y_{j,k,3} \leq \sigma_{3,j} \cdot f_j & \forall j,k & (6) \\
y_{j,k,4} \leq (\sigma_{4,j} - \sigma_{3,j}) \cdot f_j & \forall j,k & (7) \\
f_j = 0 & \exists j \notin P & (8) \\
\sum_{j=1}^{N} f_j * \nu_j <= \phi & & (9) \\
f_j = 0 & \exists j \in (\nu_j = 0) & (10) \\
f_j \in \{0,1\} & \forall j & (11)
\end{array}$$

Equations 2 and 3 balance input and output sediment flow at node j. In equation 2, flow in node j equals flow coming from its children nodes (i_s) plus flow produced in the node j itself (Effective Accumulation at node j, EA_j). These equations consider that a node can be either reforested ($f_j = 1$, $\delta_{i,j}$, β_i,$y_{i,j,4}$) or not ($1 - f_j = 1$, $\gamma_{i,j}$, α_i,, $y_{i,j,2}$). Although flow larger than 0 can be assigned to segments $y_{i,j,1}$ and $y_{i,j,3}$, these segments are not considered in equation 2 in order to model the problem in such a way that sediment flow in node i is retained when it is lower than $\sigma_{i,1}$ or $\sigma_{i,3}$. On the other hand, equation 3 computes flow delivered from node j to k, where k is the parent node. Even though in this equation all segments are summed, whereas segments $y_{j,k,1}$, $y_{j,k,2}$ and $y_{j,k,5}$ are used (different from 0) when the jth node is not

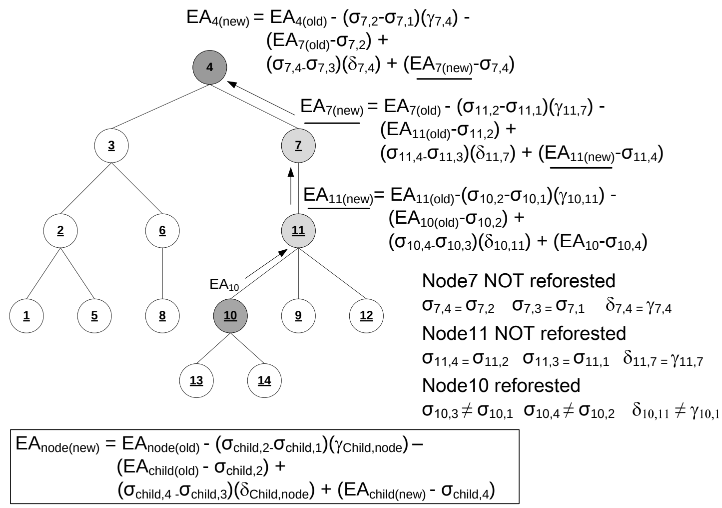

Fig. 2. Effect of reforesting one individual node

reforested, segments $y_{j,k,3}$, $y_{j,k,4}$ and $y_{j,k,5}$ are used when the jth node is reforested.

The amount of flow delivered along each segment is constrained by equations 4 and 5 whether the jth node is not reforested ($1 - f_j = 1$). The same task is performed by equations 6 and 7 in case the jth node is reforested ($f_j = 1$).

Equation 8 assures that the solution is composed by nodes belonging to the set of availability P, where e.g. cells belonging to riverbeds are not in the set P. Whereas the original formulation for sediment flow minimization [1] restricts the number of reforested cells to a given number, equation 9 restricts the number of reforested cells according to the reforesting cost of every selected cell (ν_k) and the total available budget (ϕ).

C. Effect of Reforesting an Individual Node

The tree structure is applied to balance the input and output of every node, and assures the convergence of flow to the root. Under the tree representation each node can reach the root through a unique path. This path implicitly establishes an equation to relate each node with the root. Figure 2 shows that the effect of reforesting node 10 spreads to the nodes in the path reaching the root (nodes 10, 11, 7, 4). The influence of other nodes which do not belong to the path is also taken into account since the input and output flow in every node are controlled by the *mass balance constraints* (equations 2 and 3). Therefore, the relation between every node with the root are implicitly expressed by a sequence of recursive equations (figure 2).

Since the effect of reforesting one node spreads to the root, minimization of sediment load is achieved by reforestation of nodes which contribute with the highest amount of sediment flow to the watershed outlet. Hence, the sediment reduction achieved by every reforested node can be seen as a profit.

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TABLE I
PARAMETERS APPLIED TO TEST THE IP FORMULATION IN TWO WATERSHEDS IN SOUTH DAKOTA

Parameter	Initial Values Not reforested cells	Parameter	Test Values for Reforested Nodes A	B	C	D
Flow Production (α_j)	Erosion	New F. Production (β_j)	Erosion	Erosion	Erosion	Erosion
Flow Factor (γ_j)	Slope	New Flow Factor (δ_j)	0,15	0,15	0,25	0,25
Breakpoint 1 ($\sigma_{1,j}$)	0,5	New Breakpoint 1 ($\sigma_{3,j}$)	1,0	1,5	1,0	1,5
Breakpoint 2 ($\sigma_{2,j}$)	1,0	New Breakpoint 2 ($\sigma_{4,j}$)	2,0	2,5	2,0	2,5

Budget = {10, 30, 50, 70, 90,120, 150}

In addition, the reforestation cost assigned to every node can be seen as a weight. Availability of profits and weights, and the specification of a budget constraint in equation 9 allows understanding the IP formulation in equations 1 to 10 as a 0-1 Knapsack Problem (KP). This problem requires a set of items to be packed in a knapsack of a given capacity (ϕ). Every item (j) is associated with a profit (p_j) and a weight (w_j). The profit sum of the items included in the knapsack needs to be maximized without having the weight sum exceeding ϕ. Many combinatorial problems can be reduced to KP, and the problem arises also as a subproblem in several algorithms of integer linear programming [13]. The 0-1 Knapsack problem searches for optimal solutions when two conflicting and concurrent objectives are present: maximize profit and minimize cost.

IV. Results

Table I summarizes the original data as well as the values assigned to reforested nodes in every watershed. Four sets of test values (A, B, C, D in table I) for reforested nodes are specified. Every test set is evaluated for budgets in the set $X = \{10, 30, 50, 70, 90, 120, 150\}$. Therefore, 28 tests are performed in every watershed. The IP model is implemented by means of Lingo language v7.

Two small watersheds in South Dakota in the USA, made up of 536 (Watershed 1) and 299 (Watershed 2) cells respectively, are used to test the IP formulation for minimizing sediment load at the outlet (root) by means of reforestation under a budget constraint. Raster maps in these regions are composed by cells of $30m$ x $30m$. The following data are taken from the demonstration set accompanying the Grass 6.3.0 windows software: 1) *Flow Direction* (FD), 2) Erosion representing *Flow Production* (α), 3) a normalized slope map for *Flow Factor* (γ), 4) riverbeds for *Streams*, and 5) distances to roads. These are the original maps with information for the starting year (0) when no reforestation is performed.

Table II shows the results obtained in the watershed 1. Left hand side of this table shows the objective values (sediment load at the outlet), computation times, and number of reforested cells obtained with the IP formulation constrained to budgets in the set X. The number of cells identified by means of this model (# *Selected Cells* in table II) is used in a modified formulation which restricts the selection of the cells

TABLE II
RESULTS OBTAINED IN WATERSHED 1

IP Formulation Budget given: Budget	Objective V.	Time [s]	# Selected Cells	IP Formulation # Cells given: Objective V.	Time [s]
Basin 1 $\delta_{i,j}$=0.15 $\forall$i,j $\sigma_{3,j} = 1.0$ $\sigma_{4,j} = 2.0$					
10	1557.63	82	10	1556.71	57
30	1540.37	2064	28	1537.98	52
50	1526.97	484	42	1523.88	51
70	1516.73	***	52	1514.00	52
90	1507.13	***	63	1503.02	52
120	1493.64	3556	76	1490.56	54
150	1481.18	***	88	1478.98	51
Basin 1 $\delta_{i,j}$=0.25 $\forall$i,j $\sigma_{3,j} = 1.0$ $\sigma_{4,j} = 2.0$					
10	1558.63	65	10	1556.21	76
30	1543.13	1653	28	1536.58	56
50	1531.07	333	42	1521.78	60
70	1521.83	***	52	1511.40	50
90	1513.21	***	62	1501.08	52
120	1501.11	2384	76	1486.76	53
150	1489.86	6428	88	1475.59	51
Basin 1 $\delta_{i,j}$=0.15 $\forall$i,j $\sigma_{3,j} = 1.5$ $\sigma_{4,j} = 2.5$					
10	1552.63	97	10	1551.21	86
30	1527.85	265	27	1524.15	92
50	1508.69	585	42	1500.80	60
70	1493.76	***	52	1485.43	69
90	1479.49	4752	62	1470.12	61
120	1459.95	121	75	1450.32	65
150	1441.31	***	87	1432.15	57
Basin 1 $\delta_{i,j}$=0.25 $\forall$i,j $\sigma_{3,j} = 1.5$ $\sigma_{4,j} = 2.5$					
10	1553.63	20	10	1552.21	289
30	1530.30	1550	27	1526.85	257
50	1512.06	311	42	1505.00	341
70	1498.13	***	52	1490.63	311
90	1484.81	4308	61	1477.75	318
120	1466.55	***	75	1457.82	58
150	1449.18	***	87	1440.85	69

*** Process stopped after 3 hours

according to a given number instead to a budget constraint. The results obtained with this formulation are presented in the right hand side of table II. The same configuration is applied in table III in order to present the results of Watershed 2.

From tables II and III it can be noticed that the objective value decreases with increases on the available budget. The *objective values* in tables II and III increase with increases of the *new flow factor* (δ), and when the breakpoints ($\sigma_{3,j}$, $\sigma_{4,j}$) generate narrow segments in the transfer function of reforested cells. High flow factors and narrow segments imply that less sediment is retained in the cell. This behavior is present in the IP formulation constrained either to budget or number of cells.

The sediment load at the watersheds outlet in tables II and III reveal that more flow is retained by means of reforestation of a predefined area (predefined number of cells) rather than reforestation subject to a budget restriction. Moreover, models constrained by a reforestation area require very short computation times. There are computation times in the left hand side of tables II and III which are visibly high, most likely, these times depend on the available budget as well as on the particular data present in the raster maps. In turn this

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TABLE III
RESULTS OBTAINED IN WATERSHED 2

IP Formulation Budget given				IP Formulation # Cells given	
Budget	Objective V.	Time [s]	# Selected Cells	Objective V.	Time [s]
Basin 2 $\delta_{i,j}$=0.15 $\forall$i,j $\sigma_{3,j}=1.0$ $\sigma_{4,j}=2.0$					
10	771.84	108	10	769.80	11
30	753.76	1163	28	748.96	14
50	739.49	2779	42	733.20	11
70	728.07	***	52	722.15	11
90	717.00	***	61	712.32	12
120	700.98	275	76	696.19	12
150	686.00	1255	90	681.48	11
Basin 2 $\delta_{i,j}$=0.25 $\forall$i,j $\sigma_{3,j}=1.0$ $\sigma_{4,j}=2.0$					
10	772.84	40	10	770.73	12
30	756.51	898	28	751.74	11
50	743.55	2291	42	737.35	12
70	733.17	***	52	727.27	13
90	723.08	6876	62	717.34	12
120	708.54	2300	76	703.68	12
150	694.95	3153	91	689.41	12
Basin 2 $\delta_{i,j}$=0.15 $\forall$i,j $\sigma_{3,j}=1.5$ $\sigma_{4,j}=2.5$					
10	766.98	20	10	764.75	10
30	740.89	6381	28	735.22	9
50	720.36	***	41	713.94	12
70	703.97	***	51	697.91	11
90	687.97	***	60	683.64	12
120	664.80	***	76	658.55	11
150	642.63	203	91	635.47	12
Basin 2 $\delta_{i,j}$=0.25 $\forall$i,j $\sigma_{3,j}=1.5$ $\sigma_{4,j}=2.5$					
10	767.98	10	10	765.73	16
30	743.42	347	27	739.29	18
50	724.19	49	41	717.95	12
70	708.88	***	51	702.92	11
90	694.01	***	61	688.07	11
120	671.98	***	76	666.06	12
150	651.15	215	91	644.48	12

*** Process stopped after 3 hours

Fig. 3. Standard deviation and average of the euclidean distance, slope and erosion in the reforested cells in Watershed 1. Budget given.

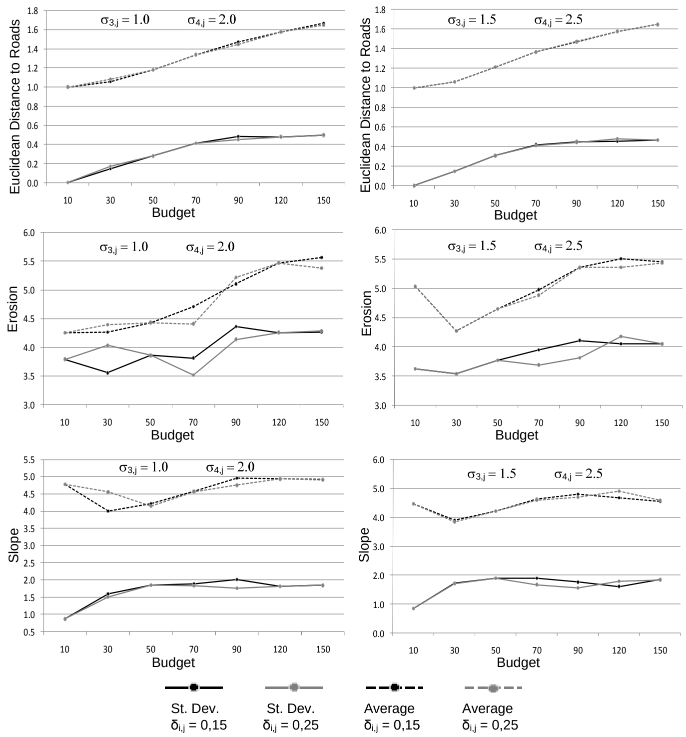
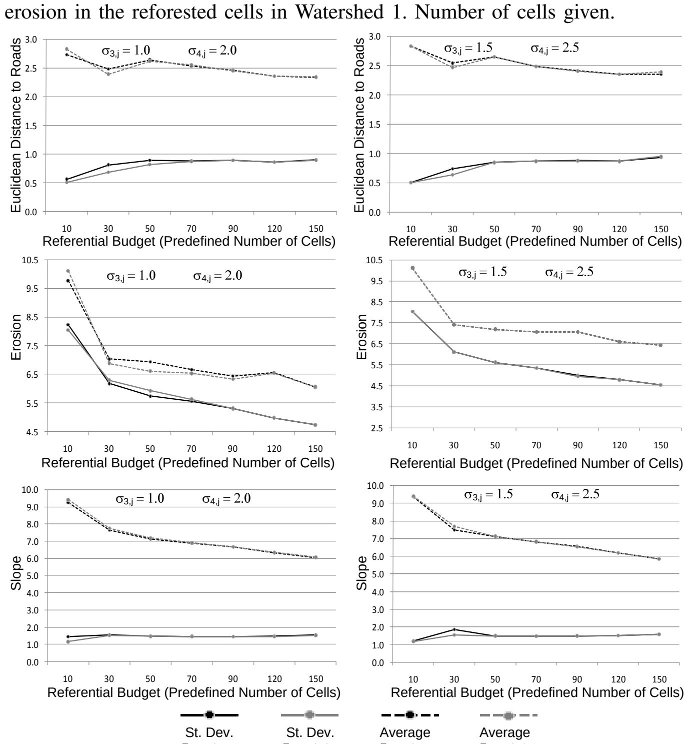


Fig. 4. Standard deviation and average of the euclidean distance, slope and erosion in the reforested cells in Watershed 1. Number of cells given.



determines the number of optimal or near to optimal solutions to be evaluated. This supports the fact that the inclusion of a budget as part of the IP formulation converts it to a knapsack problem, i.e. the problem becomes more complex.

Figures 3 and 4 describe the characteristics of the cells selected to be reforested in the tests in Watershed 1. *Referential budget* in figure 4 refers to *# Selected Cells* in table II. By making use of the standard deviation as well as the average of distance, erosion and slope of the cells selected to be reforested, the figure describes in detail the results in table II. Whereas figure 3 details the results of the IP formulations with a budget constraint, figure 4 encompasses the results for the IP model constrained by a given number of cells. Left hand side of figures 3 and 4 present the results obtained when the sediment delivery function breakpoints $\sigma_{3,j}$ and $\sigma_{4,j}$ for reforested cells are equal 1.0 and 2.0 respectively. Right hand side of figures 3 and 4 make use of values 1.5 and 2.5 as breakpoints. Results obtained in the watershed 2 (table III) show a behavior similar to the presented in figures 3 and 4 for the watershed 1.

Figures 3 present the standard deviations and averages of distance, erosion levels and slope of cells selected to be

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reforested under a budget constraint. That figure shows that distances and erosion levels increases with budget. Although slope is more stable than distances and erosion, low budgets prioritize reforesting cells with high slopes. Since average of erosion increases more quickly than the standard deviation, the reforested cells are forming more homogeneous patches (i.e. with cells with similar values) from the erosion point of view when budget increases (see figure 3). With regard to slope, patches also increase the homogeneity when the budget increases.

When a predefined number of cells is specified in the IP formulation, levels of erosion and slope tend to decrease with increases of the number of cells. This behavior is more notorious in the tests for Watershed 1. In both watersheds, levels of patches homogeneity remains stable from the erosion point of view for different budgets. Since standard deviations of distance and slope tends to unchange while averages decrease, the resulting patches tends to be more heterogeneous from distance and slope point of view when budget increases.

When there are no budget restrictions and a predefined area needs to be reforested, selected cells are those with high erosion levels and facilitating sediment flow under the initial conditions.

V. Conclusions

In the course of this paper an Integer Programming (IP) formulation has been extended for locating optimal sites for minimizing sediment load at a watershed outlet by means of reforestation under a budget constraint. The formulation is based on the general Network Flow (NF) model and simulate spatial interactions for locating cells minimizing sediment flow reaching a watershed outlet.

Inclusion of a budget constraint change the characteristics of the reforested cells compared to the characteristics of the cells selected when the number is known in advance. Whereas budget restrictions lead to situations where cells close to the roads are preferred to be reforested, IP models constrained by the number of cells gives preference to reforest the steepest cells with high levels of erosion production.

Computation times required by the tests allow concluding that the inclusion of a budget constraint makes the IP model more complex. Since section III-C explains that the effect of reforesting one individual cell can be seen as a profit to the outlet cell, the inclusion of a budget and distances to roads representing weights make the problem suitable to be considered as a 0-1 KP. In order to work with larger watersheds than the ones tested in this paper, heuristic solutions methods for 0-1 Knapsack Problems should be explored. The IP formulation in this paper will serve as an optimality reference to evaluate those heuristic methods in small sized problems.

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