

A New Clustering Algorithm On Nominal Data Sets

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Abstract—This paper presents a new clustering technique named as the Olary algorithm, which is suitable to cluster nominal data sets. This algorithm uses a new code with the name of the Olary code to transform nominal attributes into integer ones through a process named as the Olary transformation. The number of integer attributes we get through the Olary transformation is usually different from that of the original nominal attributes. Meanwhile, an extension of the Olary algorithm, which we call the ex-Olary algorithm, is introduced. Furthermore, we provide a useful way to estimate the number of underlying clusters by the use of a new kind of diagram, which is called Number of Clusters versus Distance Diagram (NCDD for short).

Index Terms—Clustering, nominal, the Olary code, the Olary transformation, the ex-Olary algorithm, NCDD

I. INTRODUCTION

Advances in communication systems and high performance computers enable us to easily collect valuable information from the Internet and to construct databases that store huge amount of information [1]. As a result, data mining has becoming more and more popular and important in the past decades. Among various kinds of technologies in the area of data mining, clustering has received considerable attention.

Different from the classification technique, clustering is an un-supervised learning method in the purpose of discovering the underlying patterns and structures of a given data set. Today, there are many kinds of clustering algorithms and there are mainly five categories [2]: hierarchical clustering, partitioned clustering, density-based clustering, grid-based clustering and model-based clustering. Specially, in partitioned clustering, each cluster is represented by its center point.

The application of clustering algorithms to scientific research faces many challenges. First, almost all the algorithms require pre-specified parameters, such as the number of clusters k , a small positive real number δ that is useful when testing the terminal conditions, a positive integer seed and so on. Second, different data sets contain different types of data points as well as different underlying structures. Usually it is not an easy job to select the most suitable clustering algorithm for a given data set without priori knowledge. Third, most clustering algorithms are only suitable for numeric data sets. However, in practical applications, we face a lot of nominal data sets.

In this paper we provide a new clustering algorithm named as the Olary algorithm, which can be used to cluster nominal data sets. Besides, we will discuss a useful method to estimate the number of underlying clusters by the use of the NCDD. The

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remaining parts are arranged as follows. In section two, we will introduce a new code with the name of the Olary code, which is used to transform nominal attributes into binary integer ones. In section three we will discuss how to compute the distance between two data points by the use of the Olary transformation. Section four contains the running process of the Olary algorithm in detail, and section five gives some experiments to show the performance of the Olary algorithm. Section six suggests an extension of the Olary algorithm, which is called the ex-Olary algorithm. What's more, a useful way of estimating the number of underlying clusters by the use of the NCDD will be discussed. Finally, in section seven, we draw conclusions and suggest some possible directions for further researches.

II. THE OLARY CODE

In this section we will introduce a new code named as the Olary code. First, let's see some examples. The 1-length Olary code has two different values, 1 and 0. The 2-length Olary code has three different values, which are 10, 11 and 01. The 3-length Olary code has four different values and they are 101, 100, 110 and 010, respectively.

Now we can present the definition of the Olary code as follows. Assume k is a positive integer ($k=1, 2, 3, \dots$). The k -length Olary code has $k+1$ different values and all the values are binary integer sequences. We can get these values in the following manner. If k is odd, the first value is 1010...101. Else if k is even, the first value is 1010...1010. We can see that the first value contains k binary integer numbers and the first number is always 1. Here 1 is the opposite number of 0 and vice versa. Replacing the k -th number of the first value with its opposite number, we receive the second value of the k -length Olary code. Turning the $(k-1)$ -th number of the second value into its opposite number, we get hold of the third value of the k -length Olary code. In a word, when i is larger than one, we gain the i -th value of the k -length Olary code just by replacing the $(k-i+2)$ -th number of the $(i-1)$ -th value with its opposite number.

The Olary code can be considered as a coding system. The k -length Olary code has $(k+1)$ different values. From the way of getting the $(k+1)$ values of the k -length Olary code, we know there is exactly one different number between the i -th value and the $(i+1)$ -th value ($i=1, \dots, k$). As a special case, turning all the numbers of the first value into their opposite numbers, we get the $(k+1)$ -th value of the k -length Olary code.

III. THE DISTANCE METRIC

Before calculating the distance between two data points, a transforming process must be done. In other words, all the original nominal attributes must be transformed into integer ones by the use of the Olary code. And each integer attribute

takes 1 or 0 as its value. Usually the number of integer attributes is different from that of the original nominal ones. And in some cases it changes greatly. In the following, we will discuss the process of this transformation, which we call the Olary transformation.

We use the (k-1)-length Olary code to transform a nominal attribute with k different values, because the (k-1)-length Olary code has exactly k different values. For example, suppose an attribute with the name of *legsNumber* has four different values and they are *two legs*, *four legs*, *six legs* and *eight legs*, respectively. So we should use the 3-length Olary code to do the transformation. First, randomly push the four different values of the nominal attribute *legsNumber* into a stack named as *stack1*. Second, randomly push the four values of the 3-length Olary code into another stack with the name of *stack2*. Third, pop the top value *v1* of *stack1* and pop the top value *v2* of *stack2*. Use *v2* to substitute *v1*. Repeat the third step until both the two stacks are empty. By now, the Olary transformation on the attribute *legsNumber* is finished. This single nominal attribute has been transformed into three integer ones, each of which takes 1 or 0 as its value.

Now we can give the distance metric used in the Olary algorithm. Consider two data points named as *p1* and *p2*, respectively. Perform the Olary transformation on the two points. Assume *m* is the number of integer attributes each of the two points has after the Olary transformation. Here *p1.value(i)* is used to denote the value of the *i*-th integer attribute of *p1*. And *p2.value(i)* is the value of the *i*-th integer attribute of *p2*. Then we can compute the distance between *p1* and *p2* as follows.

```

for i = 1 to m
    if p1.value(i) ≠ p2.value(i)
        distance = distance + 1;
    end if
end for
return distance;

```

IV. THE OLARY ALGORITHM

In this section we will discuss the Olary algorithm, whose main mechanism is the same as the simple K-means. The two algorithms have different ways of calculating the “distance” between two data points. The simple K-means uses the Euclidean distance as shown in (1). Here *l* is the number of dimensions of *x* and *y*. However, in the Olary algorithm, we choose a new distance metric as discussed in section three. The running process of the Olary algorithm is shown as the following steps.

$$\|x - y\|^2 = \sqrt{\sum_{i=1}^l (x_i - y_i)^2} . \quad (1)$$

choose a new distance metric as discussed in section three. The running process of the Olary algorithm is shown as the following steps.

STEP1. Transform all the data points in the original data set through the process of the Olary transformation. We get hold of a new data set named as *instances*, which contains only binary integer attributes.

STEP2. Initialize relative parameters, including the number of clusters *k* and a positive integer *seed*. What’s more, initialize a third parameter named as *maxIterations*, which is also a positive integer and denotes the maximum number of iterations before the Olary algorithm terminates.

STEP3. Use the function *initCenters(instances, k, seed)* as shown in Fig. 1 to finish the initialization of the *k* center points. Here we use two integer arrays *centers1[k][i]* and *centers2[k][i]* to store the center points.

STEP4. Compute the distances from each point in the data set *instances* to the current *k* center points. If the current point has the smallest distance from the *i*-th center point, then assign it to the *i*-th cluster. Use an array named as *index[i]* to store the clustering indexes of all the points in *instances*.

STEP5. Copy all the elements of the array *centers1[k][i]* into *centers2[k][i]*. Then compute the *k* new center points according to the process *computeCenters(instances, index[], k)* as shown in Fig. 2. Store the *k* new centers into *centers1[k][i]*.

STEP6. If the contents of the two arrays *centers1[k][i]* and *centers2[k][i]* are the same or *maxIterations* is smaller than the current number of iterations, the Olary algorithm stops. Else go back to STEP4 and continue the iteration.

Here we use *N* to denote the number of data points. And *instance(i)* is the *i*-th point. What’s more, *centers1[i]* is the center point of the *i*-th cluster. Assume *m* equals to *seed*. Choose the *m*-th point as the first center point.

```

for i = 2 to k
    maxDistance = 0;
    for j = 1 to N
        if instance(j) has not been chosen as a center point
            currentDistance = 0;
            for s = 1 to i-1
                currentDistance += the distance between
                    instance(j) and centers1[s];
            end for
            if currentDistance > maxDistance
                maxDistance = currentDistance;
                n = j;
            end if
        end for
        choose instance(n) as the i-th center point;
    end for

```

Figure 1. function *initCenters(instances, k, seed)*

V. EXPERIMENTS

In this section we illustrate the performance of the Olary algorithm described in the previous sections. All the data sets used here are from the UCI data set repository.

The first one is the zoo data set, which has 16 attributes, including 15 ones and an integer one. The integer attribute

suggests the number of legs, with three integer values: 0, 2 and 4. So here we consider it as a nominal attribute. We selected 74 data points from the zoo data set to form a new set with the name of zoo1. The zoo1 data set contains three clusters and its underlying structure is shown in Tab. I.

In the Olary algorithm, the random property of the Olary transformation means one nominal data set can be transformed into many different integer ones. Does this property have great influence on the final clustering result? From the results shown in Tab. II we draw the conclusion that the Olary algorithm gains very good results on the zoo1 data set and the random property of the Olary transformation has little influence on the

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Here we assume the number of attributes is m. And we use
instances(s).value(j) to denote the value of the j-th attribute
of the s-th data point. N is the number of data points.

for i = 1 to k (k is the user-specified number of clusters)
  for j = 1 to m
    count0 = 0;
    count1 = 0;
    for s = 1 to N
      if index[s]=i
        if instances(s).value(j)==0
          count0 ++;
        else
          count1 ++;
        end if
      end if
    end for
    if count0 >= count1
      centers1[i][j] = 0;
    else
      centers1[i][j] = 1;
    end for
  end for
end for
    
```

Figure 2. the process *computeCenters(instances, index[], k)*

TABLE I. THE UNDERLYING STRUCTURE OF THE ZOO1 DATA SET

cluster ID	IDs of data points
1	1,2,4,5,6,7,10,11,15,17,20,22,23,24,25, 26,29,30,35,36,37,38,39,40,41,42,49,50, 51,52,53,54,55,56,59,60,65,69,70,72,73
2	3,8,9,13,16,28,32,47,48,58,63,66,68
3	12,14,18,19,21,27,31,33,34, 43,44,45,46,57,61,62,64,67,71,74

final result. What's more, considering the single integer attribute in the zoo1 data set as a nominal one is suitable here.

Here is another question. Can we gain satisfactory clustering results by randomly initializing the center points instead of using the function *initCenters(instances, k, seed)*? In one experiment, we initialized the centers by the use of three randomly

selected points from the data set and the corresponding result is shown in Tab. III. With the help of the underlying structure, we know this result is not acceptable at all. So we draw the conclusion that the function *initCenters(instances, k, seed)* is of great importance to gain good clustering results.

The second data set is the sponge data set, which contains 42 nominal attributes and 3 integer ones. Two of the 3 integer attributes have five values: 0, 1, 2, 3 and 4. And the third one

TABLE II. RESULTS OF DIFFERENT TRANSFORMATIONS

transformation	correct rate
trans I(0→10, 2→11, 4→01)	97.3%
trans I(0→10, 2→01, 4→11)	100%
trans I(0→11, 2→10, 4→01)	100%
trans II(0→10, 2→11, 4→01)	97.3%

TABLE III. RESULT GAINED BY RANDOMLY INITIALIZING CENTERS

cluster ID	IDs of data points
1	2,6,7,10,15,20,22,23,25,26,29,30, 42,51,56,65,69,70,72
2	1,4,5,11,17,24,35,36,37,38,39,40, 41,49,50,52,53,54,55,59,60,73
3	3,8,9,12,13,14,16,18,19,21,27,28,31,32, 33,34,43,44,45,46,47,48,57,58,61,62, 63,64,66,67,68,71,74

TABLE IV. THE UNDERLYING STRUCTURE OF THE SPONGE1 DATA SET

cluster ID	IDs of the data points
1	2,3,4,5,6,7,8,9,38,39
2	1,37,40,45,46,48,49
3	12,13,14,15,16,17,18,19,20,21, 22,23,24,25,29,36
4	10,11,26,27,28,30,31,32,33,34, 35,41,42,43,44,47

TABLE V. RESULTS OF TWO DIFFERENT TRANSFORMATIONS

transformation	correct rate
trans I	89.8%
trans II	93.9%

has four values: 0, 1, 2 and 3. Here we also consider them as nominal attributes. Many of the attributes in the sponge data set have a lot of values. Moreover, this data set has a high dimensionality. So the sponge data set is much more complex than the zoo data set. We selected 49 points to form a new set with the name of sponge1. The underlying clustering structure of sponge1 is shown in Tab. IV. After the Olary transformation, each point has 96 binary integer attributes, while each point has only 45 attributes in the original sponge data set. In other words, the number of attributes changes greatly.

Now we examine whether the random property of the Olary transformation is suitable when using the Olary algorithm to cluster the sponge1 data set. From the results shown in Tab. V we draw the conclusion that the Olary algorithm gains

very good results on the sponge1 data set and the difference between the two results gained on two different integer data sets due to the random property of the Olary transformation is very small, although the sponge1 data set is rather complex. As a result, we draw the conclusion that the random property of the Olary transformation is suitable when using the Olary algorithm to cluster the sponge1 data set. What's more, experiments show that the random property of the Olary transformation is also suitable for transforming the 3 integer attributes, which are considered as nominal ones.

TABLE VI. RESULT GAINED BY RANDOMLY INITIALIZED CENTERS

cluster ID	IDs of data points
1	2,3,4,5,6,7,8,9,10,11,26,27,28,30,31,32,33,34,35,38,39,41,42,43,44,47
2	12,13,14,15,16,18,19,20,22,23,24,25,36
3	29
4	1,17,21,37,40,45,46,48,49

```

for i = 1 to N
    dis = the distance between the i-th point and the
    corresponding center point centers1[index[i]];
    sumDistance = sumDistance + dis;
end for
return sumDistance;
    
```

Figure 3. the calculation of *sumDistance*

TABLE VII. RESULTS OF THE OLARY ALGORITHM ON SPONGE1

IDs of initial center points	correct rate
28,41,1,13	87.8%
27,28,1,17	81.6%
9,17,1,14	83.7%

In one experiment we randomly initialized the center points and the result is shown in Tab. VI. Looking back at the underlying structure of the sponge1 data set in Tab. IV, we come to the point that the clustering result in Tab. VI is not acceptable at all. This reflects the importance of the process *initCenters(instances, k, seed)*, which is used to initialize all the center points when we use the Olary algorithm to cluster a given data set.

VI. ESTIMATE THE NUMBER OF UNDERLYING CLUSTERS

In this section we introduce an extension of the Olary algorithm. Meanwhile, we provide a useful way of estimating the number of underlying clusters by the use of the NCDD. We have discussed some relative theories in [3]. However, the method we introduce here is different from that in [3].

A. the ex-Olary algorithm

We gain the following information after running the Olary algorithm:

- The center points of the resulting clusters are stored in the array *centers1[k][j]*, and here k is the user-specified number of clusters.
- The array named as *index[N]* stores the clustering index of each point, and here N is the number of points in the original data set.

Now we introduce a new parameter named as *sumDistance*, the calculation of which is shown in Fig. 3. We can see the value of *sumDistance* is the sum of the distances from each point to the corresponding center point.

TABLE VIII. VALUES OF SUMDISTANCE

number of clusters	values of Distance	
	zoo1	sponge1
1	348.0	1015.0
2	208.4	560.0
3	110.2	512.2
4	94.5	458.4
5	88.3	430.8
6	82.8	404.5
7	74.9	389.6
8	69.6	371.4
9	63.6	351.4
10	61.5	329.8
11	59.6	314.0
12	57.3	299.4
13	56.0	285.9
14	52.8	275.1
15	50.2	265.8

The value of *sumDistance* changes as the clustering result varies. And we gain different results when the user-specified number of clusters k changes. So the value of *sumDistance* changes as k changes. Furthermore, for a fixed value of k, we can still get different results if the initialization of the k centers differs, although there is usually little difference as long as the results are acceptable. Thus, *sumDistance* usually changes as the initial k center points vary. We introduce an extension of the Olary algorithm as shown in Fig. 4, and we call it the ex-Olary algorithm. From Fig. 4 we know the ex-Olary algorithm tends to gain a global minimum of *sumDistance* for a fixed value of k. Assume that N is the number of points. We can get N values of *sumDistance* by setting the parameter *seed* to be each ID of the N data points, corresponding to N clustering

results. And the smallest one of the N values is the global minimum that the ex-Olary algorithm tries to get. Just randomly choose one result as the final result if the global minimum corresponds to several initializations and clustering results.

Setting k to be 3, we run the ex-Olary algorithm to cluster the zoo1 data set. The data set after the Olary transformation is the same as trans I in Tab. II. Corresponding to the global minimum of *sumDistance*, the correct rate of the clustering result is 97.3%, while the correct rate of the result in Tab. II is also 97.3%. Now let's turn to the sponge1 data set, which is rather complex. The data set used here after the Olary transformation is the same as trans I in Tab. V. We did several experiments on this data set by the use of the Olary algorithm and the results are shown in Tab. VII. The correct clustering rate by the use of the ex-Olary algorithm is 87.8%, which is the same as the best one in Tab. VII. By now, we can draw a conclusion that the clustering result of the ex-Olary algorithm is close to the best result that the Olary algorithm can gain. What's more, the distance metric we use is just to count the number of integer attributes having different values between data points after the Olary transformation. As a result, the running time of the ex-Olary algorithm is usually acceptable.

Here we use N to denote the number of points in the original data set.

```

minValue = 0.0;
minSeed = 0;
for i = 1 to N
    seed = i;
    run the Olary algorithm;
    compute the value of sumDistance as shown in Fig. 3;
    if minValue > sumDistance
        minValue = sumDistance;
        minSeed = i;
    end if
end for

now we know setting seed to be minSeed, we can get the
global minimum of sumDistance by the use of the Olary
algorithm;
```

Figure 4. the ex-Olary algorithm

B. Estimate the Number of Clusters by the Use of the NCDD

In order to estimate the number of underlying clusters in a given data set, we introduce a new variable *Distance*, the calculation of which is shown in Fig. 5. The ex-Olary algorithm tries to get the global minimum of the parameter *sumDistance*. Here the global minimum means the smallest value of the N values of *sumDistance* got by running the Olary algorithm N times, each time setting *seed* to be a different ID of the N points in the data set. However, the value of the new variable *Distance* is the average of the N values of *sumDistance*. The variable *Distance* is a better estimate of the sum of the distances from each point to the corresponding center point than *sumDistance*. Then a new kind of diagram can be drawn. The horizontal axis of the diagram is the number of user-specified

number of clusters, and the vertical axis is the corresponding values of the parameter *Distance*. We call this kind of diagram Number of Clusters versus Distance Diagram, NCDD for short. In the following, we will estimate the number of underlying clusters by the use of NCDD along with similar theories having been discussed in [3].

For both the zoo1 and the sponge1 data sets, the values of *Distance* are shown in Tab. VIII. The NCDD for the zoo1 data set is shown in Fig. 6. Here k is the user-specified number of clusters and m is the number of underlying clusters. When k is smaller than m, at least one resulting cluster contains more than one underlying clusters. The distances between data points from different clusters are usually much larger than those of points from the same cluster. So values of *Distance* are very large. As k increases, resulting clusters consisting of more than one underlying clusters split and the values of *Distance* usually decrease sharply. When k is smaller than 3, the value of *Distance* decreases significantly as k increases in Fig. 6.

When k is larger than m, each resulting cluster contains at most one underlying cluster. So the value of *Distance* is usually small. What's more, some underlying clusters split as k increases. The value of *Distance* decreases more and more slowly as k increases, basing on the fact that distances between data points in the same cluster differ little. In other words, the value of *Distance* tends to converge. As shown in Fig. 6, when k is larger than 3, *Distance* tends to converge as k increases.

Here we use N to denote the number of points in the original data set.

```

for i = 1 to N
    seed = i;
    run the Olary algorithm;
    compute the value of sumDistance as shown in Fig. 3;
    Distance = Distance + sumDistance;
end if
end for
Distance = Distance/N;
return Distance;
```

Figure 5. the calculation of *Distance*

When k equals to m-1, there is exactly one resulting cluster that contains two underlying clusters. As k increases to m, this resulting cluster splits. As a result, the value of *Distance* usually decreases significantly. When k is m+1, one underlying cluster will split into two sub-clusters. The value of the decrease of *Distance* is usually small, because data points in the same cluster are quite similar to each other. As a result, we can draw the conclusion that the value of $|\text{value}(m-1) - \text{value}(m)|$ is usually much larger than that of $|\text{value}(m) - \text{value}(m+1)|$, where $\text{value}(i)$ denotes the value of *Distance* when k equals to i. This is a significant property, which is of great importance when estimating the number of underlying clusters. In Fig. 6, $|\text{value}(2) - \text{value}(3)|$ is much larger than $|\text{value}(3) - \text{value}(4)|$, indicating that the correct number of underlying clusters is 3. Furthermore, a resulting NCDD similar to that in Fig. 6 suggests good clustering results, and vice versa.

Now let's turn to the NCDD of the sponge1 data set shown in Fig. 7. This diagram seems not as good as that in Fig. 6, suggesting that the clustering results are not as satisfactory as those on the zoo1 data set. However, the results are acceptable, and we can estimate the correct number of underlying clusters as follows. When k is smaller than 4, the value of *Distance* decreases sharply as k increases. When k is larger than 4, the *Distance* decreases more slowly as k increases. What's more, the value of $|\text{value}(3) - \text{value}(4)|$ is significantly larger than the value of $|\text{value}(4) - \text{value}(5)|$, which is the important property at the point of k being equal to the correct number of underlying clusters. Basing on these facts, we can draw the conclusion that the number of underlying clusters in the sponge1 data set is 4, which is the correct number in fact.

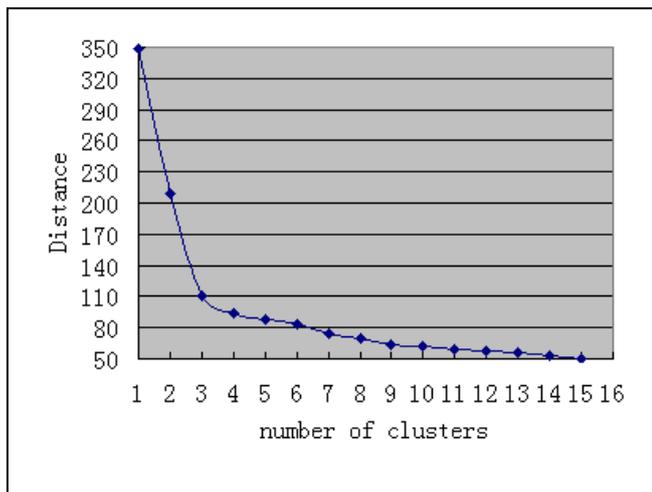


Figure 6. the NCDD of the zoo1 data set

VII. CONCLUSIONS

In this paper we give a discussion of the Olary algorithm, which is suitable to cluster nominal data sets. A new code with the name of the Olary code is introduced. And nominal attributes can be transformed into binary integer ones through the process of the Olary transformation. The number of integer attributes is usually different from that of the nominal ones. For example, the sponge1 data set has 96 integer attributes after the Olary transformation, while the number of the original attributes is only 45. Experiments show that the Olary algorithm has good performances on nominal data sets, including high-dimensional ones such as the sponge1 data set. What's more, the random property of the Olary transformation is suitable.

The ex-Olary algorithm, which is an extension of the Olary algorithm, tries to find the global minimum of *sumDistance* for a fixed value of user-specified number of clusters. As a result, the ex-Olary algorithm usually gains very good clustering results. Because we just have to count the number of attributes with different values when computing the distances between data points, the running time is usually acceptable.

Moreover, we introduce the NCDD, which is used to estimate the number of underlying clusters in a given data set. The value of the parameter *Distance* decreases as the number of clusters increases in the NCDD, as long as the clustering result is acceptable. Assume k is the user-specified number of clusters. And $\text{value}(i)$ denotes the value of *Distance* when k equals to i . Then we can estimate the number of underlying clusters by the use of the NCDD as follows.

Choose x that satisfies the following properties as the number of underlying clusters. When k is smaller than x , *Distance* decreases sharply as k increases. When k is larger than x , the value of *Distance* obviously tends to converge as shown in Fig. 6, as long as the clustering result is satisfactory. If the result is not very good, *Distance* decreases more slowly than it does in the case of k being smaller than x , as shown in Fig. 7. What's more, the value of $|\text{value}(m-1) - \text{value}(m)|$ is much larger than that of $|\text{value}(m) - \text{value}(m+1)|$.

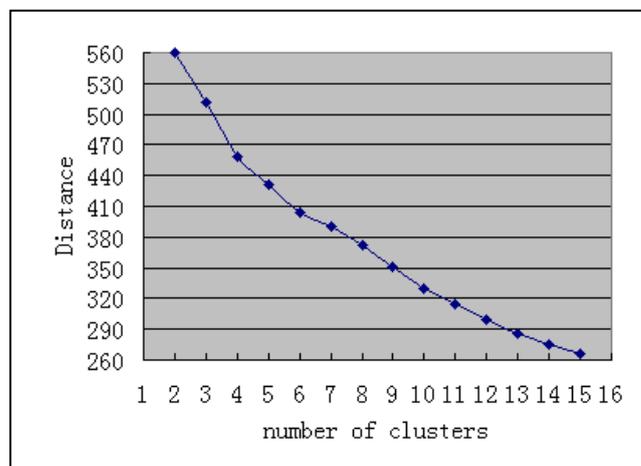


Figure 7. the NCDD of the sponge1 data set

In the future, further researches need to be done. Does any special mathematical theory hide behind the Olary algorithm? Why can the algorithm gain good clustering results by the use of the Olary transformation? Is the parameter with the name of *maxIterations* indispensable? In other words, does the Olary algorithm terminate after finite iterations without the use of this parameter? All these questions can guideline our further researches.

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