The Joint Impact of FBC and CFO in an Asynchronous MC-CDMA System

*Joy Iong-Zong Chen, Pin Xun Lia, Jhih Yuan Wang, Chi Chung Yu

***Abstract*— The factor of CFO (carrier frequency offset) definitely degrades the final results of system performance evaluation for an asynchronous MC-CDMA (multi-carrier coded-division multiple-access) system is one of the most frequently discussed issues in every kind of assuming situations. However, the other one important parameter of FBC (fading branch correlation) is much rare for implying in the study of MC-CDMA system, since they are almost considered as independent each other for the reason of algebra simplification. The issue of exploring aggregately to the matter of which parameter, CFO or FBC, is mainly dominating the system performance of an MC-CDMA system would like to be studied in this paper. The calculation of system performance with BER (bit error rate) for an MC-CDMA system is by simultaneously taking CFO and FBC, which are symbolized as ε and δ, respectively, into account for the investigation in this paper. On the basis of adopting the same quantity to the serve of simulation, that is, values of ε and δ are equally assigned to 0.4, ..., 1.0 for the CFO and FBC, respectively. Moreover, some formulas which function both of the CFO and FBC parameters are provided to this investigation. The simulation results from the evaluation of an MC-CDMA system are illustrated with the CFO and FBC at the same graph for the purpose of comparison.**

***Keywords*: BER;CFO (carrier frequency offset);FBC (fading branch correlation);MC-CDMA system**

I. Introduction

The phenomena of CFO (carrier frequency offset) is mainly caused by the reason of frequency mismatch, which can be caused by Doppler shift due to the vehicle motion or the frequency differences, between the transmitter and receiver oscillator [1]. In multicarrier wireless systems, CFO is going to give rise to ICI (inter-carrier interference) which thereby incurs the degradation of system performance for an MC-CDMA (multi-carrier coded-division multiple-access) system. In the past, there are several researches focus and discuss about the issues of CFO. Most recently, in [2] the multiplexing) system subject to CFO. A pdf (probability density function) of MAI (multiple access interference) plus background noise was not only derived for the asynchronous uplink CDMA system, but it was also extended to examine the impact of the CFO on the BER (bit error rate) performance of an MC-CDMA system over frequency-selective fading channel by the authors of [3]. On the basis of considering the CFO error in different kinds of propagation fading channels, the performance of an OFDM system was evaluated in [4].

Furthermore, in order to maintain high bandwidth efficiency, an MC-CDMA system are subject to correlated fading for different subcarriers because the frequency spacing between adjacent subcarriers of an MC-CDMA system is not enough, thus, usually it is suffering correlated fading among the spatially separated receive antennas. The FBC (fading branch correlation) problem occurred in an MC-CDMA system combining with MRC (maximum ratio combining) scheme has ever been investigated and published in several researches. In [5] the MGF (moment generating function) methods was adopted to analyze the BER of an MC-CDMA system running with the cases of independent and correlated Nakagami-*m* fading. The performance of an MC-CDMA system with MRC diversity working over correlated Nakagami-*m* fading in a multiple-cell environment was calculated in [6]. The rest of the paper is organized as follows. The system of an MC-CDMA system is established in section II. The first, second moment and the system BER are evaluated in section III. In section IV the results are presented. There is a brief conclusion drawn in section V.

II. System Model of an MC-CDMA

On the basis of Fig. 1 the block diagram of an MC-CDMA system, where in there exist *K* simultaneous subscribers with *N* subcarriers within a signal cell, that is, there are *M* orthogonal subcarriers shared by $k+1$ uplink subcarribers. A signature sequence chip with a spreading code, which is assumed has the equivalent spreading factor to the number of subcarriers, of length *L* is applied for BPSK (binary phase shift keying) to modulate each of the subcarriers. The technique mentioned above is the same as the behavior of OFDM (orthogonal frequency division multiplexing) on a direct-sequence spread-spectrum signaling when the frequency of the subcarrier is $1/T_S$ *Hz*, where T_s is the symbol duration. In order to ignore the ISI (inter-symbol interference) between contiguous OFDM symbols, the length of the CP (cyclic prefix) has been considered longer than the maximum access delay time, and assume that a perfect phase correlation can be obtained. Thus, after the zero*th* order interpolation the nondiscrete time signal comes from the parallel-to-serial converter is given as

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Joy I. Z. Chen is with the Department of Electrical Engineering, Dayeh University168 University, Rd., Dasuen, Changhua 51505 Taiwan R.O.C. (corresponding author to provide phone: 86-4-851-1888 ext. 2523； e-mail: jchen@mail.dyu.edu.tw)
Pin Xun Lia is a Master student of the Department of Electrical Engineering, Dayeh University168 University, Rd., Dasuen, Changhua 51505 Taiwan R.O.C.(e-mail: sportsleep@hotmail.com)
Jhih Yuan Wang is a Master student of the Department of Electrical Engineering, Dayeh University168 University, Rd., Dasuen, Changhua 51505 Taiwan R.O.C.(e-mail: tape125@hotmail.com)
Chi Chung Yu is with the Department of Electrical Engineering, Dayeh University168 University, Rd., Dasuen, Changhua 51505 Taiwan R.O.C. (e-mail: yu480720@yahoo.com.tw)

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$$S_0(t)=\sum_{v=0}^{M-1} s_{0,v}\cdot P_{T_C}(t-vT_c)=\frac{1}{M}\sum_{v=0}^{M-1}\sum_{m=0}^{M-1} \tag{1}$$

where $P_{T_C}(t)$ is defined as the unit amplitude pulse over the interval of chip time $[0,\ T_C]$, and

$$s_{0,v}=\frac{1}{M}\sum_{m=0}^{M-1}\xi_{0,m}\cdot\exp\left[j\frac{2\pi mv}{M}\right],\quad v=0,1,...,M-1 \tag{2}$$

where M is the number of subcarrier, and $\xi_{0,m}$ is defined as

$$\xi_{0,m}=b_{0,i}\cdot\alpha_0\cdot C_{0,i,m}\ ,\quad m=0,1,...M\text{ -1} \tag{3}$$

where $b_{0,i}\in[-1,\ 1]$ denotes the data bit of the referenced user during the ith user's signal, α_0 indicates the amplitude of the referenced user, and $C_{0,i,m}$ represents the mth chip of referenced subscriber during the ith bit interval. The subscript i will be omitted here after. Assigning the delay time, τ_R, as the time delay during each user's bit interval for an uplink asynchronous system. The received signal at the receiver input during the bit interval that is impaired by the CFO is written by

$$r(t)=\sum_{k=0}^{K}\gamma_{v,k}\cdot p_{T_C}(t-t_k) \tag{4}$$

where $t_k=vT_c+tk,\ k=0,1,...,K-1,$ and

$$\gamma_{v,k}=\frac{1}{M}\sum_{v=0}^{M-1}\exp\left[j2\pi\frac{mv}{M}\right]\sum_{m=0}^{M-1}\xi_{k,m}\cdot\exp\left[j2\pi\frac{\sqrt{\varepsilon_k}}{M}\right] \tag{5}$$

where $\xi_{k,m}$ is shown in (3), ε_k denotes the normalized frequency offset and it's defined as $\varepsilon_k=f_{k,0}/f_\Delta$, where $f_{k,0}$ is the frequency offset of user k, and $f_\Delta=(MT_C)^{-1}$. When the carrier frequency offset that is involved in the received signal for the referenced subscriber, hence, it can be obtained as the *vth* FFT (fast Fourier transform) input corresponding to the MAI (multiple access interference) without noise and expressed as

$$\begin{aligned}\gamma'_{0,v}&=\int_{vT_c}^{(v+1)T_c}\frac{1}{T_c}r(t)\cdot\rho(t-vT_c)dt,\\&=\frac{1}{M}\sum_k\sum_v\sum_m b_{ki}\cdot\alpha_k\cdot C_{kim}\exp\left[j2\pi\frac{(m+\varepsilon_k)v}{M}\right],\qquad v=0,1,...,M-1\end{aligned} \tag{6}$$

The received signal shown in the previous equation will be passed into the FFT block, then the complex-valued M samples, which are sampled within an OFDM symbol at the time instant $t_n=iT_s/M$, $i=0,1,...M-1,$ of $\gamma'_{0,v}$ accompanied by the AWGN at the output of FFT can be determined as

$$\begin{aligned}\eta_{0,n}&=\gamma'_{0,v}(t_n)=\sum_{v=0}^{M-1}\gamma'_{0,v}\exp(-j\frac{2\pi vn}{M})+N_{AWGN}\\&=\mathrm{D_n}+I_{MAI}+I_{CFO}+N_{AWGN}\end{aligned} \tag{7}$$

where the contribution of AWGN is expressed in the last component by N_n, D_n is the desired signal component of the referenced subscriber, and the I_{MAI} and I_{CFO} denote the MAI and the CFO, respectively. The ICI (inter-carrier interference) caused by the CFO generated at the v-th subscriber of the referenced user can be shown and easily expressed with a geometric series as [8]

$$\begin{aligned}I_{CFO,v}&=\sum_{q=0}^{M-1}\frac{\exp[j2\pi(v+\varepsilon^k)q]}{M}\\&=\frac{\sin[\pi(v+\varepsilon^k)]\cdot\exp[j\pi(M-1)(v+\varepsilon^k)/M]}{M\cdot\sin[\pi(v+\varepsilon^k)]}\end{aligned} \tag{8}$$

In addition to determine the result of the signals come from both the transmitter and the receiver for an MC-CDMA system, now the correlated-Rayleigh channel of the small-term channel is to be shown. Considering the channel is with the multipath delay and the signals of different users are assumed as independent of each other, then the path gain α_i, $i=0,1,...M-1,$ is Rayleigh distributed. The jpdf (joint probability density function) of the assumed correlated channel shown in [9] is adopted and the means to deal with the correlated fading channel in each branch is derived as

$$\begin{aligned}f_{\alpha_0,\cdots,\alpha_{(L-1)}}(\alpha_0,\cdots,\alpha_{(L-1)})&=\prod_{i=1}^{L}(\frac{\alpha_i}{\sigma_i^2})\exp\{-\sum_{i=1}^{L}\frac{\alpha_i^2}{2\sigma_i^2}\}\times\sum_{n=0}^{\infty}\frac{(1/2)_n}{n!}\\&\times\left\{\sum_{i<j}C_{ij}h\left[\frac{\mathrm{L}_g(\alpha_j^{'}/(2\sigma_j^{'2}),1)}{1}\times\frac{\mathrm{L}_g(\alpha_j^{'}/(2\sigma_j^{'2}),1)}{1}\right]+\cdots\right.\\&\left.+C_{12..(2L)}h\left[\frac{\mathrm{L}_g(\alpha_j^{'}/(2\sigma_j^{'2}),1)}{1}\times\frac{\mathrm{L}_g(\alpha_{2L}^{'}/(2\sigma_{2L}^{'2}),1)}{1}\right]\right\}^n\end{aligned} \tag{9}$$

where $\mathrm{L}_g(v,w)$ is the generalized Laguerre polynomial of degree g, defined as [6]

$$\mathrm{L}_g(v,w)=\mathrm{L}_g^{(w-1)}(v)\times g!=\frac{\left(-\frac{d}{dv}\right)^g\left[v^gP(v)\right]}{P(v)} \tag{10}$$

, and $\{\cdots\}^n$ is a symbol of the nth power of a multinomial. Traditionally it exists a little troublesome to directly obtain the jpdf of L corrected variables with Rayleigh distribution instead of the methods with MGF (moment generating function) or CHF (characteristic function).

III. The Calculation of Moments and BER

In order to compute the *SNR* for the signal at the output of the decision maker, the expectation (first moment) of the desired signal and the variance (second moment) of the interference and AWGN should be calculated firstly. The statistical calculation for the MC-CDMA system within single-cell environment will be analyzed below. Thereafter, the error probability with coherent technical demodulation conditioned on the instantaneous *SNR* for an asynchronous MC-CDMA system working in the environment simultaneously accompanying the CFO and FBC is given as [10]

$$P_r(error\,|\,\alpha_l,\ l=0,1,\cdots,L-1)=Q\left(\sqrt{SNR}\right)=Q\left(\sqrt{\frac{(\xi_s)^2}{(\sigma_T^2)}}\right) \tag{11}$$

where $(\xi_s)^2$ and (σ_T^2) present the first moment of the desired signal and the second moment of all the interference, which includes the three terms shown in (7), respectively, and $Q(\cdot)$ is the well-known Macuamm Q-function, which can be alternately expressed as [11]

$$Q(t)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}e^{(\frac{-t^2}{2\sin^2\theta})}d\theta \tag{12}$$

After the desired signal and the total interference are determined, the average error probability for an MC-CDMA system in correlated-Rayleigh fading channels can be accomplished by averaging $P_r(error\,|\,\alpha_l,\ l=0,1,\cdots,L-1)$ over L variates with the jpdf shown in (9), and denoted as

$$\begin{aligned}P_{av}&=\overbrace{\int_0^\infty\int_0^\infty\cdots\int_0^\infty}^{L}P_r(error\,|\,\alpha_l,\ l=0,\ 1,\cdots,\ L-1)\\&\times f_{\alpha_0,\cdots,\alpha_{(L-1)}}(\alpha_0,\cdots,\alpha_{(L-1)})\cdot d\alpha_0 d\alpha_1\cdots d\alpha_{L-1}\end{aligned} \tag{13}$$

Once the jpdf can be obtained, the previous equation involving L-fold integration can be evaluated. Furthermore,

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the second moment of AWGN within cellular environment without any other interfering can be determined as

$$\left(\sigma_{AWGN}^2\right)=\sum_{c=0}^{C-1}\left[\left(\sigma_{AWGN}^2\right)\right]=\frac{MN_{AWGN,\,0}}{4T_b}\Omega_i^{(0)} \tag{14}$$

where C denotes the number of cells, it isn't involved in previous equation, since the macro diversity isn't considered in this paper, and $\Omega_i^{(0)}=E[(\alpha_i^{(0)})^2]$, $i=0,1,...,M-1$, represents the average power of the ith path for the 0th user (referenced user). In order to consider the correlation factor in analysis for an asynchronous MC-CDMA system, the correlation coefficient should be taken into the account of assumption, and it becomes as

$$\begin{aligned}E[(\sum_{l=0}^{L-1}\alpha_l^{(k)})^2]=2N\sum_{l=0}^{L-1}(\Omega_l^{(k)})+\Big[2N(N-1)\Gamma^2(1.5)\\ \times\sum_{i\neq j}(\Omega_i^{(k)})^{1/2}(\Omega_j^{(k)})^{1/2}\,{}_2F_1(-\frac{1}{2},-\frac{1}{2};1;\lambda_{ij})\Big]\end{aligned} \tag{15}$$

where a quasi-Gaussian correlation model of equally spaced linear array with an arbitrary correlation coefficient, λ_{ij}, is adopted here, being given as [12]

$$\lambda_{ij}=\exp[-0.5\eta(i-j)^2(\delta)^2],\ i,j=0,\cdots,L-1 \tag{16}$$

where $\eta\cong 21.4$ is a coefficient which chosen from setting this correlation model equal to the Bessel correlation model [35] with a -3 dB point, and δ is the normalized distance between two neighboring branches, with λ denotes the wavelength of the carrier frequency. The parameter δ is applied to determine the threshold level of correlation. The assigned values of δ are arranged in the interval (0.4, 1), in which $\delta=0.4$ and $\delta=1$ represent two extreme conditions, *i.e.*, fully correlated and uncorrelated branches, respectively.

For all K interfering users the amount of AWGN including the CFO component accompanies with all the subcarrier can be calculated as $I_{AWGN}=\sum_{k=0}^{K-1}\mathrm{I}_{CFO}^2\cdot\left(\sigma_{AWGN}^2\right)$, where $\mathrm{I}_{CFO}^2=\sum_{g=0}^{M-1}(I_{CFO,g})^2$. Next, the second moment of the MAI plus the ICI for the referenced user caused by the other interfering user conditioned on the fading gain over the frequency selective fading channel, which is with correlated Rayleigh distributed, can be obtained as

$$\sigma_{MAI.ICI}^2=\frac{1}{3n}\sum_{k=0}^{K}\left(\sum_{g=0}^{M}\mathrm{I}_{CFO,g}^2\cdot(\alpha_g^{(k)})^2\Omega_i^{(k)}-(\alpha_0^{(0)})^2\Omega_0^{(0)}I_{CFO,0}\right) \tag{17}$$

where $\alpha_g^{(k)}$ is one of the fading gain of the referenced user. Thus, the total second moment, σ_T^2, can be obtained by combining (14) and (15) and expressed as

$$\begin{aligned}\sigma_T^2=\frac{1}{3n}\Bigg\{\sum_{k=0}^{K}\left(\sum_{g=0}^{M}\mathrm{I}_{CFO,\,g}^2\cdot(\alpha^k)^2\Omega_n^{(k)}-(\alpha^0)^2\Omega_0^{(0)}I_{CFO,\,0}\right)\\ +\sum_{g=0}^{M-1}\mathrm{I}_{CFO,\,g}^2\cdot\left(\frac{MN_0}{4T_b}\Omega_n^{(0)}\right)\Bigg\}\end{aligned} \tag{18}$$

The average BER of an MC-CDMA system with both CFO and FBC operating can be derived by combining (13) and (16) after the second moment of the total interference arrived at, and where the derivative of signal power is not going to be shown in the paper. Based on the results, the phenomenon of correlation between different branches will be illustrated by means of numerical analysis.

IV. Results

The previously derived results of system performance for an MC-CDMA system with simultaneously including CFO and FBC are numerically evaluated in this section. There is a 3-dimension diagram shown in Fig. 2, where the vertical Axis represents the BER, while the other two Axis (called as X-Axis and Y-Axis) corresponding to express different CFO and FBC value. Both the values of CFO and FBC are assumed and assigned in the interval of (0.4, 1), that is $0.4\le(\varepsilon,\delta)\le 1$. The system BER performance of an MC-CDMA system will stay at stable status after some fixed values of CFO and FBC. For instance, in case the E_b/N_0 and subcarrier number are considered as $5\,dB$ and $M=64$, respectively, the system BER will stay at about 10^{-5} after the value of CFO arrives at $\varepsilon=0.42$ and whatever of FBC values are. It is known that the larger value is for the CFO, the more degradation of the system performance is. However, the affect of FBC to system performance is not same as that of CFO, *i.e.*, the system performance of an MC-CDMA will become superior when the value of FBC increases, since the FBC is determined by the distance between the correlated branches. Moreover, the other important viewpoint worth to note is the comparison between the results from the effect of CFO and FBC. It is easy to seize the fact that the system BER will stay at about 10^{-1} and 10^{-2} corresponding to the values of CFO and FBC are set as the same 0.66. Thus, the system performance of an MC-CDMA system is definitely deeply dominated by the factor of CFO. The mentioned facts can also be understood in the compared results come from the Fig. 3 and Fig. 4 where the plane of BER to the parameters of CFO and FBC are represented respectively. Furthermore, the results from numerical calculation of subcarrier with different values of, $M=64$, 512, and 1024 are illustrated as different layers in Fig. 2. Hence, the larger number of subcarrier, the better system performance of multi-carrier transmission systems is."

V. Conclusion s

On the basis of considering fading channel and including the variate of the local oscillator, to the best of author knowledge, it is almost few of publications to explore the system performance of an MC-CDMA system with the parameters FBC and CFO simultaneously. In this paper both of FBC and CFO are adopted as the parameters for evaluation the BER performance of an MC-CDMA system. Some results from figuring out the two factors, FBC and CFO, are shown in 3-dim curves at the same time. It is easily to realize the fact that the system performance of an MC-CDMA system is still mainly dominated by the effect of CFO. It is known that the solution of solving CFO and FBC has been proposed by some publications, there are still many spaces for being able to be investigated. In the future, the authors are keeping in exploring and developing the maximal performance algorithms for solving the critical problems.

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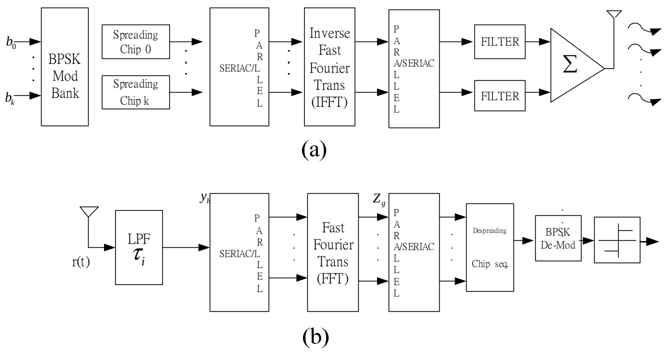

Fig. 1 The system block diagram of an MC-CDMA system, (a) The transmitter, (b) The receiver

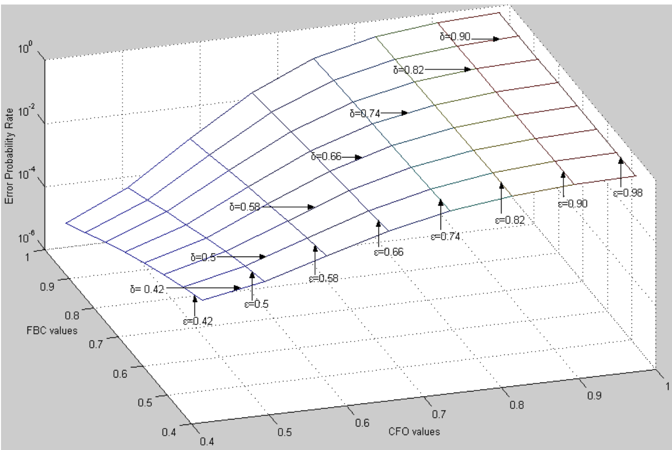

Fig. 2 The plots of BER vs both different CFO (ε) and FBC (δ) values

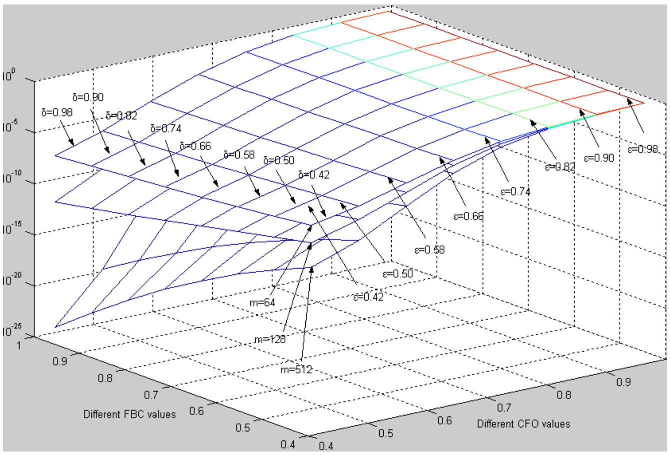

Fig. 3 The plots of BER corresponding to different values of CFO (ε), FBC (δ), and fading parameters

VI. Conclusion

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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