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Voronoi Based Coverage Control for Nonholonomic Mobile Robots

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Abstract—**In this paper we deal with the problem of covering an environment using a group of mobile robots with nonholonomic kinematic and dynamic constraints. In comparison with standard coverage control procedures, we develop a combined controller for Voronoi-based coverage approach in which kinematic and dynamic constraints of the actual mobile sensing robots are incorporated in the controller design. The stability of the entire systems is guaranteed using Lyapunov stability theory, and numerical simulations are provided approving the effectiveness of the proposed method.**

Index Terms—**Distributed coverage control, Sensor networks, Nonholonomic mobile robots, Voronoi tessellation.**

I. Introduction

NOWADAYS, sensor network has a broad application in environmental sampling, ecosystem monitoring, and military surveillance. The coverage problem is a fundamental issue of a sensor network system. Researchers have proposed various solutions to a lot of interesting sensor network coverage problems. In [1] coverage controllers are categorized in three common kinds, a Voronoi controller, which is geometric in nature, a minimum variance controller, which has a probabilistic interpretation, and a potential field controller.

Cortes et. al. [2] proposes a decentralized control law for multi-robot coverage of an area partitioned into Voronoi diagram, in the sense that continually driving the robots toward the centroids of their Voronoi cells. A recent text that presents much of this work in a cohesive fashion is [3] and an excellent overview is given in [4]. Different extensions of the framework devised in [2] have been proposed in the literature. In [5] the problem of limited-range interaction between agents was addressed. In [6] the basic approach was extended to deal with the agents with limited energy. The problem of the online learning of the distribution density function, while moving toward the optimal locations, was addressed in [7], [8]. In [1] the authors propose a cost function form for coverage problems that can be specialized to fit different distributed sensing and actuation scenarios. The cost function is shown to subsume several different kinds of existing coverage cost functions. There has been another extension to heterogeneous groups of finite size robots and non-convex environments in[9].

There are also a number of other notions of multi-robot sensory coverage (e.g.[10,11,12,13]). Nevertheless, in this paper, we use the notion of an optimal sensing configuration developed by [2].

Standard approaches to Voronoi based coverage control assume simple integrator dynamics for the robots, yielding the ability of traversing both smooth and non-smooth trajectories for robots. They do not address the kinematic and dynamic constraints of physical nonholonomic mobile robots in developing coverage algorithms. However, most of the actual robots such as differential drive ones suffer from kinematic nonholonomic constraints confining the plausible motions of the robot. Once an appropriate feedback velocity control inputs are designed for kinematic steering system, one should take into account the specific dynamic vehicles to convert a steering system command into control inputs for the actual vehicle [14].

Stabilization and tracking control of nonholonomic mobile robots has been a subject of intense research in the past years [15,16,17]. Many approaches have been proposed to address this issue of nonholonomic stabilization. As pointed out by Kim and Tsiotras [18], the majority of nonholonomic control laws are based on kinematic models [19,20,21]. Stabilization of dynamic models for nonholonomic systems has also been addressed in [22,23,24,25,26]. A review on some of the existing results addressing tracking problems for nonholonomic systems are reported in[27] .

A popular way of implementing a kinematic control law to a dynamic nonholonomic system is by backstepping [28] the velocity control commands to acceleration input. Backstepping has been used in translating kinematic controllers into equivalent dynamic ones in [14,29,30].

In this paper we extend the contributions in kinematic and dynamic control of single nonholonomic mobile robots to the Voronoi-based locational optimization framework introduced in [2], and propose a control law with the aim of coverage control problem. After including a kinematic velocity controller in the coverage problem, we seek to incorporate the dynamics of the robots into the coverage controller design based on the backstepping approach of [14].Using Lyapunov stability theory, we prove that the control law causes the network to converge to a near optimal sensing configuration.

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The remainder of the paper is organized as follows. We describe problem setup along with some background on nonholonomic mobile robots and locational optimization problem in section II. In section III we present the proposed controller and prove its stability. Numerical simulation results are described in section IV. Finally, we conclude the paper in Section V.

II. Problem Setup

Consider we want to deploy a group of N nonholonomic mobile robots in a bounded, convex environment $D \subset \mathbb{R}^2$. In the following, we first describe the characteristics of the sensing mobile robots, and then depict the Voronoi based coverage approach with some background on locational optimization problem.

A. Nonholonomic Mobile Robots

Let each of the robots be a two-wheeled mobile robot moving on a horizontal plane as shown in Fig. 1. Let $q_i \in Q$ be the configuration of the $i'th$ robot described by generalized coordinates in the global frame as

$$q_i = [p_i^T \quad \theta_i]^T \tag{1}$$

where $p_i = (x_i, y_i) \in \mathbb{R}^2$ is the position of the point C_i of the $i'th$ robot in the global coordinate frame {O, X, Y} and $\theta_i \in (-\pi, \pi]$ is the orientation of that, measured from X-axis of that frame. The vehicle is subjected to an independent velocity constraint of the form

$$A^T(q_i)\dot{q}_i = [\sin(\theta_i) \quad -\cos(\theta_i) \quad 0]\dot{q}_i = 0 \tag{2}$$

Defining a full rank matrix $S(q)$, such that $A^T(q_i)S(q) = 0$, a vector $\mathbf{v}$ exists satisfying

$$\dot{q}_i = S(q_i)\mathbf{v}_i \tag{3}$$

where, $\mathbf{v}_i = \begin{bmatrix} v_i & \omega_i \end{bmatrix}^T$ with $\dot{\theta}_i = \omega_i$ (the angular velocity) and $v_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2}$ (the linear velocity) of the $i'th$ robot. It is easy to verify that the kinematic equations of motion (3) of point C_i in terms of its linear velocity and angular velocity are[31]

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{4}$$

The Lagrange formalism is used to find the dynamic equations of the mobile robots. The dynamical equations of an n-dimensional mobile robot can be expressed in the matrix form [31]:

$$\begin{aligned} M_i(q_i)\dot{v} + V_{m_i}(q_i, \dot{q}_i)\mathbf{v} + F_i(\dot{q}_i) + G_i(q_i) \\ = B_i(q_i)\boldsymbol{\tau}_i - S^T(q_i)\lambda \end{aligned} \tag{5}$$

where $M(q_i) \in \Re^{n \times n}$ is a symmetric, positive definite

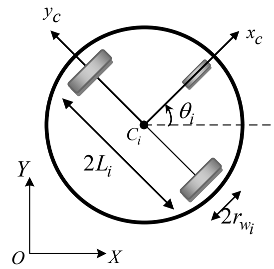

Fig. 1. Nonholonomic mobile robot configuration

inertia matrix, $V_m(q_i, \dot{q}_i) \in \Re^{n \times n}$ is the centripetal and Coriolis matrix, $F(\dot{q}_i) \in \Re^{n \times 1}$ denotes the surface friction, $G(q) \in \Re^{n \times 1}$ is the gravitational vector, $B(q_i) \in \Re^{n \times 1}$ is the input transformation matrix, $\boldsymbol{\tau}_i \in \Re^{n \times 1}$ is the input vector, and $\lambda_i \in \Re^{m \times n}$ is the vector of constraint forces.

One can rewrite the kinematic and dynamic equations of the mobile base (i.e. (5), (6)) by differentiating (5), substituting the result in (6) and multiplying by S^T as [14]:

$$\bar{M}_i(q_i)\dot{\mathbf{v}}_i + \bar{V}_{m_i}(q_i, \dot{q}_i)\mathbf{v}_i + \bar{F}_i(\dot{q}_i) + \bar{\tau}_d = \bar{B}_i \boldsymbol{\tau}_i \tag{6}$$

$$\bar{\boldsymbol{\tau}}_i = \bar{B}_i \boldsymbol{\tau}_i \tag{7}$$

the parameters of which for the mobile base in figure 1 can be obtained as [14]:

$$\bar{M}_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & I_i \end{bmatrix}, \quad \bar{B}_i = \frac{1}{r_{wi}} \begin{bmatrix} 1 & 1 \\ L_i & -L_i \end{bmatrix} \begin{bmatrix} \tau_{R_i} \\ \tau_{L_i} \end{bmatrix},$$

$$\bar{V}_{m_i} = 0, \tag{8}$$

in which m and I represent the mass and inertia of the robot i, and L_i and r_{wi} are shown in figure 1. It should be underlined that the matrix $\dot{\bar{M}}_i - 2\bar{V}_{m_i}$ has a skew-symmetric property[14].

Once the desired velocity control inputs for the kinematic model, denoted by $\mathbf{v}_{d_i}$, are obtained, one should convert $\mathbf{v}_{d_i}$ to the control torque inputs $\boldsymbol{\tau}_i$ in order to incorporate the dynamics of the physical mobile robot platforms.

B. Locational Optimization

In the following, we state some basic definitions and results from locational optimization that will be useful in this work. More thorough discussions were given by[2] and [8].

Let an arbitrary point in D be denoted by $\tilde{p}$. Let $\{V_1, ..., V_N\}$ be the Voronoi partition of D, for which the robots' positions are the generator points. Specifically,

$$V_i = \{\tilde{p} \in D \mid \|\tilde{p} - p_i\| \le \|\tilde{p} - p_i\|, \forall j \ne i\} \tag{10}$$

Let the unreliability of the sensor measurement be denoted by a quadratic function $f(x)$ specifically, $f(\|\tilde{p} - p_i\|) = 1/2(\|\tilde{p} - p_i\|)^2$ describes how unreliable is the measurement of the information at $\tilde{q}$ by a sensor at p_i. This

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form of $f(\|\tilde{p}-p_i\|)$ is physically appealing since it is reasonable that sensing will become more unreliable farther from the sensor [32].

Define the sensory function to be a continuous function $\phi: D \rightarrow R^+$ (where R^+ is the set of strictly positive real numbers). The sensory function should be thought of as a weighting of importance over D. We want to have many robots where $\phi(\tilde{p})$ is large, and few where it is small. Here, we assume that $\phi(\tilde{p})$ is known by the robots in the network.

As a measure of the system performance, we define the coverage functional as follows

$$\mathcal{H}(p_1, \ldots, p_N) = \sum_{i=1}^{N} \int_{V_i} \frac{1}{2} (\|\tilde{p}-p_i\|)^2 \phi(\tilde{p}) d\tilde{p} \tag{11}$$

The mass, first moment, and centroid of a Voronoi region V_i are defined as [2]

$$M_{V_i} = \int_{V_i} \phi(\tilde{p}) d\tilde{p}, \quad L_{V_i} = \int_{V_i} \tilde{p}\, \phi(\tilde{p})\, d\tilde{p}$$

$$C_{V_i} = \frac{L_{V_i}}{M_{V_i}} \tag{12}$$

respectively. Thus, M_{V_i} and C_{V_i} have properties intrinsic to physical masses and centroids. The moments and the Voronoi regions themselves depend on the robot positions. Remarkably, despite this dependency, a standard result from locational optimization [2] is that

$$\frac{\partial \mathcal{H}}{\partial p_i} = -\int_{V_i} (\tilde{p}-p_i) \phi(\tilde{q})\, d\tilde{q} = -M_{V_i} \left(C_{V_i} - p_i \right) \tag{13}$$

Equation (13) implies that critical points of $\mathcal{H}$ correspond to the configurations such that $p_i = C_{V_i}$ for all i, that is, each agent is located at the centroid of its Voronoi region. This brings us to the concept of optimal coverage as follows: A robot network is said to be in a (locally) optimal coverage configuration if every robot is positioned at the centroid of its Voronoi region, $p_i = C_{V_i}$ for all i, which is called *centroidal Voronoi configuration* [2,7,32].

III. Coverage With Nonholonomic Mobile Robots

In this section we investigate the coverage control for a group of nonholonomic mobile robots. Before we proceed, let suppose that the following assumptions hold:

Assumption 1: Every robot has complete knowledge of its own dynamics.

Assumption 2: The robots have the ability to compute their own Voronoi partitions in a distributed manner [2].

Assumption 3: The robots work in such conditions that the torque disturbances can be neglected, that is $\bar{\tau}_{d_i} = 0, \forall i.$

A. Kinematic control

In order to design a kinematic controller, we first need to presume the following assumption, which will be relaxed in section (III-B).

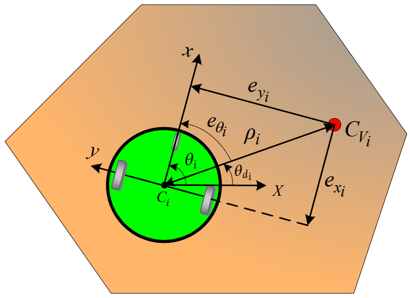

Fig. 2. Position and orientation errors in partition i

Assumption 4: perfect velocity tracking holds such that $\mathbf{v}_i = \mathbf{v}_{d_i}, \forall i \in \mathcal{N}$.

Define position errors for the $i'th$ robot as

$$x_{e_i} = x_i - C_{V_i,x}$$
$$y_{e_i} = y_i - C_{V_i,y}, \tag{14}$$

the desired orientation of motion for $(x_{e_i}, y_{e_i}) \neq (0,0)$ as

$$\theta_{d_i} = \text{Atan2}(-y_{e_i}, -x_{e_i}), \tag{15}$$

and the orientation error as

$$e_{\theta_i} = \theta_i - \theta_{d_i}. \tag{16}$$

The kinematic error dynamics can be written independent of the inertial coordinate frame by Kanayama transformation [33]:

$$\begin{bmatrix} e_{x_i} \\ e_{y_i} \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) \\ -\sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \begin{bmatrix} x_{e_i} \\ y_{e_i} \end{bmatrix} \tag{17}$$

where e_{x_i} and e_{y_i} are the error variables in mobile coordinate system which is attached to the $i'th$ robot.

We propose the following auxiliary velocity control law for the $i'th$ robot:

$$v_{d_i} = -k_{v_i} \rho_i \cos(e_{\theta_i}) \tag{18.a}$$

$$\omega_{d_i} = -e_{\theta_i} \tag{18.b}$$

where $\rho_i = \sqrt{e_{x_i}{}^2 + e_{y_i}{}^2}$.

Theorem 1: Consider a group of N nonholonomic mobile robots whose kinematic models are described through (4). Let the assumptions (1) through (4) hold. Under control law (18), it is guaranteed that the whole system is asymptotically stable and the robots positions converge to a centroidal Voronoi configuration.

Proof: Consider the Lyapunov function candidate as $V = \mathcal{H}$. The time derivative of V along the trajectories of the error dynamics then can be obtained as follows

$$\dot{\mathcal{H}} = \sum_{i=1}^{N} \frac{\partial \mathcal{H}}{\partial q_i} \dot{q}_i$$

Using (13) and the fact that $\dfrac{\partial \mathcal{H}}{\partial \theta_i} = 0$

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$$\dot{\mathcal{H}} = \sum_{i=1}^{N} \frac{\partial \mathcal{H}}{\partial p_i} \dot{p}_i = \sum_{i=1}^{N} M_{V_i} (p_i - C_{V_i}) \dot{p}_i$$
$$= \sum_{i=1}^{N} M_{V_i} v_i [x_{e_i} \quad y_{e_i}].[\cos(\theta_i) \quad \sin(\theta_i)]^T$$
$$= -\sum_{i=1}^{N} k_{v_i} \rho_i \cos(e_{\theta_i}) e_{x_i}$$

Using the fact that $\cos(e_{\theta_i}) = \frac{e_{x_i}}{\rho_i}$ one can conclude that

$$\dot{\mathcal{H}} = -\sum_{i=1}^{N} k_{v_i} \rho^2{}_i \cos^2(e_{\theta_i}) \quad (19)$$

which is clearly non-positive. Due to the convexity of the region D, one can conclude that each of the Voronoi centroids C_{V_i} lies in the interior of the $i'th$ partition and so in the interior of the region D. Looking at the control law (18.a) one can easily see that it provides the robots with bidirectional linear velocities the sign of which depends on the value of $\cos(e_{\theta_i})$. One can then realize that the robots always move toward the interior of the region D and never leave it even if their initial local x-axis (x_c) are outward it. Therefore, D is a positive invariant set for the trajectories of the closed loop system. Since this set is closed and bounded, one can make use of LaSalle's invariance principle to infer that the robots positions converge to the largest invariant subset of the set $S = \{(\rho_i = 0) \vee (\cos e_{\theta_i} = 0), \forall i \in \mathcal{N}\}$. For each robot, in the case that $\cos e_{\theta_i} = 0$ and e_{x_i} or $e_{y_i} \neq 0$, according to (18.b) $|\omega_i| = \pi/2$, so the set $\{\cos e_{\theta_i} = 0\}$ is a non-invariant set except the case that the $i'th$ robot is located on the centroid of its Voronoi partition. On the other hand, $\rho_i = 0$ only if both e_{x_i} and e_{y_i} are equal to zero. Therefore, the largest invariant set contained in S is the set $\Omega = \{e_{x_i} = e_{y_i} = 0, \forall i \in \mathcal{N}\}$. Moreover, for every invariant set in Ω, it should be $\omega_i = 0$ which yields $\theta_i = 0$. Therefore, under control law (18), the closed loop system is asymptotically stable and the robots positions converge to a set of centeroidal Voronoi configuration. □

Remark 1: Convergence of θ_i to θ_{d_i} can be made arbitrarily exponentially fast by the selection of [34]:

$$w_i = -k_{w_i} e_{\theta_i} + \dot{\theta}_{d_i} \quad (20)$$

where

$$\theta_{d_i} = \frac{e_{x_i} \dot{e}_{y_i} - \dot{e}_{x_i} e_{y_i}}{\rho^2}, \quad (21)$$

which results in

$$\dot{\theta}_i - \dot{\theta}_{d_i} = -k_{\omega_i} (\theta_i - \theta_{d_i}). \quad (22)$$

One can also make use of a sufficiently smooth estimate of $\dot{\theta}_{d_i}$, namely $\hat{\dot{\theta}}_{d_i}$, which can be computed using the following estimations of $\dot{e}_{x_i}$ and $\dot{e}_{y_i}$:

$$\hat{\dot{e}}_{x_i} = \frac{e_{x_i}(t + \Delta t) - e_{x_i}(t)}{\Delta t} \quad (23.a)$$

$$\hat{\dot{e}}_{y_i} = \frac{e_{y_i}(t + \Delta t) - e_{y_i}(t)}{\Delta t} \quad (23.b)$$

for some small $\Delta t > 0$ [27].

Remark2: The controller (18) may drive the robot i to a singular configuration in which $\cos(e_{\theta_i}) = 0$. This condition can occur in two cases. The first one does when $e_{x_i} = e_{y_i} = 0$, which can be conducted with zero control inputs. The second occurs when $e_{x_i} = 0$ and $e_{y_i} \neq 0$, introducing a $\pi/2$ radian turn with respect to the current orientation of the robot, which violates from the nonholonomic constraint. One way to deal with this problem is to assume that the desired positions are located in such a way that they do not introduce such sharp turn. This assumption can hold if the robots initiate with some considerations on their initial orientations, which we do not investigate in this paper. But even if this assumption is not satisfied, each robot can modify the desired orientation so that θ_{d_i} is replaced with the following perturbed version [27]:

$$\tilde{\theta}_{d_i} = \theta_{d_i} + \tilde{\varepsilon}_{\theta_i} \quad (24)$$

where $\tilde{\varepsilon}_{\theta_i} \geq 0$ is some small perturbation value. This condition guarantees that the system avoids singularities.

Remark3: Here we assume that the dimensions of the robots and the area being covered are selected such that the robots do not collide with each other. Note that once the dimension of the robots is selected small enough with respect to that of the region D, the robots do not collide according to the fact that they all move toward their centroidal positions of their Voronoi partitions. We will discuss the collision avoidance problem in detail in our future work.

B. *Dynamic control*

Now we consider the case that the perfect velocity tracking assumption does not hold. Considering $\mathbf{u}_i$ as an auxiliary input, a suitable control input for velocity following is given by the computed-torque nonlinear feedback control input [14]

$$\bar{\boldsymbol{\tau}}_i = \bar{B}_i^{-1}(\bar{M}_i(q_i)\mathbf{u}_i + \bar{V}_{m_i}(q_i, \dot{q}_i)\mathbf{v}_i + \bar{F}_i(\mathbf{v}_i)),$$

which converts the dynamic control problem into:

$$\dot{q}_i = S(q_i)\mathbf{v}_i \quad (25.a)$$

$$\dot{\mathbf{v}}_i = \mathbf{u}_i. \quad (25.b)$$

One can define the auxiliary velocity error as:

$$e_{d_i} = \mathbf{v}_{d_i} - \mathbf{v}_i \quad (26)$$

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$$e_{d_i} = \begin{bmatrix} v_i + k_{v_i}\rho_i \cos(e_{\theta_i}) \\ w_i + k_{w_i} e_{\theta_i} \end{bmatrix}$$

Differentiating (26) and using (6) and (7), one can write the mobile robots dynamics in terms of velocity tracking error and its derivative:

$$\bar{M}_i(q_i)\dot{e}_{d_i} = -\bar{V}_{m_i}(q_i,\dot{q}_i)e_{d_i} - \bar{\boldsymbol{\tau}}_i + f_i(\mathbf{x}_i) \quad (27)$$

where

$$f_i(\mathbf{x}_i) \triangleq \bar{M}_i(q_i)\dot{\mathbf{v}}_{d_i} + \bar{V}_{m_i}(q_i,\dot{q}_i)\mathbf{v}_{d_i} + \bar{F}_i(\dot{q}_i) \quad (28)$$

is the nonlinear mobile robot function and the vector $\mathbf{x}_i$ is defined as $\mathbf{x}_i = [\mathbf{v}_i^T \ \mathbf{v}_{d_i}^T \ \dot{\mathbf{v}}_{d_i}^T]^T$. Proposing the auxiliary nonlinear control input $\mathbf{u}_i$ to be [15]

$$\mathbf{u}_i = \dot{\mathbf{v}}_{d_i} + Ke_{d_i} \quad (29)$$

where K is a positive definite and diagonal matrix defined by $K = kI_2$, one can obtain the following torque input for the $i'th$ robot

$$\boldsymbol{\tau}_i = \bar{B}_i^{-1}(\bar{M}_i(q_i)Ke_{d_i} + f_i(x_i)). \quad (30)$$

Theorem 2: Consider a group of N nonholonomic mobile robots whose kinematic and dynamic models are described through (4), (6) and (7). Let the assumptions (1) through (3) hold. Under control laws (18), (28) and (30), it is guaranteed that the whole system is asymptotically stable and the robots positions converge to a centroidal Voronoi configuration.

Proof: Pick the candidate Lyapunov function as

$$V = \mathcal{H} + \frac{1}{2}e^T{}_{d_i}\bar{M}_i e_{d_i} \quad (31)$$

Differentiating V results in

$$\dot{V} = \dot{\mathcal{H}} + \frac{1}{2}e^T{}_{d_i}\dot{\bar{M}}_i e_{d_i} + e^T{}_{d_i}\bar{M}_i\dot{e}_{d_i}$$

$\dot{\mathcal{H}}$ is shown to be non-positive in (19). Substituting (30) into (27) results in the closed loop error dynamics as

$$\bar{M}_i(q_i)\dot{e}_{d_i} = -(\bar{M}_i(q_i)K + \bar{V}_{m_i}(q_i,\dot{q}_i))e_{d_i} \quad (32)$$

Substituting (32) into (31) and considering the skew symmetric property mentioned in section II, one can write:

$$\dot{V} = \dot{\mathcal{H}} - e^T{}_{d_i}(\bar{M}_i K)e_{d_i} \quad (33)$$

Since $e^T{}_{d_i}(\bar{M}_i K)e_{d_i} < 0$ is positive semi-definite, considering the same argument as the preceding theorem, one can deduce that the closed loop system is asymptotically stable and the position and velocity errors asymptotically converge to the set

$$\Omega = \{e_{x_i} = e_{y_i} = e_{d_{i,1}} = e_{d_{i,2}} = 0, \forall i \in \mathcal{N}\}. \qquad \square$$

IV. Simulation Results

The proposed distributed coverage algorithm has been demonstrated via numerical simulations in Matlab environment. A team of 20 mobile robots is waiting to be deployed into a $2m \times 2m$ square environment. The robots in the network were started from random initial positions with the angle of $\theta_{i0} = \pi/2$. The Matlab numerical solver ode45 was used to integrate the equations of motion of the group of robots, and the spatial integrals in (12) required for the computation of the centroids were computed by discretizing each Voronoi region and summing contributions of the integrand over the grid. Voronoi regions were computed using a decentralized algorithm similar to that of [2]. The employed parameters for the mobile robots and controllers are summarized in Table I.

TABLE I
Simulation Parameters

Robot Parameters		Controller parameters	
m	1 kg	k_{v_i}	3
I	0.5 kg-m^2	k_{ω_i}	6
r	0.08 m	K_i	$40I_2$
r_w	0.03 m		
L	0.175m		

The simulations are carried out via two scenarios. In the first scenario, the robots are to be deployed in an environment with a Gaussian sensory function,

$$\phi(\tilde{p}) = (1/\sigma\sqrt{2\pi})\exp(-(\tilde{p}-\mu)^2/2\sigma^2) \quad (34)$$

where $\mu = (1,1)^T, \sigma = 0.18$ .The initial positions of the robots are shown in figure 3(a), while the final configuration of the robots as well as the trajectories of them during the simulation run the in the through the evolution are shown in Figure 3(b). In the second scenario, a bimodal Gaussian distribution function is considered as

$$\phi(\tilde{p}) = (1/\sigma_1\sqrt{2\pi})\exp(-(\tilde{p}-\mu_1)^2/2\sigma_1^2) + (1/\sigma_2\sqrt{2\pi})\exp(-(\tilde{p}-\mu_2)^2/2\sigma_2^2)$$

(35)

the parameters of which are selected as $\mu_1 = (1/3,1/3)^T$, $\mu_2 = (5/3,5/3)^T, \sigma_1 = \sigma_2 = 0.18$. The trajectories of the robots positions together with the final configuration in this scenario are shown in Figure 3(c).

The centers of the contributing Gaussian functions and the centroids of the Voronoi partitions are marked with red o's and blue $\times$'s, respectively. The performance of the proposed controller is clearly demonstrated in the simulation results.

V. Conclusion And Future Work

In this study we considered the coverage problem of mobile sensing robots subject to nonholonomic kinematic and dynamic constraints and introduced an extension to the Voronoi-based standard coverage problem for single integrator agents. The Lyapunov based stability analysis showed that the robots finally converge to the centroidal Voronoi configuration and the whole system is stable. The proposed method has been successfully verified in numerical simulations. Future work will focus on guaranteeing collision avoidance among the robots and the extension of

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the proposed controller for coverage in unknown environments and with nonholonomic mobile robots having unknown dynamic parameters.

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