Fuzzy Goal Programming for Aggregate Production and Logistics Planning

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Abstract— **In this research, fuzzy goal programming model for aggregate production and logistics planning with interval demand and uncertain production capacity is proposed. Two fuzzy goals are considered in the model; profit goal and change of workforce level goal. In conventional aggregate production planning (APP) models, logistics planning is not included. Even it is a critical criterion that creates extra cost. Moreover, demand is considered as crisp demand, which is not realistic. Actual demand is uncertain in nature and does not exactly equal to forecast demand. So, APP with interval demand that the best solution of possible demand can be selected is proposed in this research. Uncertain capacity is also considered in the proposed model. The proposed model can extremely increase profit and reduce change of workforce level. Furthermore, uncertain demand and production capacity are also cooperated, which make the model more realistic for the industrial applications. A case study of a real factory is illustrated to show the effectiveness of the proposed model.**

***Index Terms*— Fuzzy goal, APP, interval demand, logistics planning**

I. Introduction

Aggregate production planning (APP) is concerned with matching supply and demand of forecasted and varying customer orders over the medium term, often from 3 to 18 months in advance [1], [2]. An APP problem is about determining the maximize profit and minimize workforce, and inventory levels for each period of the planning horizon for a given set of production resources and constraints [3]. Generally, multiple objectives are considered such as maximize profit, minimize late orders, and minimize workforce level changes [3]. These objectives conflict in nature. Both deterministic and stochastic models have been proposed for modeling APP problems [4]-[6]. One of the most effective methods for solving multiple objectives problem is "Goal Programming", (GP) [7]-[10]. However, considerable uncertainty was ignored. Stochastic models of APP can deal with uncertainty but they were hard to solve and statistical estimations proved inefficient because of lacking of statistical observation [11]. Heuristic approaches also have been presented [12]-[13]. However, problem constraints are not considered.

A suitable way to model imprecise data of APP problem is to use fuzzy set. Fuzzy set concept has been applied to APP in many literatures [14]-[16]. Many fuzzy goal programming (FGP) models were also proposed for solving multiple objective decision making problem with fuzzy environment [2], [17]-[21]. Most of them consider fuzzy goals, fuzzy capacity or fuzzy coefficients using conventional APP model [22]-[23], which logistics planning is not included. Moreover, uncertain demand may not exactly equal to forecast demand which is normally used as target level of demand for each period. Demand may deviate in a small range from this target value. If the appropriate level of demand, which suits for the actual capacity, can be selected from the possible interval then the appropriate production plan can be generated.

This paper considers a case study for APP application of a manufacturing company. This company produces plastic parts for automotive and electronic industries. It has some problems due to existing APP based on human experience and crisp information such as insufficient workforce level, shipment delay, excessive inventory level and unsatisfied demand. There are two issues to be concerned. Firstly, they feel uncomfortable to estimate the demand in each period as a constant using forecast demand. If they under-estimate the demand, an opportunity loss of sales and profit will occur. On the other hand, if they over-estimate the demand especially during peak demand periods, costly overtime, unnecessary subcontracting, and inventory holding will occur. Secondly, the company feels that the production output is not limited by the fixed capacity. In reality, the capacity can be deviated in a small range of a negative or a positive direction due to machine breakdown, adjustability or improvement of machine capacity. Moreover, conventional APP models do not concern about transportation cost. So, in this research the FGP model for aggregate production and logistics planning with interval demand and uncertain production capacity is proposed to solve the problem of this manufacturing company.

II. Model Formulation

A. Problem Description and Notations

APP model is developed to satisfy the case study problem. The company produces *n* types of products based on forecast demand in each planning horizon period (*t*): Two objective functions are considered in this case; to maximize profit and to minimize changes of workforce.

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1. Notations of parameters and variables

1.1 Indices:

- i number of product types, $i = 1,2,...,n$.
- t number of periods in the planning horizon, $t = 1,2,...,T$.

1.2 Input parameters:

- Pr_i selling price per unit of product type i, (Baht/unit).
- CI_i inventory carrying cost per unit of product i, (Baht/unit).
- CB_i backorder cost per unit of product i, (Baht/unit).
- CP_i production cost per unit of product i, (Baht/unit).
- CS_i subcontract cost per unit of product i, (Baht/unit).
- CT_i transportation cost per trip of product i, (Baht/trip).
- COn_t overtime cost per unit for normal working day in period t, (Baht/unit).
- $COh1_t$, $COh2_t$ overtime cost per unit for holiday during 8:00 am -5:00 pm and after 5:00 pm in period t, (Baht/unit).
- CW_t average salary per worker in period t, (Baht/worker).
- CH_t hiring and training cost per worker in period t, (Baht/worker).
- CF_t downsizing cost per worker in period t, (Baht/worker).
- PH_i production capacity rate of product i. (units/hour).
- RH_t maximum number of allowable regular hours per worker in period t, (hours/worker).
- D_{it} forecast demand of product i in period t, (units).
- $Smax_i$, $Bmax_i$ maximum subcontract and backorder quantities of product i, (units/month).
- $Imax$ maximum inventory level in each period, (units/month).
- $Wmax_i$, $Wmin_i$ maximum and minimum workforce level of product i in each period, (workers).
- $PTmax_i$ maximum quantities of product i in each trip of transportation, (units/trip).
- $Onmax_t$ maximum number of allowable overtime hours per worker for normal working day in period t, (hours/worker).
- $Oh1max_t$, $Oh2max_t$ maximum number of allowable overtime hours per worker for holiday during 8:00 am - 5:00 pm and after 5:00 pm in period t, (hours/worker).
- $Imax$ maximum inventory level, (units).
- $Dmax_i$, $Dmin_i$ maximum and minimum quantities of forecast demand of product i in each period, (units).
- I_{i0}, B_{i0} initial number of inventory and backorder level of product i, (units).
- W_0 initial number of workers in period t, (workers).

1.3 Decision Variables:

- d_{it} forecast demand in an interval of product i in period t, (units).
- B_{it} backorder quantities of product i in period t, (units).
- I_{it} inventory level of product i in period t, (units).
- P_{it} regular production of product i in period t, (units).
- O_{it} overtime production of product i in period t, (units).
- POn_{it} overtime production for normal working day of product i in period t, (units).
- $POh1_{it}$, $POh2_{it}$ overtime production for holiday during 8:00 am - 5:00 pm and after 5:00 pm of product i in period t, (units).
- S_{it} subcontracted production of product i in period t, (unit).
- NT_{it} number of trip for transportation normal delivery of product i in period t, (trips).
- NTB_{it} number of trip for transportation backorder delivery of product i in period t, (trips).
- W_t regular production workers in period t, (workers).
- W_{it} regular production workers of product i in period t, (workers).
- WO_{it} overtime workers of product i in period t, (workers).
- WOn_{it} overtime workers for normal working day of product i in period t, (workers).
- $WOh1_{it}$, $WOh2_{it}$ overtime workers for holiday during 8:00 am - 5:00 pm and after 5:00 pm of product i in period t, (workers).
- H_t hired workers in period t, (workers).
- F_t fired workers in period t, (workers).

2. Objective functions

Two objective functions are considered in the proposed model.

2.1 Maximization of profit objective (z_1): Profit comes from revenue of actual demands sent to customer minus costs of backordering, production, overtime, subcontracting, inventory and costs related to workforces. This objective is the main objective for all companies.

$$\begin{aligned} Max\ z_1 = & \sum_{i=1}^{n}\sum_{t=1}^{T} Pr_i D_{it} - \sum_{i=1}^{n} Pr_i B_{iT} - \sum_{i=1}^{n}\sum_{t=1}^{T} CB_i B_{it} \\ & - \sum_{i=1}^{n}\sum_{t=1}^{T} CP_i (P_{it} + O_{it}) - \sum_{i=1}^{n}\sum_{t=1}^{T} CS_i S_{it} \\ & - \sum_{i=1}^{n}\sum_{t=1}^{T} CI_i I_{it} - \sum_{t=1}^{T} CW_t W_t - \sum_{t=1}^{T} CH_t H_t \\ & - \sum_{t=1}^{T} CF_t F_t - \sum_{i=1}^{n}\sum_{t=1}^{T} CT_i (NT_{it} + NTB_{it}) \\ & - \sum_{i=1}^{n}\sum_{t=1}^{T} (COn_t POn_{it} + COn_t POn_{it} + COn_t POn_{it}). \end{aligned} \tag{1}$$

2.2 Minimization change of workforce levels (z_2): Change of workforce level means the total numbers of hiring and firing in every period. This objective related to human resource management and morale of workers.

$$Min\ z_1 = \sum_{t=1}^{T} (H_t + F_t). \tag{2}$$

3. Constraints

3.1 Product balance constraints: Production, overtime subcontract and backorder quantities of current period equal to demand and inventory level of current period plus backorder quantities minus inventory level of the previous

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period as shown in (3).

$$P_{it} + O_{it} + S_{it} + B_{it} = D_{it} + I_{it} + B_{it-1} - I_{it-1} \quad , \forall i\ \forall t. \qquad (3)$$

3.2 Production constraints: Production of regular time should not greater than production quantities generated by worker during regular time, which can be represented by (4).

$$P_{it} \le W_{it} RH_t PH_i \, , \qquad \forall i\ \forall t. \qquad (4)$$

3.3 Overtime constraints: Overtime for normal working day, overtime for holiday during 8:00 am -5:00 pm and after 5:00 pm for each product in each period are represented by (5)-(7), respectively. These overtime productions should not greater than overtime production quantities generated by overtime worker for each product in each period. Total overtime production in each period is summarized as shown in (8).

$$POn_{it} \le WOn_{it} On_t PH_i \, , \qquad \forall i\ \forall t. \qquad (5)$$
$$POh1_{it} \le WOh1_{it} Oh1_t PH_i \, , \qquad \forall i\ \forall t. \qquad (6)$$
$$POh2_{it} \le WOh2_{it} Oh2_t PH_i \, , \qquad \forall i\ \forall t. \qquad (7)$$
$$O_{it} = POn_{it} + POh1_{it} + POh2_{it} \, , \qquad \forall i\ \forall t. \qquad (8)$$

3.4 Backorder and subcontract constraints: Backorder and subcontract quantities should not exceed the maximum allowable limit as shown in (9), (10).

$$B_{it} \le Bmax_i \, , \qquad \forall i\ \forall t. \qquad (9)$$
$$S_{it} \le Smax_i \, , \qquad \forall i\ \forall t. \qquad (10)$$

3.5 Inventory constraints: The inventory level cannot exceed the maximum allowable limit since there are limited warehouse spaces that can be shown as (11).

$$\sum_{i=1}^{n} I_{it} \le Imax \, , \qquad \forall t. \qquad (11)$$

3.6 Workforce constraints: Number of workers in each period is equal to the number of workers in previous period plus workers being hired at that period minus the number of workers being laid off at that period as shown in (12). Equation (13) shown that the total number of workers in period *t* is equal to the summation of the workers for all product. The number of worker for product *i* in period *t* should not less than the minimum number of workers and should not greater than the maximum number of workers of each product in each period as shown in (14).

$$W_t = W_{t-1} + H_t - F_t \, , \qquad \forall t. \qquad (12)$$
$$W_t = \sum_{i=1}^{n} W_{it} \, , \qquad \forall t. \qquad (13)$$
$$Wmin_i \le W_{it} \le Wmax_i \, , \qquad \forall i\ \forall t. \qquad (14)$$

3.7 Overtime workforce constraints: The total number of overtime worker in normal working day for all products in every period should not less than the total number of regular workers of all products in every period as shown in (15). The total number of overtime worker in holiday during 8:00 am – 5:00 pm for all products in every period should not less than the total number of workers after 5:00 pm for all products in every period and should not greater than the number of workers in regular time of all products in every period as shown in (16). Overtime workers are able to transfer from workers of one product to another product.

$$\sum_{i=1}^{n}\sum_{t=1}^{T} Won_{it} \le \sum_{i=1}^{n}\sum_{t=1}^{T} W_{it} \, , \qquad (15)$$
$$\sum_{i=1}^{n}\sum_{t=1}^{T} Woh2_{it} \le \sum_{i=1}^{n}\sum_{t=1}^{T} Woh1_{it} \le \sum_{i=1}^{n}\sum_{t=1}^{T} W_{it} \, , \qquad (16)$$

3.8 Transportation constraints: Number of trip for transportation normal delivery of product *i* in period *t* should not less than the demand of product *i* in period *t* divided by the maximum capacity for each trip of product *i* as shown in (17). Number of trip for transportation backorder delivery of product *i* in period *t* should also not less than backorder quantities of product *i* in period *t* divided by maximum capacity for each trip of product *i* as shown in (18).

$$NT_{it} \ge D_{it}/PTmax_i \, , \qquad \forall i\ \forall t. \qquad (17)$$
$$NTB_{it} \ge B_{it}/PTmax_i \, , \qquad \forall i\ \forall t. \qquad (18)$$

B. Fuzzy Goal Programming (FGP) Model with Interval Demand and Fuzzy Production Capacity Constraint.

FGP normally uses to solve multiple objective decision making problems [15], [16]. In this research Preemptive Fuzzy Goal Programming (P-FGP) has been applied. Two fuzzy goals are concerned; profit and change of workforce level. P-FGP is suitable for this problem since the first goal (profit goal) is extremely important than the second goal (change of workforce level).

Defining membership function of each goal is based on the Positive-Ideal Solution (PIS) and the Negative-Ideal Solution (NIS) [20]. The PIS is the best possible solution when each objective function is optimized. The NIS is the feasible and worst value of each objective function. Triangular membership function is considered as shown in Fig. 1 [15]-[16], [21]. Membership function $\mu(Z_k)$ can be written as (19).

$$\mu(Z_k) = \begin{cases} 0 & \text{, if } Z_k \le \tau_k - \Delta_k \\ 1 - \left(\frac{\tau_k - Z_k}{\Delta_k}\right) & \text{, if } \tau_k - \Delta_k \le Z_k \le \tau_k \\ 1 - \left(\frac{Z_k - \tau_k}{\Delta_k}\right) & \text{, if } \tau_k \le Z_k \le \tau_k + \Delta_k \\ 0 & \text{, if } Z_k \ge \tau_k + \Delta_k \end{cases}, \qquad (19)$$

where,

$$\tau_k = PIS_k \, , \qquad \forall k. \qquad (20)$$
$$\Delta_k = |PIS_k - NIS_k| , \qquad \forall k. \qquad (21)$$

Profit goal ($\tilde{z}_1$) and change of workforce level goal ($\tilde{z}_2$) can be written as (22), (23).

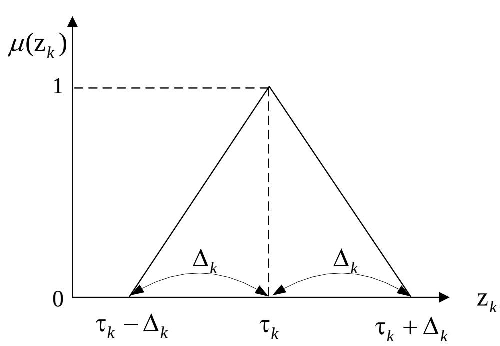

Fig.1. The membership function of *k*th fuzzy goal.

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$$\tilde{z}_1 = \sum_{i=1}^{n}\sum_{t=1}^{T} Pr_i d_{it} - \sum_{i=1}^{n} Pr_i B_{iT} - \sum_{i=1}^{n}\sum_{t=1}^{T} CB_i B_{it} - \sum_{i=1}^{n}\sum_{t=1}^{T} CP_i (P_{it} + O_{it}) - \sum_{i=1}^{n}\sum_{t=1}^{T} CS_i S_{it} - \sum_{i=1}^{n}\sum_{t=1}^{T} CI_i I_{it} - \sum_{t=1}^{T} CW_t W_t - \sum_{t=1}^{T} CH_t H_t - \sum_{t=1}^{T} CF_t F_t - \sum_{i=1}^{n}\sum_{t=1}^{T} CT_i (NT_{it} + NTB_{it}) - \sum_{i=1}^{n}\sum_{t=1}^{T} (COn_t POn_{it} + COn_t POn_{it} + COn_t POn_{it}). \quad (22)$$

$$\tilde{z}_2 = \sum_{t=1}^{T} (H_t + F_t). \quad (23)$$

In (22), d_{it} is introduced for determination of demand within an interval of possible demand quantities for product i in period t that should not less than the minimum number of demands and should not greater than the maximum number of demands of each product in each period as shown in (24). Then, (3), (17) has been changed to (25), (26).

$$Dmin_{it} \le d_{it} \le Dmax_{it}, \quad \forall i\ \forall t. \quad (24)$$

$$P_{it} + O_{it} + S_{it} + B_{it} = d_{it} + I_{it} + B_{it-1} - I_{it-1}, \forall i\ \forall t. \quad (25)$$

$$NT_{it} \ge d_{it}/PTmax_i, \quad \forall i\ \forall t. \quad (26)$$

Production capacity (PH_i) is also considered as fuzzy capacity, $(\widetilde{PH_\iota})$; $\widetilde{PH_\iota} = (PH_i^-, PH_i, PH_i^+)$. PH_i^+, PH_i, PH_i^- represent optimistic, most-likely and pessimistic production capacity rate of product *i*, (units/hour). So, (4)-(7) can be rewritten as:

$$P_{it} \le W_{it} RH_t \widetilde{PH_\iota}, \quad \forall i\ \forall t. \quad (27)$$

$$POn_{it} \le WOn_{it} On_t \widetilde{PH_\iota}, \quad \forall i\ \forall t. \quad (28)$$

$$POh1_{it} \le WOh1_{it} Oh1_t \widetilde{PH_\iota}, \quad \forall i\ \forall t. \quad (29)$$

$$POh2_{it} \le WOh2_{it} Oh2_t \widetilde{PH_\iota}, \quad \forall i\ \forall t. \quad (30)$$

P-FGP with interval demand and fuzzy production capacity can be shown as:

Lexicographically maximize $\{\alpha_1, \alpha_2\}$, (31)

subject to

$$\alpha_k^* \le \mu(Z_k), \quad k = 1, 2. \quad (32)$$

$$Z_k + \Delta_k(\delta_k^- - \delta_k^+) = \tau_k, \quad k = 1, 2. \quad (33)$$

$$\alpha_k + \delta_k^- - \delta_k^+ \le 1, \quad k = 1, 2. \quad (34)$$

$$\alpha_k \le \mu(Z_k), \quad k = 1, 2. \quad (35)$$

$$\delta_k^-, \delta_k^+ \ge 0 \quad k = 1, 2. \quad (36)$$

(9)-(16), (18), (24)-(30),

where α_k is the satisfaction level of the objective *k*th $\alpha_k \epsilon [0,1]$ and α_k^* is the desired level of satisfaction level of objective *k*th.

Wang (1997) suggested converting $\widetilde{PH_\iota}$ using most-likely criterion. Then,

$$\widetilde{PH_\iota} = (4PH_i + PH_i^- + PH_i^+)/6 \quad \forall i. \quad (37)$$

In the proposed method, three models can be generated based on optimistic, most-likely and pessimistic criteria because decision maker may need more information than just know only most-likely case. For most-likely criterion, Wang (1997)'s method is applied by substitute $\widetilde{PH_\iota}$ with (37). For optimistic and pessimistic criteria, PH_i^- and PH_i^+ are used, respectively. So, three solutions are obtained from the P-FGP model for optimistic, most-likely and pessimistic criteria.

III. A Case Study

A case study is presented to demonstrate the proposed model. The company under consideration is a plastic injection factory for automotive and electronic parts. The planning horizon is 6 months. There are 5 groups of products (A, B,…,E) by customers. Regular production is 8 hours per shift. Two shifts a day.

Average inventory cost per unit (CI_i) is 0.0076 Baht. The maximum allowable inventory level in each period (*Imax*) is 100,000 units. The maximum allowable backorder level ($Bmax_i$) is twenty percentage of forecast demand.

Initial workforce level (W_0) is 248 workers. Other information is given in the following tables.

TABLE I
THE BASIC DATA FOR EACH PRODUCT TYPE

Product	A	B	C	D	E
Pr_i	47.00	0.85	30.00	20.00	8.00
CB_i	9.40	0.17	6.00	4.00	1.60
CS_i	39.38	0.00	34.26	0.00	0.00
CP_i	19.42	0.49	15.66	5.92	1.62
CT_i	1,200	0*	1,200	100	800
COn_i	8.31	0.25	2.74	1.03	0.48
$COh1_i$	11.08	0.33	3.65	1.37	0.64
$COh2_i$	16.62	0.50	5.48	2.06	0.96
PH_i	6	140	8	30	80
$Smax_i$	50,000	0	100,000	0	0
I_{i0}	0	15,600	0	3,200	20,800
B_{i0}	28,000	0	66,000	0	0
$Wmax_i$	136	24	60	48	32
$Wmin_i$	68	12	30	24	16
$PTmax_i$	20,000	0	10,000	10,000	35,000

*Product B currently uses mill-run system so transportation cost is not concerned.

TABLE II
THE BASIC DATA IN EACH PERIOD

Period	1	2	3	4	5	6
CW_t	5,600	5,400	5,200	5,800	5,800	6,000
CH_t	4,560	3,800	3,990	4,560	4,560	4,180
CF_t	16,800	16,200	15,600	17,400	17,400	18,000
RH_t	384	320	336	384	384	352
$Onmax_t$	144	120	126	144	144	132
$Oh1max_t$	112	160	160	96	112	144
$Oh2max_t$	42	60	60	36	42	54

TABLE III
CRISP DEMAND DATA IN EACH PERIOD

Period	D_A	D_B	D_C	D_D	D_E
1	568,000	448,000	496,400	744,800	1,988,000
2	484,500	473,200	349,200	460,800	1,288,000
3	424,000	532,000	423,600	552,200	1,433,600
4	384,000	464,000	469,600	668,800	1,688,800
5	368,000	468,600	421,200	682,600	1,788,600
6	320,400	404,200	444,800	724,800	1,866,000

TABLE IV
INTERVAL DEMAND DATA IN EACH PRODUCT

Product	A	B	C	D	E
$Dmax_i$	568,000	532,000	496,400	744,800	1,988,000
$Dmin_i$	320,400	404,200	349,200	460,800	1,288,000

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TABLE V
PRODUCTION CAPACITY DATA IN EACH PRODUCT

Product	A	B	C	D	E
PH_i^-	4	94	6	21	54
PH_i^+	8	187	11	40	107

TABLE VI
P-FGP WITH CRISP DEMANDS AT $\alpha_1 = 0.8$

Period	B_A	B_B	B_C	B_D	B_E
1	113,600	0	99,280	4,002	0
2	41,216	0	69,840	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0

Period	I_A	I_B	I_C	I_D	I_E
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0

Period	S_A	S_B	S_C	S_D	S_E
1	50,000	0	100,000	0	0
2	50,000	0	12,214	0	0
3	50,000	0	0	0	0
4	29,964	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0

Period	P_A	P_B	P_C	P_D	P_E
1	313,344	432,400	92,160	473,321	983,040
2	261,120	473,200	76,800	394,434	819,200
3	274,176	532,000	80,640	414,156	860,160
4	313,344	464,000	92,160	473,321	983,040
5	313,344	468,600	92,160	473,321	983,040
6	287,232	404,200	84,480	433,878	901,120

Period	O_A	O_B	O_C	O_D	O_E
1	119,056	0	270,960	264,277	984,160
2	245,764	0	289,626	70,368	468,800
3	141,040	0	412,800	138,044	573,440
4	40,692	0	377,440	195,479	705,760
5	54,656	0	329,040	209,279	805,560
6	33,168	0	360,320	290,922	964,880

Period	NT_A, NTB_A	NT_B, NTB_B	NT_C, NTB_C	NT_D, NTB_D	NT_E, NTB_E
1	24, 6	-	35, 10	46, 1	37, 0
2	28, 2	-	50, 7	74, 0	57, 0
3	16. 0	-	42, 0	55, 0	51, 0
4	19, 0	-	47, 0	68, 0	41, 0
5	18, 0	-	42, 0	67, 0	48, 0
6	21, 0	-	44, 0	72, 0	53, 0

Period	W_A	W_B	W_C	W_D	W_E
1	136	12	30	41	32
2	136	12	30	41	32
3	136	12	30	41	32
4	136	12	30	41	32
5	136	12	30	41	32
6	136	12	30	41	32

Period	WOn_A	WOn_B	WOn_C	WOn_D	WOn_E
1	138	0	0	28	85
2	251	0	0	0	0
3	187	0	0	8	57
4	47	0	97	45	61
5	63	0	69	48	70
6	42	0	44	73	91

Period	$WOh1_A$, $WOh2_A$	$WOh1_B$, $WOh2_B$	$WOh1_C$, $WOh2_C$	$WOh1_D$, $WOh2_D$	$WOh1_E$, $WOh2_D$
1	0, 0	0, 0	208, 251	43, 0	0, 0
2	68, 0	0, 0	132, 251	15, 0	37, 0
3	0, 0	0, 0	228, 251	23, 0	0, 0
4	0, 0	0, 0	251, 251	0, 0	0, 0
5	0, 0	0, 0	251, 72	0, 0	0, 0
6	0, 0	0, 0	251, 56	0, 0	0, 0

PIS_1 = 197,198,233 Baht, PIS_2 = 0 worker, NIS_1 = 165,149,771 Baht, NIS_2 = 40 workers, Δ_1 = 32,048,462, Δ_2 = 40, α_2 = 0.92.
1st goal = 190,788,540 Baht, 2nd goal = 3 workers.

TABLE VII
P-FGP WITH INTERVAL DEMANDS AT $\alpha_1 = 0.8$

Period	d_A	d_B	d_C	d_D	d_E
1	404,216	532,000	349,200	744,800	1,988,000
2	431,783	532,000	349,200	744,800	1,988,000
3	427,502	532,000	349,200	744,800	1,988,000
4	502,731	532,000	349,200	744,800	1,988,000
5	498,490	532,000	349,200	744,800	1,988,000
6	481,820	532,000	349,200	744,800	1,988,000

Period	B_A	B_B	B_C	B_D	B_E
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0

Period	I_A	I_B	I_C	I_D	I_E
1	0	0	0	0	0
2	0	0	26,041	0	0
3	0	0	100,000	0	0
4	0	0	25,299	0	0
5	0	0	0	0	0
6	0	0	0	0	0

Period	S_A	S_B	S_C	S_D	S_E
1	50,000	0	0	0	0
2	50,000	0	0	0	0
3	50,000	0	0	0	0
4	50,000	0	0	0	0
5	50,000	0	0	0	0
6	50,000	0	0	0	0

Period	P_A	P_B	P_C	P_D	P_E
1	313,344	516,400	92,160	444,257	983,040
2	261,120	532,000	76,800	370,215	819,200
3	274,176	532,000	80,640	388,725	860,160
4	313,344	532,000	92,160	444,257	983,040
5	313,344	532,000	92,160	444,257	983,040
6	287,232	532,000	84,480	407,236	901,120

Period	O_A	O_B	O_C	O_D	O_E
1	68,872	0	323,040	297,343	984,160
2	120,663	0	298,441	374,585	1,168,800
3	103,326	0	342,519	356,075	1,127,840
4	139,387	0	182,339	300,543	1,004,960
5	135,146	0	231,741	300,543	1,004,960
6	144,588	0	264,720	337,564	1,086,880

Period	NT_A, NTB_A	NT_B, NTB_B	NT_C, NTB_C	NT_D, NTB_D	NT_E, NTB_E
1	22, 0	-	35, 0	74, 0	57, 0
2	20, 0	-	35, 0	74, 0	57, 0
3	24, 0	-	35, 0	74, 0	57, 0
4	25, 0	-	35, 0	74, 0	57, 0
5	25, 0	-	35, 0	74, 0	57, 0
6	21, 0	-	35, 0	74, 0	57, 0

Period	W_A	W_B	W_C	W_D	W_E
1	136	12	30	39	32
2	136	12	30	39	32
3	136	12	30	39	32
4	136	12	30	39	32
5	136	12	30	39	32
6	136	12	30	39	32

Period	WOn_A	WOn_B	WOn_C	WOn_D	WOn_E
1	80	0	15	69	85
2	168	0	0	0	81
3	137	0	0	0	112
4	161	0	0	0	87
5	156	0	0	5	87
6	183	0	0	0	66

Period	$WOh1_A$, $WOh2_A$	$WOh1_B$, $WOh2_B$	$WOh1_C$, $WOh2_C$	$WOh1_D$, $WOh2_D$	$WOh1_E$, $WOh2_E$
1	0, 0	0, 0	249, 249	0, 0	0, 0
2	0, 0	0, 0	140, 249	78, 0	31, 0
3	0, 0	0, 0	174, 249	74, 0	0, 0
4	0, 0	0, 0	144, 249	104, 0	0, 0
5	0, 0	0, 0	165, 249	83, 0	0, 0
6	0, 0	0, 0	137, 249	78, 0	34, 0

PIS_1 = 229,058,460 Baht, PIS_2 = 0 worker, NIS_1 = 136,687,324 Baht, NIS_2 = 52 workers, Δ_1 = 92,371,135, Δ_2 = 52, α_2 = 0.99.
1st goal = 210,584,233 Baht, 2nd goal = 1 worker.

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Using the proposed P-FGP model with interval demand, Decision Maker (DM) can set the satisfaction level of the first objective that can be alleviated and then the best solution can be found at high degree of satisfaction level (ex. α_1^*=0.8, $\alpha_2 = 0.99$) for most-likely case, which is better than single objective optimization and multiple objective optimization with crisp demand. The proposed model is better than single objective optimization due to consideration of both goals under acceptable level of the first goal. It also has advantages over multiple objective optimization problems with crisp demands. In this study P-FGP with crisp demand and interval demand are compared. In the first goal, profit is increased from 190,788,541 to 210,584,233 Baht and change of workforce level is reduced from 3 to 1 worker. These advantages exist because all of backorder quantities are eliminated and transportation cost for backorder is also eliminated. The total workforce level for P-FGP with interval demands is fewer than the total workforce level for P-FGP with crisp demands. Hiring and firing is also fewer. So, costs related to workforce level are reduced and change of workforce level is also reduced. However, inventory level for P-FGP with interval demands is greater than inventory level for P-FGP with crisp demands. These benefits can be occurred because the appropriate demands are selected from possible demand intervals for generation the APP. The results of decision variables are shown in Table VI, VII. In the proposed model fuzzy capacity is also considered so additional results of pessimistic and optimistic cases can be generated that. This information can help DM to decide the production plan when the situation changes. The results for each case are shown in Table VIII.

TABLE VIII
SUMMARY OF RESULTS

Cases	Z_1	Z_2
Maximum Z_1 with crisp demands	197,198,233	40
Minimum Z_2 with crisp demands	165,149,771	0
Maximum Z_1 with interval demand	229,058,460	52
Minimum Z_2 with interval demand	136,687,324	0
FGP for crisp demand	190,788,541	3
FGP for most likely capacity with interval demand	210,584,233	1
FGP for pessimistic capacity with interval demand	171,829,124	25
FGP for optimistic capacity with interval demand	228,207,552	0

IV. CONCLUSION

Preemptive fuzzy goal programming (P-FGP) model for aggregate production and logistics planning with interval demand and uncertain production capacity is proposed for solving the problem of the case study. Two fuzzy goals; profit goal and change of workforce level goal were considered. P-FGP model with interval demand has advantages over single objective optimization and P-FGP model with crisp demands because the better solution can be found for both profit and change of workforce level by setting the appropriate demand from a possible demand interval. P-FGP with interval demand can reduce costs of backorder, transportation, hiring, firing and subcontract. The fuzzy production capacity is also considered. Three models of P-FGP are generated based on optimistic, pessimistic and most-likely criteria. These can give more information for DM when the situation changes.

Further study might consider uncertainty of cost coefficients. Interactive approach is also attractive for DMs.

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