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New Approach for Fuzzy Control and Analysis of Queueing System with Flexible Service Time

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Abstract **– Fuzzy control of queues is a new method for controlling cost of the system, number of customers waiting in the queue and many other aspects of queues. This paper proposes fuzzy control approach for reducing cost of the system. $M/F/1$ (Poisson arrival and fuzzy service rate) with flexible service time is examined in this paper. All the fuzzy mathematics rules in this paper are based on Zadeh's extension principle, Mamdani implication, possibility and probability concept. At the end of this paper a simple numerical example is presented.**

I. INTRODUCTION

Queueing theory is a classical mathematical method for studying the queues such as average waiting time and average number of customers in the system [2], [3]. Fuzzy queueing and fuzzy control of queues would be a new approach for surveying the queueing systems. If we let service time be expressed by possibility rather than probability, then fuzzy queueing method would be a more realistic approach than classical queueing theory methods. $M/F/1$ (Poisson arrival and fuzzy service rate) with flexible service time is examined in this paper, and the objective is to employ a fuzzy policy to reduce the costs of the system and control the average number of customers in the queue. All the fuzzy mathematics rules in this paper are based on Zadeh's extension principle [1], Mamdani implication, possibility and probability concept. The results of this paper can be extended to other queueing systems.

II. $M/F/1$ QUEUES IN FUZZY ENVIROMENT

Consider we have a queueing system with Poisson arrival, one server and fuzzy service time. The arrival rate is λ and the discipline is first in first served. Suppose that the service time is a fuzzy set denoted by $\widetilde{S}$. Therefore $\widetilde{S}$ = $\{t \in \Re^+, \mu_s(t) > 0\}$. Imagine after each service completion we have a new state. The number of customers that a person whose service is just completed sees in the system is considered as a state of the system. Therefore the Probability that we move from state i to j is identical to the probability that $j-i+1$ customers enter the system during the service time t. The probability of i arrivals during service time t is denoted by $\widetilde{P}_i$. So this is a Markov chain and since all the probabilities are fuzzified, it's considered as fuzzy Markov chain. We can show this fuzzy Markov chain as follows [5]:

$$\widetilde{P}_{ij} = \begin{bmatrix} \widetilde{P}_0 & \widetilde{P}_1 & \widetilde{P}_2 & \widetilde{P}_3 & . & . & . \\ \widetilde{P}_0 & \widetilde{P}_1 & \widetilde{P}_2 & \widetilde{P}_3 & . & . & . \\ 0 & \widetilde{P}_0 & \widetilde{P}_1 & \widetilde{P}_2 & . & . & . \\ 0 & 0 & \widetilde{P}_0 & \widetilde{P}_1 & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \end{bmatrix}$$

We need to solve the stationary equations. The stationary equations are as follows:

$\widetilde{P}_0\,\pi_0 + \widetilde{P}_0\,\pi_1 = \pi_0$, $\widetilde{P}_1\,\pi_0 + \widetilde{P}_1\,\pi_1 + \widetilde{P}_0\,\pi_3 = \pi_1$ and so on. The results are as follows:

$$\pi_0 = 1 - \lambda t,$$

$$\pi_1 = (1-\lambda t)\left[\exp(\lambda t) - 1\right],$$

$$\pi_n = (1-\lambda t)\sum_{k=1}^{n}(-1)^{n-k}\exp(k\lambda t)\times \left[\frac{(k\lambda t)^{n-k}}{(n-k)!} + \frac{(k\lambda t)^{n-k-1}}{(n-k-1)!}\right], \qquad n \geq 2$$

π_i represents the fraction of time in long run that the state of system is i. Now we can compute L by the following formula:

$$L = \sum_i i\widetilde{P}_i = \frac{\lambda t(2-\lambda t)}{2(1-\lambda t)}$$

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and therefore

$$W = \frac{L}{\lambda} = \frac{t(2-\lambda t)}{2(1-\lambda t)}.$$

We understand from above formulas that L and W are also fuzzy sets. So,

$$L = \left\{ \frac{\lambda t(2-\lambda t)}{2(1-\lambda t)}, \mu_s(t) \right\} \text{ and}$$

$$W = \left\{ \frac{t(2-\lambda t)}{2(1-\lambda t)}, \mu_s(t) \right\}$$

III. FUZZY CONTROL

In this part we want to employ a policy to control the service time of queueing system in fuzzy environment. As Heyman has proved the optimal policy for costs, we just keep the server on as long as there is at least one customer in the system [4]. When system gets empty we turn the server off and the only thing we need to specify is when turn the server on. Suppose that in our system we have switching cost denoted by SC (whenever we turn the server on), Holding cost rate per customer (HC) and server running cost rate (RC). Also suppose that we have a flexible server which can switch from one service time to another one. The important fact in the real world is that running cost rate depends on the service rate (higher service rate=higher service running cost). So we should try to employ a policy to control the service time after turning on the server. Our fuzzy controller has two outputs. One is d_1 which indicates whether or not the server is on and the other is d_2 which determines the service time. Now we need to know our inputs to be able to define our control logic.

If there is no switching cost , obviously it will be optimal to turn the server on as soon as one customer enters to the system. But since we have a big switching cost in most of the cases, we turn the server on when the accumulated holding cost (AHC)gets big enough to comptete with switching cost (SC):

$$AHC = \sum_i HCS_i \,,$$

where S_i is the number of customers in the system in the ith unit of time.

Since we have the average arrival rate λ , therefore after first unit time we have average λ customers in the system. The average accumulated holding cost after first unit of time will be $AHC = HC\lambda/2$. After n units of time the average accumulated holding cost will be

$$AHC = HC\lambda/2 + 3HC\lambda/2 + 5HC\lambda/2 + \ldots + (2n-1)HC\lambda/2 = n^2\lambda HC/2 \,.$$

Since when AHC gets equal to switching cost SC , we turn the server on (d_1 is on), therefore

$$n^2\lambda HC/2 = SC \rightarrow n = \sqrt{\frac{2SC}{\lambda HC}}.$$

TABLE I. Rule base

L (length of the queue)	*HC* (holding cost rate per customer)	d_1	d_2
Z	*Z*	*NO*	–
S	*Z*	*NO*	–
M	*Z*	*NO*	–
B	*Z*	*YES*	*M*
Z	*S*	*NO*	–
S	*S*	*YES*	*B*
M	*S*	*YES*	*B*
B	*S*	*YES*	*M*
Z	*M*	*NO*	–
S	*M*	*YES*	*B*
M	*M*	*YES*	*M*
B	*M*	*YES*	*M*
Z	*B*	*NO*	–
S	*B*	*YES*	*M*
M	*B*	*YES*	*M*
B	*B*	*YES*	*M*

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So when accumulated holding cost gets equal to SC, the length of the queue is approximately

$$L = n\lambda = \lambda\sqrt{\frac{2SC}{\lambda HC}}.$$

Therefore when the queue length is $\lambda\sqrt{2SC/\lambda HC}$, we turn the server on ($d_1 = on$). Also we know that the higher the h, the easier it is to make a decision to turn the server on.

Now we should define the membership function of h, L and d_2 (service time). We computed that when length of the queue gets $\lambda\sqrt{2SC/\lambda HC}$ we turn the dormant server on. So $\lambda\sqrt{2SC/\lambda HC}$ is considered BIG. Since our normalized interval is $[0,6]$, we should scale $\lambda\sqrt{2SC/\lambda HC}$ to 6, so the scaling factor will be

$$\frac{6}{\lambda\sqrt{\frac{2SC}{\lambda HC}}}.$$

The scaling factor for HC is $6/SC$. The membership functions for L and HC will be like figure I and II, respectively.

We computed

$$L = \sum_i i\widetilde{P}_i = \frac{\lambda t(2-\lambda t)}{2(1-\lambda t)}.$$

The service time (t) by which L is BIG ($L = \lambda\sqrt{2SC/\lambda HC}$), is also considered as BIG. So we should solve the following equation, which is:

$$\frac{\lambda t(2-\lambda t)}{2(1-\lambda t)} = \lambda\sqrt{\frac{2SC}{\lambda HC}}.$$

We denote bigger t we get from above equation by A and consider it as BIG. Therefore the scaling factor for d_2 is $6/A$ (figure III).

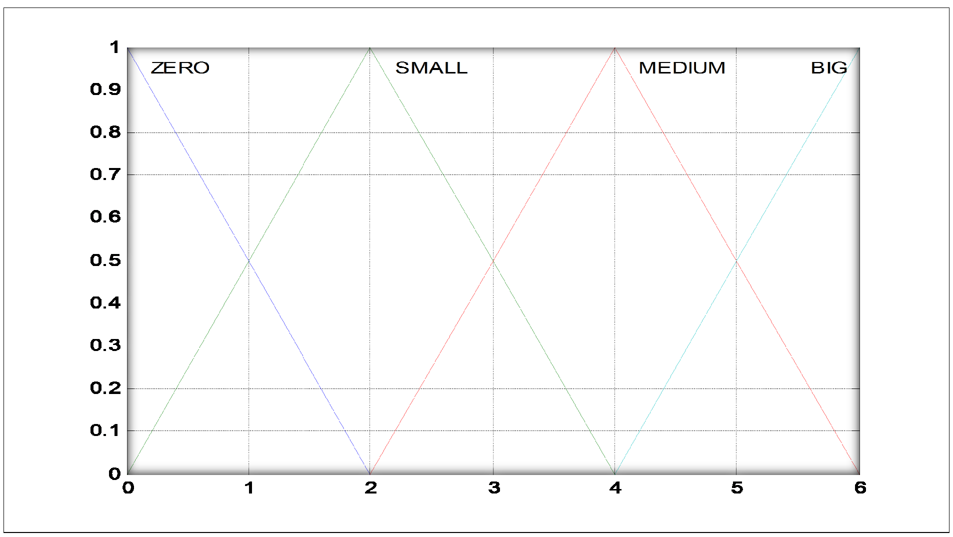

FIGURE I. Membership function: normalized input variable L

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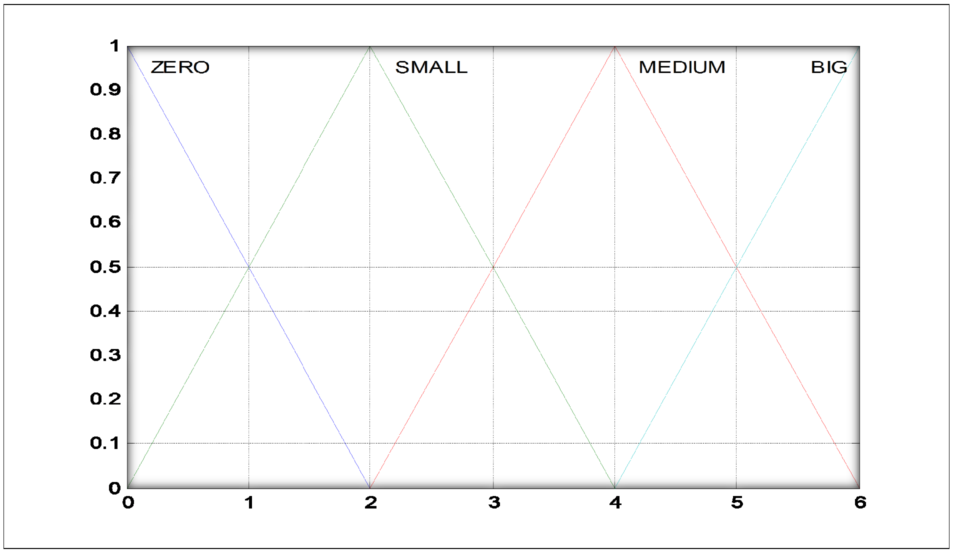

FIGURE II. Membership function: normalized input HC

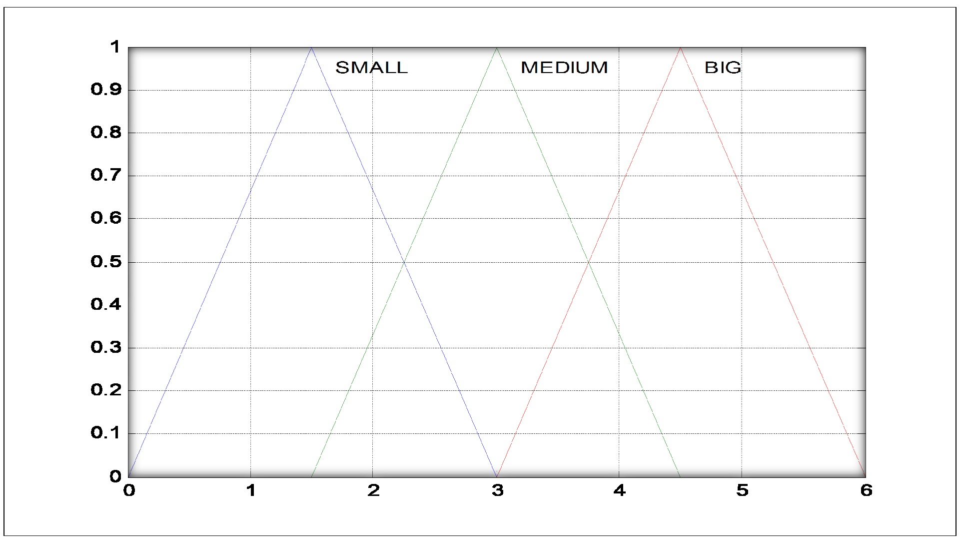

FIGURE III. The Membership function: normalized output d_2

IV. NUMERICAL EXAMPLE

Suppose that we have a queueing system $M/M/1$ with arrival rate $\lambda = 1$, holding cost rate $h = 88.88888$ and switching cost $SC = 100$. Suppose that the length of the queue is 1 ($L = 1$). The scaling factor for HC is $6/SC$. So 88.88888 is scaled down to

$$\frac{88.88888 \times 6}{100} \approx 5.333333.$$

Also the scaling factor for L is

$$\frac{6}{\lambda\sqrt{\dfrac{2SC}{\lambda HC}}} = 4.$$

So $L = 1$ is scaled to 4.

According to figure II, $HC = 5.3333$ is $MEDIUM$ with grade 0.33333 and is BIG with grade 0.66666. Also from figure I, we understand that $L = 4$ is $MEDIUM$ with grade 1. d_1 will be YES if all fuzzy outputs are YES. From table 1 (fuzzy rules) and Mamdani implication we have the followings:

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If HC is $MEDIUM$ with grade 0.33333 and L is $MEDIUM$ with grade 1, then d_1 is YES with grade 0.33333.

If HC is BIG with grade 0.66666 and L is $MEDIUM$ with grade 1, then d_1 is YES with grade 0.6666. Because all the decisions on d_1 is YES, then the final decision on d_1 is also YES.

According to Mamdani implication and fuzzy rule base in table 1, the decision on d_2 (service time) is as follows:

If HC is $MEDIUM$ with grade 0.33333 and L is $MEDIUM$ with grade 1, then d_2 is $MEDIUM$ with grade 0.33333.

If HC is BIG with grade 0.6666 and L is $MEDIUM$ with grade 1, then $d2$ is $MEDIUM$ with grade 0.6666.

According to figure III and above decisions, the peak values and heights of the fuzzy set are as follows: $e_1 = 3$, $e_2 = 3$, $f_1 = 0.33333$, $f_2 = 0.6666$. Now by the height method of defuzzification, our final decision on d_2 is

$$d_2 = \frac{\sum_{i=1}^{2} f_i e_i}{f_i} = 2.99 \approx 3 .$$

This is a normalized service time (t). For computing the original service time:

$$\frac{\lambda t(2-\lambda t)}{2(1-\lambda t)} = \lambda\sqrt{\frac{2SC}{\lambda HC}} \rightarrow \frac{2t-t^2}{2-2t} = 1.5 .$$

$$\rightarrow A = 4.3027$$

So the scaling factor is

$$\frac{6}{A} = 1.3945 \rightarrow t = \frac{3}{1.3945} = 2.15.$$

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