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Effect of Non-uniform Temperature and Magnetic Field on Convection Driven by Surface Tension and Buoyancy

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***Abstract*—Effect of a non-uniform basic temperature gradient and magnetic field on the onset of Bénard-Marangoni convection in a horizontal micropolar fluid layer bounded below by a rigid plate and above by non-deformable free surface subjected to a constant heat flux is studied. The lower rigid surface and the upper non-deformable surface are assumed to be perfectly insulating. Six different non-uniform basic state temperature profiles are considered. The resulting eigenvalue problem is solved using the Rayleigh-Ritz technique, and the influence of various parameters on the onset of convection is discussed.**

***Index Terms*—Bénard-Marangoni convection, magnetic field, non-uniform temperature, micropolar fluid.**

I. Introduction

THE theory and modelling of materials processing in the microgravity environment has stirred a huge interest recently. The introduction of micronsized magnetically-inert suspended particles is the most popular thinking for the past two decades. These micronsized magnetically-inert suspended particles have identified themselves to the point of camouflage which is with the magnetically responding carrier fluids. These points are described to the applications involving magneto convection. The micropolar fluid theory which takes into account the inertial characteristics of the substructure particles which are allowed to undergo rotation has been proposed by Eringen[1] and was developed by Eringen [2].

There has been several investigations dealing with thermal instability of micropolar fluid heated from below. Siddheswar and Pranesh [3] studied the effect of non-uniform temperature gradient and magnetic field on Benard-Marangoni convection in micropolar fluid and they found that the disturbance can be stabilized with a sufficiently strong magnetic field. Effect of suction-injection-combination and magnetic field on Benard convection in micropolar fluid has been discussed in Pranesh [4]. The effects of non-uniform temperature profiles on Marangoni convection in micropolar fluid confined between an upper free, constant heat flux boundary, and a lower rigid isothermal boundary is addressed by Rudraiah and Siddheshwar [5]. They found that micropolar fluid heated from below is more stable compared to the viscous fluid. Recently, the effect of non-uniform temperature gradient and magnetic field on Marangoni convection in a micropolar fluid layer with a prescribed heat flux at its lower boundary has been discussed by Fu et al. [6].

The instabilities of Bénard-Marangoni convection have been investigated in many previous works. However, little research has been studied in the case of a constant heat flux at lower boundary in which no perturbation in the heat flux is allowed. Very recently, Isa et al. [7] and [8] have analyzed the effect of non-uniform temperature and magnetic field on Marangoni and Bénard-Marangoni convection in a horizontal viscous fluid layer subject to a constant heat flux from below. Motivated by these previous works, we aim to study the effect of non-uniform temperature gradient and magnetic field on Bénard-Marangoni convection in a micropolar fluid subject to a constant heat flux at a lower boundary. The linear stability theory is applied and the resulting eigenvalue problem is solved using the single-term Galerkin expansion.

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II. Mathematical Formulation

We consider an infinite horizontal layer of Boussinesquian electrically conducting micropolar fluid of thickness d. The lower boundary is assumed to be rigid, while the upper free surface which is in contact with air and subjected to temperature-dependent surface tension forces is assumed to be flat and nondeformable. We use Cartesian coordinates with two horizontal $x-$ and $y-$ axis located at the lower solid boundary and a positive $z-$ axis is directed towards the free surface. The magnetic field, H_0 acts in the $z-$ direction. Let ΔT be the temperature difference between lower and upper boundaries of the fluid. The interface at the upper boundary has a temperature dependent surface tension $\sigma(T)$ given by

$$\sigma = \sigma_0 - \sigma_1(T - T_0), \tag{1}$$

where σ_0 is the unperturbed value of σ and $\sigma_1 = -\left(d\sigma/dT\right)_{T_0}$.

We follow the governing equations presented by Siddheshwar and Pranesh [3]:
Continuity equation

$$\nabla \cdot q = 0. \tag{2}$$

Conservation of linear momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla\right) \vec{q}\right] = -\nabla P - \rho \vec{g} \hat{k} \; + \; (2\zeta + \eta) \nabla^2 \vec{q} + \zeta \nabla \times \vec{\omega} \; + \; \mu_m \left(\vec{H} \cdot \nabla\right) \vec{H}. \tag{3}$$

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Conservation of angular momentum

$$\rho_0 I\left[\frac{\partial\vec{\omega}}{\partial t}+(\vec{q}\cdot\nabla)\vec{\omega}\right]=\left(\lambda^{'}+\eta^{'}\right)\nabla(\nabla\cdot\vec{\omega})+\eta^{'}\nabla^2\vec{\omega}+\zeta(\nabla\times\vec{q}-2\vec{\omega}). \quad (4)$$

Conservation of energy

$$\frac{\partial T}{\partial t}+\left(\vec{q}-\frac{\beta}{\rho_0 C_\nu}\nabla\times\vec{\omega}\right)\cdot\nabla T=\chi\nabla^2 T. \quad (5)$$

Equation of state

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right]. \quad (6)$$

Magnetic induction equation

$$\frac{\partial\vec{H}}{\partial t}+(\vec{q}\cdot\nabla)\vec{H}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\gamma_m\nabla^2\vec{H}. \quad (7)$$

where $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, $\vec{H}$ is the magnetic field, $P=p+(\mu_m/2)H^2$ is the hydromagnetic pressure, ρ is the density, ρ_0 is the density of the fluid at a reference temperature $T=T_0$, $\vec{g}$ is the acceleration due to gravity, ζ is the coupling viscosity coefficient or vortex viscosity, η is the shear kinematic viscosity coefficient, I is the moment of inertia, $\lambda^{'}$ and $\eta^{'}$ are the bulk and shear spin viscosity coefficient, β is the micropolar heat conduction coefficient, C_ν is the specific heat, χ is the thermal conductivity, α is the coefficient of thermal expansion and $\gamma_m=1/\mu_m\sigma_m$ is the magnetic viscosity (where μ_m is the magnetic permeability and σ_m is the electrical conductivity). We note that all material coefficients are all positive quantities and restricted on the assumption of the Clausius-Duhem inequality (Eringen [2]).

Equations (2) - (7) are solved subject to the boundary conditions appropriate to a rigid and thermally perfect insulating wall at the lower boundary and by a free surface at the upper boundary of the micropolar fluid. The free surface is subject to an adiabatic condition (constant heat flux). Since the shear stress for micropolar fluid is no different from that of viscous fluid, the boundary condition for the flat free boundaries in respect of Newtonian fluids are also appropriate for micropolar fluid. In addition, we assume the spin-vanishing boundary condition at the boundaries.

The basic state is given by

$$\vec{q}_b=0,\ \vec{\omega}_b=0,\ \vec{H}_b=H_0\hat{k},$$
$$p=p_b(z),\ \rho=\rho_b(z),\ -\frac{d}{\Delta T}\frac{dT_b}{dz}=f(z). \quad (8)$$

In equation (8), the non-uniformity in T_b finds its origin in transient heating or cooling at the boundaries of the fluid layer (Rudraiah and Siddheshwar [5]). Nondimensional temperature gradient $f(z)$ must satisfy the condition

$$\int_0^1 f(z)dz=1.$$

To investigate the effect of the non-uniform temperature gradient on the convection, six types of basic temperature profile are chosen and these have been presented in Table I as model $i=1-6$ (see Rudraiah and Siddheshwar [5]).

We express the perturbation quantities as normal modes in the form

$$[W,\,\Omega,\,T,\,H_z]=[W(z,t),\,G(z,t),\,T(z,t),\,H_z(z,t)]\exp\left[i(lx+my)\right], \quad (9)$$

TABLE I
REFERENCE STEADY-STATE TEMPERATURE GRADIENT (δ =DIRAC DELTA-FUNCTION, ϵ = TIME DEPENDENT THERMAL DEPTH PARAMETER).

Model (i)	Reference steady-state temperature gradient	$f(z)$
1	Linear	1
2	Inverted parabola	$2(1-z)$
3	Parabola	$2z$
4	Step function	$\delta(z-\epsilon)$
5	Piecewise linear (heated from below)	ϵ^{-1} for $0\le z<\epsilon$, 0 for $\epsilon<z\le 1$.
6	Piecewise linear (cooled from above)	0 for $0\le z<1-\epsilon$, ϵ^{-1} for $1-\epsilon<z\le 1$.

where l and m are horizontal components of the wave number $\vec{a}$. The amplitudes of the perturbations of velocity, spin and temperature defined to be $W(z,t), G(z,t)$ and $T(z,t)$ respectively. This expression is used in the linearized version of the basic equation. Then, we make the resulting equations dimensionless by using the following definitions:

$$(x^*,y^*,z^*)=\frac{(x,y,z)}{d},\ \vec{q}^{\,*}=\frac{\vec{q}^{\,'}}{\chi/d},$$
$$\vec{\omega}^{\,*}=\frac{\vec{\omega}^{\,'}}{\chi/d^2},\ T^*=\frac{T^{'}}{\Delta T},\ \vec{H}^{\,*}=\frac{\vec{H}^{\,'}}{H_0}. \quad (10)$$

We assume the principle of exchange of stability is valid and our present analysis deals with stationary convection. Then we yield

$$(1+N_1)\left(D^2-a^2\right)^2W+N_1\left(D^2-a^2\right)G-a^2RT-QD^2W=0, \quad (11)$$

$$N_1\left(D^2-a^2\right)W-N_3\left(D^2-a^2\right)G+2N_1G=0, \quad (12)$$

$$\left(D^2-a^2\right)T+f(z)\left(W-N_5G\right)=0, \quad (13)$$

where $D\equiv d/dz$ and the asterisks have been dropped for simplicity. From (11) - (13), we have three non-dimensional groups given by:
Coupling Parameter

$$N_1=\frac{\zeta}{\zeta+\eta}\ \ (0\le N_1\le 1)$$

Couple Stress Parameter

$$N_3=\frac{\eta^{'}}{(\zeta+\eta)\,d^2}\ \ (0\le N_3\le m)$$

Heat Conduction Parameter

$$N_5=\frac{\beta}{\rho_0 C_\nu d^2}\ \ (0\le N_5\le n)$$

where m and n are finite, positive real numbers. The range of values of N_1, N_3 and N_5 is guided by the Clausius-Duhem inequality (Eringen [2]).

Equations (11) - (13) subject to

$$W=DW=DT=G=0 \text{ at } z=0$$
$$W=D^2W+a^2MT=DT=G=0 \text{ at } z=1 \quad (14)$$

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where $M = \sigma_T \Delta T d/\mu\chi$ is the Marangoni number. Equation (14) refers to the case of rigid, adiabatic conditions at the lower rigid boundary $(z = 0)$, and constant heat flux at the free surface $(z = 1)$ of the micropolar fluid. The condition on G is the spin-vanishing boundary condition.

To solve the problem, we use a single term Galerkin technique to find the critical eigen value. Firstly, we multiply equations (11), (12) and (13) by W, G, and T, respectively. From the resulting equations, we perform the integration by parts with respect to z from 0 to 1 and we use the boundary conditions (14). Then, we take $W = AW_1$, $G = BG_1$ and $T = CT_1$ in which A, B and C are constants and $W_1 = z^2 - z^3$, $G_1 = z - z^2$ and $T_1 = 1$ are trial functions. The trial functions will be generally chosen in such a way that they satisfy all the boundary conditions except the one given by $D^2W + a^2MT = 0$ at $z = 1$, but the residual from this is included in the residual from the differential equations. Finally, we obtain the following equation for the eigen value:

$$M = \left[\frac{8b_1\left[(1+N_1)\,b_2 + 14Q\right] - 7N_1^2 b_3^2}{420\,(1+N_1)\,b_4}\right] - \frac{R}{12\,(1+N_1)}, \tag{15}$$

where
$b_1 = N_3 b_3 + 2N_1$,
$b_2 = 420 + 28a^2 + a^4$,
$b_3 = 10 + a^2$,
$b_4 = 2b_1 \left\langle f(z)\left(z^2 - z^3\right)\right\rangle - N_5 N_1 b_3 \left\langle f(z)\left(z - z^2\right)\right\rangle$.

In b_4, $\langle ... \rangle$ denotes integration with respect to z between $z = 0$ and $z = 1$.

III. Results and Discussions

In this paper, we study the effect of six types of basic temperature profiles ($i = 1-6$) and vertical magnetic field on the onset of Bénard-Marangoni convection with non-slip and adiabatic condition at the bottom boundary of the micropolar fluid layer. The single Galerkin procedure provides a good method for establishing this problem. Table II show the critical Marangoni number M_c for different non-uniform temperature profiles when $Q = 10$. From Table II, it can be seen that the increasing values of critical Marangoni number is due to the increasing of N_1 and increase in N_1 indicates the increase in the concentration of microelements, and as a result coupling parameter stabilized the system. The corresponding effect of the large magnetic field ($Q = 1000$) and coupling parameter on the onset of convection may be viewed in Table III. In Table III, M_c for step function, piecewise linear (cooled from above) and parabola profiles decrease monotonically with the increasing of N_1. So, it prove that the critical M_c for micropolar will always less than the Newtonian value for these 3 temperature profiles.

IV. Conclusion

In the present paper the problem on combined effect of the non-uniform temperature gradient and magnetic field on the onset of Bénard-Marangoni convection in a micropolar fluid layer heated from below is investigated. We found that the critical Marangoni number increases with increasing Chandrasekhar number for all non-uniform temperature considered. We can conclude that it is possible to delay the onset of convection by the application of an inverted parabola profile and imposed of magnetic field. As expected, the electrically conducting micropolar fluid layer heated from below is more stable compared to the Newtonian fluid.

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TABLE II
CRITICAL MARANGONI NUMBER M_c FOR VARIOUS VALUES OF N_1 AND R WHEN $Q = 10$, $N_3 = 2.0$ AND $N_5 = 1.0$.

Q	N_1	a_c	R	$(M_c)_4$	$(M_c)_6$	$(M_c)_3$	$(M_c)_5$	$(M_c)_1$	$(M_c)_2$
10	0	0	0	36.000	46.390	53.333	60.750	64.000	80.000
			200.000	19.334	29.723	36.667	44.083	47.333	63.333
			432.000	0	10.390	17.333	24.750	28.000	44.000
			556.679		0	6.943	14.360	17.610	33.610
			640.000			0	7.417	10.667	27.000
			729.000				0	3.250	19.250
			768.000					0	16.000
			960.000						0
	0.5		0	40.115	51.703	60.908	73.709	76.896	104.266
			200.000	29.004	40.592	49.796	62.598	65.785	93.154
			722.074	0	11.588	20.792	33.594	36.781	64.150
			930.650		0	9.205	22.007	25.193	52.563
			1096.337			0	12.802	15.988	43.358
			1326.770				0	3.186	30.556
			1384.125					0	27.370
			1876.780						0
	1.0		0	47.587	61.349	74.817	99.648	102.250	161.447
			200.000	39.254	53.016	66.484	91.314	93.917	153.114
			1142.085	0	13.762	27.230	52.061	54.663	113.861
			1472.377		0	13.468	38.299	40.901	100.098
			1795.610			0	24.831	27.433	86.630
			2391.546				0	2.602	61.800
			2454.000					0	59.197
			3874.737						0

TABLE III
CRITICAL MARANGONI NUMBER M_c FOR VARIOUS VALUES OF N_1 AND R WHEN $Q = 1000$, $N_3 = 2.0$ AND $N_5 = 1.0$.

Q	N_1	a_c	R	$(M_c)_4$	$(M_c)_6$	$(M_c)_3$	$(M_c)_5$	$(M_c)_1$	$(M_c)_2$
10^3	0	0	0	927.001	1194.540	1373.333	1564.313	1648.000	2060.000
			200.000	910.334	1177.873	1356.667	1547.646	1631.333	2043.333
			11124.010	0	267.539	446.333	637.312	720.999	1132.999
			14334.480		0	178.793	369.773	453.460	865.460
			16480.000			0	190.979	274.667	686.667
			18771.750				0	83.687	495.687
			19776.000					0	412.000
			24720.000						0
	0.5		0	763.168	983.614	1158.729	1402.277	1462.896	1983.588
			200.000	752.057	972.503	1147.618	1391.166	1451.785	1972.476
			13737.020	0	220.446	395.562	639.109	699.728	1220.420
			17705.050		0	175.116	418.663	479.282	999.974
			20857.130			0	243.547	304.167	824.858
			25240.980				0	60.619	581.311
			26332.120					0	520.692
			35704.580						0
	1.0		0	723.344	932.535	1137.256	1514.695	1554.250	2454.079
			200.000	715.010	924.202	1128.923	1506.361	1545.917	2445.746
			17360.250	0	209.192	413.912	791.351	830.906	1730.735
			22380.850		0	204.721	582.159	621.715	1521.544
			27294.150			0	377.439	416.994	1316.823
			36352.670				0	39.555	939.384
			37302.000					0	899.829
			58897.890						0

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