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# Some Properties of Copper – Gold and Silver – Gold Alloys at Different % of Gold

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***Abstract*: The norm of elastic constant tensor and the norms of the irreducible parts of the elastic constants of Copper, Silver and Gold metals and Copper-Gold and Silver-Gold alloys at different percentages of Gold are calculated. The relation of the scalar parts norm and the other parts norms and the anisotropy of these metals and their alloys are presented. The norm ratios are used to study anisotropy of these metals and their alloys.**

**Index Terms** -Copper, Silver, Gold, Alloys, Anisotropy, Elastic Constants.

## I. ELASTIC CONSTANT TENSOR DECOMPOSITION

The constitutive relation characterizing linear anisotropic solids is the generalized Hook's law [1]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}\,,\ \varepsilon_{ij} = S_{ijkl}\sigma_{kl} \tag{1}$$

Where $\sigma_{ij}$ and $\varepsilon_{kl}$ are the symmetric second rank stress and strain tensors, respectively $C_{ijkl}$ is the fourth-rank elastic stiffness tensor (here after we call it elastic constant tensor) and $S_{ijkl}$ is the elastic compliance tensor.

There are three index symmetry restrictions on these tensors. These conditions are:

$$C_{ijkl} = C_{jikl}\,,\ C_{ijkl} = C_{ijlk}\,,\ C_{ijkl} = C_{klij} \tag{2}$$

Which the first equality comes from the symmetry of stress tensor, the second one from the symmetry of strain tensor, and the third one is due to the presence of a deformation potential. In general, a fourth-rank tensor has 81 elements. The index symmetry conditions (2) reduce this number to 81. Consequently, for most asymmetric materials (triclinic symmetry) the elastic constant tensor has 21 independent components.

Elastic compliance tensor $S_{ijkl}$ possesses the same symmetry properties as the elastic constant tensor $C_{ijkl}$ and their connection is given by [2,3,4,5]:

$$C_{ijkl}S_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right) \tag{3}$$

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Where $\delta_{ij}$ is the Kronecker delta. The Einstein summation convention over repeated indices is used and indices run from 1 to 3 unless otherwise stated.

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic constant tensor is obtained. It contains two scalars, two deviators, and one-nonor parts:

$$C_{ijkl} = C^{(0;1)}_{ijkl} + C^{(0;2)}_{ijkl} + C^{(2;1)}_{ijkl} + C^{(2;2)}_{ijkl} + C^{(4;1)}_{ijkl} \tag{4}$$

Where:

$$C^{(0;1)}_{ijkl} = \frac{1}{9}\delta_{ij}\delta_{kl}C_{ppqq}\,, \tag{5}$$

$$C^{(0;2)}_{ijkl} = \frac{1}{90}\left(3\delta_{ik}\delta_{jl} + 3\delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\left(3C_{pqpq} - C_{ppqq}\right) \tag{6}$$

$$C^{(2;1)}_{ijkl} = \frac{1}{5}\left(\delta_{ik}C_{jplp} + \delta_{jk}C_{iplp} + \delta_{il}C_{jpkp} + \delta_{jl}C_{ipkp}\right) - \frac{2}{15}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)C_{pqpq} \tag{7}$$

$$\begin{aligned} C^{(2;2)}_{ijkl} = {} & \frac{1}{7}\delta_{ij}\left(5C_{klpp} - 4C_{kplp}\right) + \frac{1}{7}\delta_{kl}\left(5C_{ijpp} - 4C_{ipjp}\right) \\ & - \frac{2}{35}\delta_{ik}\left(5C_{jlpp} - 4C_{jplp}\right) - \frac{2}{35}\delta_{jl}\left(5C_{ikpp} - 4C_{ipkp}\right) \\ & - \frac{2}{35}\delta_{il}\left(5C_{jkpp} - 4C_{iplp}\right) - \frac{2}{35}\delta_{jk}\left(5C_{ilpp} - 4C_{iplp}\right) \\ & + \frac{2}{105}\left(2\delta_{jk}\delta_{il} + 2\delta_{ik}\delta_{jl} - 5\delta_{ij}\delta_{kl}\right)\left(5C_{ppqq} - 4C_{pqpq}\right) \end{aligned} \tag{8}$$

$$C^{(4;1)}_{ijkl} = \frac{1}{3}(C_{ijkl} + C_{ikjl} + C_{iljk}) - \frac{1}{21}\left[\delta_{ij}\left(C_{klpp} + 2C_{kplp}\right) + \delta_{ik}\left(C_{jlpp} + 2C_{jplp}\right)\right.$$

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$$+\delta_{il}\left(C_{jkpp}+2C_{jpkp}\right)+\delta_{jk}\left(C_{ilpp}+2C_{iplp}\right)$$
$$+\delta_{jl}\left(C_{ikpp}+2C_{ipkp}\right)+\delta_{kl}\left(C_{ijpp}+2C_{ipjp}\right)\Big]$$
$$+\frac{1}{105}\left[\left(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)\left(C_{ppqq}+2C_{pqpq}\right)\right] \quad (9)$$

These parts are orthonormal to each other. Using Voigt's notation [1] for $C_{ijkl}$, can be expressed in 6 by 6 reduced matrix notation, where the matrix coefficients $c_{\mu\lambda}$ are connected with the tensor components $C_{ijkl}$ by the recalculation rules:

$$c_{\mu\lambda}=C_{ijkl}\,; \qquad (ij\leftrightarrow\mu=1,....,6,\, kl\leftrightarrow\lambda=1,....,6)$$

That is:

$$11\leftrightarrow 1,\,22\leftrightarrow 2,\,33\leftrightarrow 3,\,23=32\leftrightarrow 4$$
$$31=13\leftrightarrow 5,\,12=21\leftrightarrow 6.$$

II. THE NORM CONCEPT

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

$$N=\|T\|=\left\{T_{ijkl..........}.T_{ijkl..........}\right\}^{1/2}$$

Denoting rank-n Cartesian $T_{ijkl..........}$, by $T_n$, the square of the norm is expressed as [5]:

$$N^2=\|T\|^2=\sum_{j,q}\left\|T^{(j;q)}\right\|^2=\sum_{(n)}T_{(n)}T_{(n)}=\sum_{(n),j,q}T_{(n)}^{(j;q)}T_{(n)}^{(j,q)}$$

This definition is consistent with the reduction of the tensor in tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space.

Since the norm of a Cartesian tensor is an invariant quantity, we suggest the following:

Rule1. The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger the norm value, the more effective the property is.

It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. We know that, for isotropic materials, the elastic compliance tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic compliance tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N=N_s$, Hence, the ratio $\frac{N_s}{N}=1$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so we can define $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{sc}}{N}$ for septor parts, and $\frac{N_n}{N}$ for nonor parts. Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part they can also be used to asses the anisotropy degree of a material property as a whole, we suggest the following two more rules:

Rule 2. When $N_s$ is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

Rule3. When $N_s$ is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the larger the norm ratio value we have, the more anisotropic the material property is.

The square of the norm of the elastic stiffness tensor (elastic constant tensor) $\boldsymbol{C_{mn}}$ is:

$$\|N\|^2=\sum_{mn}\left(C_{mn}^{(0;1)}\right)^2+\sum_{mn}\left(C_{mn}^{(0;2)}\right)^2$$
$$+2\sum_{mn}\left(C_{mn}^{(0;1)}.C_{mn}^{(0;2)}\right)+\sum_{mn}\left(C_{mn}^{(2;1)}\right)^2+\sum\left(C_{mn}^{(2;2)}\right)^2$$
$$+2\sum_{mn}\left(C_{mn}^{(2;1)}.C_{mn}^{(2;2)}\right)+\sum_{mn}\left(C_{mn}^{(4;1)}\right)^2 \quad (10)$$

Let us consider the irreducible decompositions of the elastic stiffness tensor (elastic constant tensor) in the following elements and alloys:


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By using table1, table 2, and table 3 and the decomposition of the elastic constant tensor, we have calculated the norms and the norm ratios as is shown in table 4, table 5 and in table 6.

Table 1, Elastic Constants (GPa), [2]

| Element, Cubic System | $c_{11}$ | $c_{44}$ | $c_{12}$ |
|---|---|---|---|
| Gold, Au | 190 | 42.3 | 161 |
| Silver, Ag | 122.2 | 46.1 | 91.8 |
| Copper, Cu | 169 | 75.3 | 122 |

Table 2, Elastic Constants (GPa) [2]

| Alloy, Cubic System Copper -Gold, Cu-Au,At % Gold | $c_{11}$ | $c_{44}$ | $c_{12}$ |
|---|---|---|---|
| 0.23 | 170.0 | 74.2 | 123.3 |
| 2.8 | 169.2 | 73.9 | 123.9 |
| 10 | 174.7 | 73.1 | 131.0 |
| 50 | 188.3 | 41.5 | 150.3 |
| 80 | 191.3 | 47.5 | 156.3 |

Table 3, Elastic Constants (GPa) [2]

| Alloy, Cubic System Silver-Gold, Ag-Au,At % Gold | $c_{11}$ | $c_{44}$ | $c_{12}$ |
|---|---|---|---|
| 2 | 123.7 | 46.9 | 93.0 |
| 4 | 124.1 | 47.3 | 92.8 |
| 25 | 138.5 | 48.7 | 104.5 |
| 50 | 147.7 | 50.8 | 113.0 |
| 75 | 166.5 | 48.6 | 132.5 |

Table 4, the norms and norm ratios

| Element | $N_s$ | $N_d$ | $N_n$ | $N$ | $\frac{N_s}{N}$ | $\frac{N_d}{N}$ | $\frac{N_n}{N}$ |
|---|---|---|---|---|---|---|---|
| Gold, Au | 522.339 | 0 | 50.958 | 524.819 | 0.9953 | 0 | 0.0971 |
| Silver, Ag | 326.352 | 0 | 54.624 | 330.892 | 0.9863 | 0 | 0.1651 |
| Copper, Cu | 450.929 | 0 | 94.951 | 460.818 | 0.9785 | 0 | 0.2061 |

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Table 5, the norms and norm ratios

| Alloy, Cubic System Copper -Gold, Cu-Au,At % Gold | $N_s$ | $N_d$ | $N_n$ | $N$ | $\frac{N_s}{N}$ | $\frac{N_d}{N}$ | $\frac{N_n}{N}$ |
|---|---|---|---|---|---|---|---|
| 0.23 | 453.283 | 0 | 93.210 | 462.767 | 0.9795 | 0 | 0.2014 |
| 2.8 | 453.052 | 0 | 93.943 | 462.689 | 0.9792 | 0 | 0.2030 |
| 10 | 470.257 | 0 | 93.943 | 479.548 | 0.9806 | 0 | 0.1959 |
| 50 | 500.642 | 0 | 41.243 | 502.337 | 0.9966 | 0 | 0.0821 |
| 80 | 517.473 | 0 | 54.991 | 520.386 | 0.9944 | 0 | 0.1057 |

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Table 6, the norms and norm ratios

| Alloy, Cubic System Silver-Gold, Ag-Au,At % Gold | $N_s$ | $N_d$ | $N_n$ | $N$ | $\frac{N_s}{N}$ | $\frac{N_d}{N}$ | $\frac{N_n}{N}$ |
|---|---|---|---|---|---|---|---|
| 2 | 329.910 | 0 | 57.832 | 334.940 | 0.9850 | 0 | 0.1727 |
| 4 | 330.323 | 0 | 58.015 | 335.379 | 0.9849 | 0 | 0.1730 |
| 25 | 367.462 | 0 | 58.107 | 372.028 | 0.9877 | 0 | 0.1562 |
| 50 | 393.77 | 0 | 61.315 | 398.515 | 0.9881 | 0 | 0.1539 |
| 75 | 447.679 | 0 | 57.924 | 451.411 | 0.9917 | 0 | 0.1283 |

## III. CONCLUSION

We can conclude from table 4, by considering the ratio $\frac{N_s}{N}$ that Gold, is more isotropic than Silver, and Silver is more isotropic than Copper and by considering the value of $\boldsymbol{N}$ we found that this value is more high for Gold than Copper and this value for Copper is more high than Silver so we can say that Gold elastically is more strong than Copper, and Copper elastically is more strong than Silver.

And we can conclude from table 5 by considering the ratio $\frac{N_s}{N}$ that in the Alloy Cu-Au as the percentage of Au increases (from 0.23% to 2.8%) the anisotropy of the alloy increases, but as the percentage of Au increases (from 2.8% tom %50) the anisotropy decreases and as the percentage of Au (from 50% to 80%) the anisotropy increases, and by considering the value of $\boldsymbol{N}$ as the percentage of Au increases (from 2.8 % to 80%), the value of $\boldsymbol{N}$ increases so we can say that the alloy becomes elastically more strongest.

And we can conclude from table 6 by considering the ratio $\frac{N_s}{N}$ that in the Alloy Ag-Au as the percentage of Au increases (from 2%

and 4% to 75%) the anisotropy of the alloy decreases , and by considering the value of $\boldsymbol{N}$ as the percentage of Au increases (from 2% to 75%), the value of $\boldsymbol{N}$ increases so we can say that the alloy becomes elastically more strongest.

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