Radiation Effects on Marangoni Boundary Layer Flow Past a Flat Plate in Nanofluid

Rohana Abdul Hamid, Norihan Md. Arifin, Roslinda Mohd Nazar, and Ioan Pop

***Abstract*— The problem of Marangoni convection boundary layer flow in nanofluid with radiation effects is studied using different types of nanoparticles. The general governing partial differential equations are transformed into a set of two nonlinear ordinary differential equations using unique similarity transformation. Numerical solutions of the similarity equations are obtained using the Runge-Kutta-Fehlberg method. Three different types of nanoparticles, namely Cu, Al_2O_3, and TiO_2 are considered by using water as a base fluid with Prandtl number Pr = 6.2. The effects of radiation parameter on the heat transfer characteristics are discussed.**

***Index Terms*—Marangoni convection, boundary layer, nanofluids, radiation**

I. Introduction

Among the tasks facing by the engineer is the development of ultrahigh-performance cooling in many industrial technologies. This is where nanotechnology takes important part for further development of high performance, compact, cost-effective liquid cooling systems. Other than that, nanofluids have effective applications in many industries such as electronics, transportation, biomedical and many more [1]. Nanotechnology has been an ongoing topic of discussion in public health as some of the researchers claimed that nanoparticles could present possible dangers in health and environment [2].

For the past few years, there are numerous studies regarding nanofluids that have been done and rapidly increase of the works shows the importance of the field. An excellent collection of papers on nanofluids can be found in the review papers by Buongiorno [3], Daungthongsuk and Wongwises [4], Trisaksri and Wongwises [5], Wang and Mujumdar [6,7], and Kakaç and Pramuanjaroenkij [8].

There are also several experimental studies to better understand the mechanism of heat transfer enhancement during convection heat transfer in nanofluids [9-11]. Radiation – convection interaction problems are found in consideration of the cooling of high temperature components, convection cells and their effect on radiation from stars, furnace design where heat transfer from surfaces occurs by parallel radiation and convection, the interaction of incident solar radiation with the earth's surface to produce complex free convection patterns and thus to complicate the art of weather forecasting and marine environment studies for predicting free convection patterns in oceans and lakes [12]. Because of the reasons, researchers started to investigate the interaction between the two modes of heat transfer such as Cess [13] and Al-Amri and El-Shaarawi [14] who studied the effects of thermal radiation on the forced convection boundary layer. Marangoni convection which induced by the variations of the surface tension gradients has important application in the fields of welding and crystal growth. Moreover, the convection is necessary to stabilize the soap films and drying silicon wafers. Many researchers have investigated Marangoni convection in various geometries such as Okano et al. [15], Christopher and Wang [16], Pop et al. [17] and Magyari and Chamka [18].

The nanofluid equations model proposed by Tiwari and Das [19] has been very successfully used in several papers (Oztop and Abu-Nada [20], Muthtamilselvan *et al.* [21] and Arifin *et al*. [22]). Very recently, Arifin *et al*. [23] has studied a similarity solution for Marangoni boundary layer flow over a flat plate due to an imposed temperature gradient in a nanofluid. Marangoni flow induced by surface tension along a liquid surface causes undesirable effects in crystal growth melts in the same manner as buoyancy-induced natural convection (Christopher and Wang [16]). However, this problem has been extended by Arifin *et al.* [24] when the wall is permeable, that is there is a suction or injection effect.

In this paper, we present further similarity solutions for the Marangoni convection boundary layer flow over a flat plate in a nanofluid with radiation effects. Results are presented in graphs showing the influence of the emerging parameters.

II. Basic Equations

We consider the steady two-dimensional Marangoni boundary layer flow past a semi-infinite flat plate in a water-based nanofluid containing different type of nanoparticles, namely, copper (Cu), aluminium oxide (Al_2O_3), and titanium dioxide (TiO_2) with radiation effects. The nanofluid is assumed incompressible and the flow is assumed to be laminar. It is also assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical

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properties of the nanofluids are given in Table 1 (see Oztop and Abu-Nada [20]).

Further, we consider a Cartesian coordinate system (x, y), where x and y are the coordinates measured along the plate and normal to it, respectively, and the flow takes place at $y \geq 0$. It is also assumed that the temperature of the plate is $T_w(x)$ and that of the ambient nanofluid is T_∞. Following Golia and Viviani [25,26] and Magyari and Chamkha [18] the surface tension σ is assumed to vary linearly with temperature as

$$\sigma = \sigma_0 \left[1 - \gamma (T - T_0)\right] \quad (1)$$

where σ_0 and T_0 are the surface tension and temperature at the slit, respectively, and we assume that $T_0 \equiv T_\infty$. For most liquids the surface tension σ decreases with temperature, i.e. γ is a positive fluid property.

The steady boundary layer equations for a nanofluid in the coordinates $\bar{x}$ and $\bar{y}$ are (see Christopher and Wang [16] and Tiwari and Das [19])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y} \quad (4)$$

subject to the boundary conditions

$$v = 0, \quad T = T_0 + A x^{m+1}, \quad \mu_{\mathrm{nf}}\frac{\partial u}{\partial y} = \frac{\partial \sigma}{\partial T}\frac{\partial T}{\partial x} \quad \text{at} \quad y = 0$$
$$u = 0, \quad T = T_\infty \quad \text{as} \quad y \to \infty \quad (5)$$

Here u and v are velocity components along the x and y axes, respectively, T is the temperature of the nanofluid, m is the constant exponent of the temperature, q_r is the radiative heat flux, α_{nf} is the thermal diffusivity of the nanofluid, ρ_{nf} is the effective density of the nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid and μ_{nf} is the effective viscosity of the nanofluid, which are given by

$$\alpha_{\mathrm{nf}} = \frac{k_{\mathrm{nf}}}{(\rho C_p)_{\mathrm{nf}}}, \quad \rho_{\mathrm{nf}} = (1-\phi)\rho_{\mathrm{f}} + \phi\rho_{\mathrm{s}},$$
$$\mu_{\mathrm{nf}} = \frac{\mu_{\mathrm{f}}}{(1-\phi)^{2.5}}, \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_{\mathrm{f}} + \phi(\rho C_p)_{\mathrm{s}},$$
$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \frac{(k_{\mathrm{s}} + 2k_{\mathrm{f}}) - 2\phi(k_{\mathrm{f}} - k_{\mathrm{s}})}{(k_s + 2k_{\mathrm{f}}) + \phi(k_{\mathrm{f}} - k_{\mathrm{s}})} \quad (6)$$

where ϕ is the solid volume fraction of the nanofluid, ρ_f is the reference density of the fluid fraction,ρ_s is the reference velocity of the solid fraction, μ_f is the viscosity of the fluid fraction, k_f is the thermal conductivity of the fluid and k_s is the thermal conductivity of the solid, and $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid.

Using the Rosseland approximation for radiation [27], the radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (7)$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow such the term T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor's series about T_∞ and neglecting higher-order terms, thus

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

Using (7) and (8), (4) reduces to [28]

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}(1 + Nr)\frac{\partial^2 T}{\partial y^2} \quad (9)$$

where $Nr = 16\sigma^* T_\infty^3/(3k_{nf}k^*)$ is the radiation parameter.

We look now for a similarity solution of (2)-(4) subject to the boundary conditions (5) of the following form:

$$\psi = C_1 x^{(2+m)/3} f(\eta), \ \theta(\eta) = (T - T_\infty)/(Ax^{1+m})$$
$$\eta = C_2 x^{(m-1)/3} y \quad (10)$$

where ψ is the stream function which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Further, m, A, C_1 and C_2 are constants with A, C_1 and C_2 given by

$$A = \Delta T / L^{m+1}, \quad C_1 = \sqrt[3]{\sigma_0 \gamma A \mu_{\mathrm{f}} / \rho_{\mathrm{f}}^2},$$
$$C_2 = \sqrt[3]{\sigma_0 \gamma A \rho_{\mathrm{f}} / \mu_{\mathrm{f}}^2}, \quad (11)$$

with L being the length of the surface and ΔT is the constant characteristic temperature.

Substituting (6) and (10) into (3) and (9), we get the following ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)} f''' + \frac{2+m}{3} ff'' - \frac{1+2m}{3} f'^2 = 0 \quad (12)$$

$$\frac{1}{\Pr}(1+Nr)\frac{k_{nf}/k_f}{(1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f)}\theta'' + \frac{2+m}{3} f\theta' - (1+m) f'\theta = 0 \quad (13)$$

and the boundary conditions (5) become

$$f(0) = 0, \ \frac{1}{(1-\varphi)^{2.5}} f''(0) = -1, \ \theta(0) = 1$$
$$f'(\infty) = 0, \ \theta(\infty) = 0 \quad (14)$$

We can now determine the surface velocity $u(x,0) = u_w(x)$ as

$$u_w(x) = \sqrt[3]{\frac{(\sigma_0 \gamma A)^2}{\rho_{\mathrm{f}} \mu_{\mathrm{f}}}}\, x^{(1+2m)/3} f'(0) \quad (15)$$

A quantity of interest is also the local Nusselt number Nu_x which is defined as

$$Nu_x = \frac{x q_w(x)}{k_f[T(x,0) - T(x,\infty)]} \quad (16)$$

Where $q_w(x)$ is the heat flux from the surface of the plate and it is given by

$$q_w(x) = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (17)$$

Using (9), (15) and (16), we get

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$$Nu_x = -\frac{k_{nf}}{k_f} C_2 x^{(2+m)/3} \theta'(0) \tag{18}$$

The average Nusselt number Nu_L based on the average temperature difference between the temperature of the surface and the temperature far from the surface (ambient fluid) is given by

$$Nu_L = -\frac{6+3m}{5+4m} \frac{k_{nf}}{k_f} Ma_L^{1/3} \Pr^{-1/3} \theta'(0) \tag{19}$$

where Ma_L is the Marangoni based on L and is defined as

$$Ma_L = \frac{\sigma_T A L^{2+m}}{\mu_f \alpha_f} = \frac{\sigma_T \Delta T L}{\mu_f \alpha_f} \tag{20}$$

Also, the total mass flow $\dot{m}$ in the boundary layer per unit width is given by

$$\dot{m} = \int_0^\infty \rho_f u \, dy = \sqrt[3]{\sigma_0 \gamma \rho_f \mu_f} \, x^{(2+m)/3} f(\infty) \tag{21}$$

III. RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (12) and (13) subject to the boundary conditions (14) were solved numerically using the Runge-Kutta-Fehlberg method using Maple 12. Following Oztop and Abu-Nada [20], we considered the range of nanoparticles fraction ϕ as $0 \le \phi \le 0.2$. The Prandtl number of the based fluid (water) is kept constant at $\Pr = 6.2$. Further, it should also be pointed out that the thermophysical properties of fluid and nanoparticles (Cu, Al_2O_3, TiO_2) used in this study are given in Table 1. It is worth mentioning that, the present study reduces to that of a classical viscous (regular) fluid studied by Christopher and Wang [16] for $\phi = 0$ and Arifin et al [23] for $\phi \neq 0$ when Nr = 0. The velocity fields, i.e. the momentum equation solutions have been discussed in Arifin et al. [23] in details. This paper focuses on the heat transfer problem with radiation effect.

TABLE 1

THERMOPHYSICAL PROPERTIES OF FLUID AND NANOPARTICLES (OZTOP AND ABU-NADA [19])

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p (J / kg K)	4179	385	765	686.2
ρ(KG/ m^3)	997.1	8933	3970	4250
k(W/ m K)	0.613	400	40	8.9538

Figure 1 shows the variations of the surface temperature gradient, $-\theta'(0)$ with the radiation parameter, Nr for different nanoparticles Cu, Al_2O_3 and TiO_2 when the nanoparticles fractions are considered to be $\phi = 0$ and $\phi = 0.1$ when Pr = 6.2, m = 0 One can see that the surface temperature gradient, $-\theta'(0)$ decreases as Nr increases for all three nanoparticles. Meanwhile Fig. 2 shows the effects of ϕ on the surface temperature gradients for the nanoparticles Cu, Al_2O_3, and TiO_2 with Nr = 2 and Nr = 5. As ϕ increases, it is seen that the surface temperature gradient, $-\theta'(0)$ decrease. Fig. 3 – 5 illustrate the effects of Nr on temperature profiles for Cu, Al_2O_3, and TiO_2 nanoparticles, respectively. It is observed from these figures that for any type of nanoparticles, as the radiation parameter, Nr increases, the surface temperature gradients decrease, which is in agreement with Fig. 1.

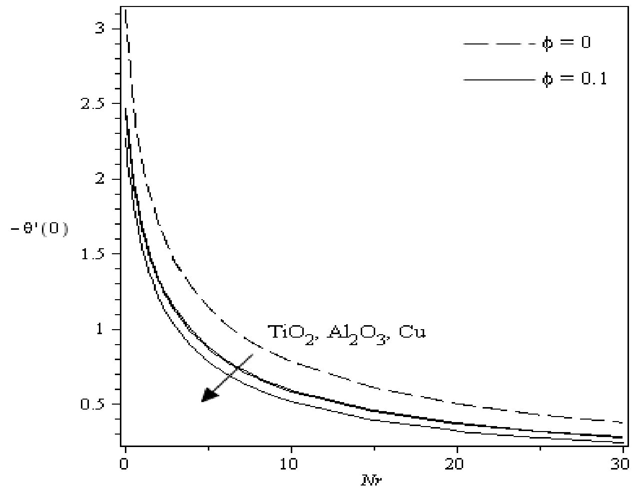

Fig. 1. Variations of $-\theta'(0)$ with Nr for different types of nanoparticles when Pr = 6.2, m = 0and different ϕ.

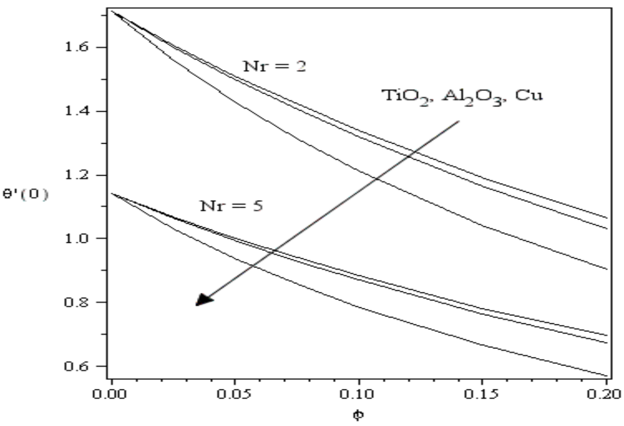

Fig. 2. Variations of $-\theta'(0)$ with ϕ for different types of nanoparticles when Pr = 6.2, m = 0 and different Nr.

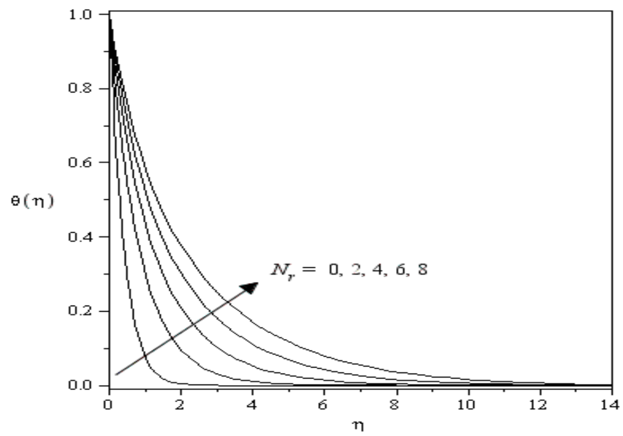

Fig. 3. Temperature profiles for Cu nanoparticles with m = 0, $\phi = 0.1$ for various Nr.

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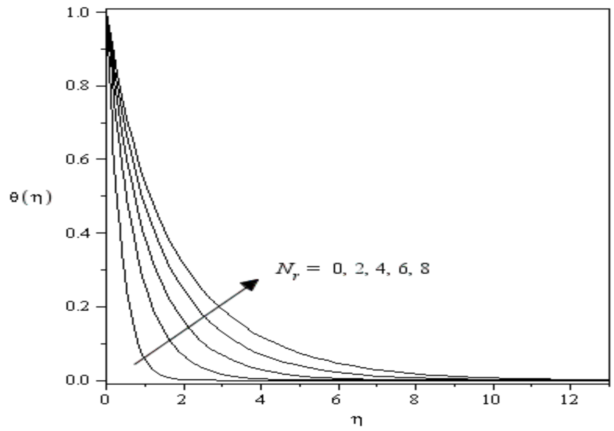

Fig. 4. Temperature profiles for Al_2O_3 nanoparticles with m = 0, $\phi = 0.1$ for various Nr.

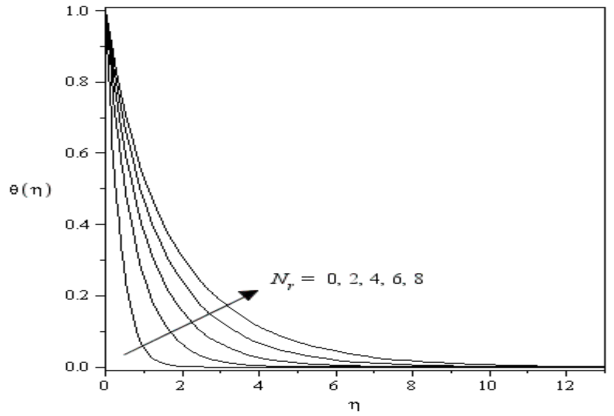

Fig. 5. Temperature profiles for TiO_2 nanoparticles with m = 0, $\phi = 0.1$ for various Nr.

IV. CONCLUSION

We have theoretically studied the problem of two-dimensional laminar Marangoni-driven boundary layer flow in nanofluids with the effects of radiation. Three different types of nanoparticles, namely Cu, Al_2O_3, and TiO_2 are considered. The governing partial differential equations were transformed into a set of two nonlinear ordinary differential equation using similarity transformation, before being solved numerically by the Runge-Kutta-Fehlberg method. Numerical results are obtained for the surface temperature gradient as well as the temperature profiles for some values of the governing parameters, namely the solid volume fraction of the nanofluid ϕ ($0 \le \phi \le 0.2$) and the Prandtl number Pr. The effects of radiation parameter on the heat transfer characteristics are studied.

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