A Note on Scheduling with Learning Effect and Past-sequence-dependent Setup Time

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Abstract — **In this paper, we present a scheduling model where both the learning effect and the setup time are considered. Under the proposed model, the learning effect is expressed as a general function of the actual processing time of jobs already processed and its scheduled position, and the setup time is past-sequence-dependent. We then provide the optimal sequences for some single-machine problems.**

Index terms — **learning effect, past-sequence-dependent setup times, scheduling, single-machine**

I. Introduction

In classical scheduling problems, the job processing times are assumed to be fixed and known. However, recent empirical studies in several industries have demonstrated that unit costs decline as firms produce more of a product and gain knowledge or experience. Moreover, Biskup [1] pointed out that repeated processing of similar tasks improves the worker skills; workers are able to perform setup, to deal with machine operations or software, or to handle raw materials and components at a greater pace. It is known as “learning effect” in the literature.

Biskup [1] was among the pioneers that brought the concept of learning effect into scheduling problems. Since then, many researchers have devoted lots of efforts on this new area. For example, Mosheiov [2] presented several examples to demonstrate that the optimal schedules of some problems with learning effect may be different from those of the classical ones without learning consideration. Lee *et al*. [3] considered a bi-criteria single-machine scheduling problem to minimize the sum of the total completion time and the maximum tardiness. Chen et al. [4] provided the branch-and-bound and heuristic algorithms for a bi-criteria two-machine flowshop scheduling problem. Koulamas and Kyparisis [5] introduced a sum-of-job-processing-time-based learning effect scheduling model in which employees learn more if they perform a job with longer processing time. They showed that the single-machine problems to minimize makespan and total completion time are polynomially solvable. In addition, they proved that the two-machine flowshop problems to minimize makespan and total completion time problems are polynomially solvable under the assumption of ordered or proportional job processing times. Wang [6] considered some single-machine problems with the effects of learning and deterioration, and proved that the makespan and sum of completion times (square) minimization problems remain polynomially solvable. They also showed that the weighted shortest processing time first (WSPT) rule and the earliest due date first (EDD) rule provide the optimal sequence for the weighted sum of completion times and the maximum lateness in some special cases. Janiak and Rudek [7] brought a new learning effect model into the scheduling field where the existing approach is generalized in two ways. First they relaxed one of the rigorous constraints, and thus each job can provide different experience to the processor in their model. Second they formulated the job processing time as a non-increasing *k*-stepwise function that in general is not restricted to a certain learning curve, thereby it can accurately fit every possible shape of a learning function. Eren [8] proposed a non-linear mathematical programming model for a single machine scheduling problem with unequal release dates and learning effects. Wang [9] dealt with the single machine scheduling problems with a time-dependent learning effect and deteriorating jobs. He showed that the makespan problem remains polynomially solvable, but the classical shortest processing time rule does not provide the optimal solution. Janiak and Rudek [10] brought into scheduling a new approach called multi-abilities learning that generalizes the existing ones and models more precisely real-life settings. On this basis, they focused on the makespan problem with the proposed learning model and provide optimal polynomial time algorithms for its special cases. Lee et al. [11] investigated a single-machine problem with the learning effect and release times where the objective is to minimize the makespan. Huang et al. [12] consider the single machine scheduling problems with time-dependent deterioration and exponential learning effect. They provided the optimal solutions for some single machine problems. Cheng et al. [13] introduced a new scheduling model in which job deterioration and learning, and setup times are considered simultaneously. They showed some single machine problems remain polynomially solvable. Biskup [14] reviewed the scheduling problems with learning effects.

Recently, Koulamas and Kyparisis [15] presented the concept of “past-sequence-dependent” (p-s-d) setup times. They provided an example in high-tech manufacturing that the setup time is proportional to the processing times of jobs already processed. In addition, Biskup and Herrmann [16] provided another example of wear-out of equipment in which the sum of the processing times of the prior jobs adds to the processing time of the actual job. In the examples above, the worker skills might improve during the manufacturing process. Recently, several researchers have started to consider both the learning effect and past-sequence -dependent setup times simultaneously. For instance, Wang et al. [17] studied the exponential time-dependent learning effect. Wang et al. [18] considered the Biskup [1] position-based learning effect model and

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provided the optimal solutions for some single machine problems. Yin et al. [19] and Wang and Li [20] considered both the position-based and sum-of-processing-time-based learning effect model, and showed some single machine problems remain polynomially solvable. Motivated by this, we propose a more general scheduling model which includes those previous models as special cases. Under the proposed model, the actual job processing time is expressed as a general function of the normal processing time of jobs already processed and its scheduled position at the same time.

The remainder of this paper is organized as follows. The solution procedures for single-machine problems to minimize makespan, total completion time, and total weighted completion time are presented in the next section. The conclusion is given in the last section.

II. Single-machine problems

A set of n jobs are ready to be processed on a single machine. For each job j, there is a normal processing time p_j and a weight w_j. Due to the learning effect, the actual processing time of job j is

$$p_{j[r]} = p_j f(\sum_{k=1}^{r-1} p_{[k]}, r) \tag{1}$$

for $r = 1, 2, \cdots, n$, if it is scheduled in the rth position in a sequence where $p_{[k]}$ denote the processing time of the job scheduled in the kth position in a sequence. It is assumed that $f:(0,+\infty)\times[1,+\infty)\to(0,1]$ is a differentiable non-increasing function with respect to both the variables x and y, $f_x(x,y_0)=\frac{\partial}{\partial x}f(x,y_0)$ is non-decreasing with respect to x for every fixed y_0 and $f(0,1)=1$. In addition, the setup time is also taken into consideration. As in Koulamas and Kyparisis (2008), the p-s-d setup time of job j if it is scheduled in the rth position of a sequence is as follows:

$$s_{j[1]} = 0 \text{ and } s_{j[r]}^A = b\sum_{l=1}^{r-1} p_{[l]}, \tag{2}$$

where b is a normalizing constant number with $0<b<1$, and $p_{[l]}$ denotes the actual processing time of a job if it is scheduled in the lth position. Throughout the paper, we will use the notation C_j , $L_j = C_j - d_j$ and $T_j = \max\{0,\ C_j - d_j\}$ to denote the completion time, the lateness and the tardiness of job j.

Before presenting the main results, we first state the lemmas that will be used in the proofs in the sequel.

Lemma 1:
$(1-\theta)(1+c)f(x_1,y_1)+\theta f(x_1+\lambda t,y_2)-f(x_1+\lambda\theta t,y_2)\le 0$ for $\theta\ge 1$, $\lambda\ge 0$, $c>0$, $t\ge 0$ and $y_1\le y_2$.

Proof: Let $F(t)=(1-\theta)(1+c)f(x_1,y_1)+\theta f(x_1+\lambda t,y_2)-f(x_1+\lambda\theta t,y_2)$

Taking the first derivative of $F(t)$ with respect to t, we have

$$F'(t)=\lambda\theta\frac{\partial}{\partial x}f(x_1+\lambda t,y_2)-\lambda\theta\frac{\partial}{\partial x}f(x_1+\lambda\theta t,y_2)$$
$$\le 0$$

since $\frac{\partial}{\partial x}f(x,y_0)$ is a non-decreasing function of x and $\theta\ge 1$. It implies that $F(t)$ is a non-increasing function. Thus,

$F(t)\le F(0)=(1-\theta)[(1+c)f(x_1,y_1)-f(x_1,y_2)]\le 0.$

This completes the proof.

Lemma 2:
$c\delta_2\lambda+\delta_2\lambda t\frac{\partial}{\partial x}f(x_1+\lambda\theta t,y_2)-\delta_1 f(x_1+\lambda t,y_2)+f(x_1,y_1)\ge 0$

for $x_1>0$, $0\le y_1\le y_2$, $0<\delta_1<\delta_2<1$, $t>0$, $c\ge 0$ and $\theta\ge 1$.

Proof: Let $F(\theta)=c\delta_2\lambda+\delta_2\lambda t\frac{\partial}{\partial x}f(x_1+\lambda\theta t,y_2)-\delta_1 f(x_1+\lambda t,y_2)$ $+f(x_1,y_1)$. We have

$$F(\theta)\ge\delta_1[f(x_1,y_2)-f(x_1+\lambda t,y_2)]+\lambda\delta_2 t\frac{\partial}{\partial x}f(x_1+\lambda t,y_2)$$

since $\delta_1<1$, f is a nonnegative, non-increasing with respect to y. By Mean Value Theorem, we have that there exists an ξ where $0<\xi<1$ such that

$$F(\theta)\ge\delta_1[\frac{\partial}{\partial x}f(x_1+\lambda\xi t,y_2)](-\lambda t)+\lambda\delta_2 t\frac{\partial}{\partial x}f(x_1+\lambda t,y_2)$$
$$\ge\delta_2\lambda t[\frac{\partial}{\partial x}f(x_1+\lambda t,y_2)-\frac{\partial}{\partial x}f(x_1+\lambda\xi t,y_2)]\ge 0$$

since $0<\delta_1<\delta_2<1$, $\lambda\ge 0$, $t>0$, and $\frac{\partial}{\partial x}f(x,y_0)$ is non-decreasing with respect to x for every fixed y_0. This completes the proof.

Lemma 3: $(\theta-1)f(x_1,y_1)+\delta_2[f(x_1+\lambda\theta t,y_2)+c\lambda\theta+cx_1/t]$ $-\delta_1[\theta f(x_1+\lambda t,y_2)+c\lambda+cx_1/t]\ge 0$

for $x_1>0$, $\theta\ge 1$, $0\le y_1\le y_2$, $\lambda\ge 0$, $t>0$, $c\ge 0$ and $0<\delta_1<\delta_2<1$.

Proof: Let
$G(\theta)=(\theta-1)f(x_1,y_1)+\delta_2[f(x_1+\lambda\theta t,y_2)+c\lambda\theta+cx_1/t]$ $-\delta_1[\theta f(x_1+\lambda t,y_2)+c\lambda+cx_1/t]$. Taking the first derivative of $G(\theta)$ with respect to θ, we have from Lemma 2 that $G'(\theta)\ge 0$. Thus, we have

$G(\theta)\ge G(1)=(\delta_2-\delta_1)[f(x_1+\lambda t,y_2)+c\lambda+cx_1/t]\ge 0$

since $0<\delta_1<\delta_2<1$. This completes the proof.

We will prove the properties using the pairwise interchange technique. Suppose that S and S' are two job schedules and the difference between S and S' is a pairwise interchange of two adjacent jobs i and j. That is, $S=(\pi,i,j,\pi')$ and $S'=(\pi,j,i,\pi')$, where π and π' each denote a partial sequence. Furthermore, we assume that there are r-1 scheduled jobs in π. In addition, let A denote the completion time of the last job in π. Under the proposed model, the completion times of jobs i and j in S and S' are

$$C_i(S)=A+b\sum_{k=1}^{r-1}p_{[k]}+p_i f(\sum_{k=1}^{r-1}p_{[k]},r), \tag{3}$$

$$C_j(S)=p_j f(\sum_{k=1}^{r-1}p_{[k]}+p_i f(\sum_{k=1}^{r-1}p_{[k]},r),r+1)+A+b\sum_{k=1}^{r-1}p_{[k]}$$
$$+b(\sum_{k=1}^{r-1}p_{[k]}+p_i f(\sum_{k=1}^{r-1}p_{[k]},r))+p_i f(\sum_{k=1}^{r-1}p_{[k]},r), \tag{4}$$

$$C_j(S')=A+b\sum_{k=1}^{r-1}p_{[k]}+p_j f(\sum_{k=1}^{r-1}p_{[k]},r), \tag{5}$$

and

$$C_i(S')=p_j f(\sum_{k=1}^{r-1}p_{[k]},r)+b(\sum_{k=1}^{r-1}p_{[k]}+p_j f(\sum_{k=1}^{r-1}p_{[k]},r))$$

$$+A+b\sum_{k=1}^{r-1} p_{[k]} + p_i f(\sum_{k=1}^{r-1} p_{[k]} + p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), r+1) \cdot \quad (6)$$

Property 1. The optimal schedule is obtained by the shortest processing time rule for the $1 | p_{j[r]} = p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), s_{psd} | C_{\max}$ problem.

Proof: Suppose $p_j \geq p_i$. To show that S dominates S', it suffices to show that $C_j(S) \leq C_i(S')$.

Taking the difference between Equations (4) and (6), we have

$$C_i(S') - C_j(S) = (p_j - p_i)(1+b) f(\sum_{k=1}^{r-1} p_{[k]}, r)$$
$$+ p_i f(\sum_{k=1}^{r-1} p_{[k]} + p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), r+1)$$
$$- p_j f(\sum_{k=1}^{r-1} p_{[k]} + p_i f(\sum_{k=1}^{r-1} p_{[k]}, r), r+1) \cdot \quad (7)$$

Substituting $x_1 = \sum_{k=1}^{r-1} p_{[k]}$, $\theta = p_j / p_i$, $c = b$, $t = p_i$, $\lambda = f(\sum_{k=1}^{r-1} p_{[k]}, r)$, $y_1 = r$ and $y_2 = r+1$ into Equation (7), we have from Lemma 1 that $C_j(S') \geq C_i(S)$. This completes the proof.

Property 2. The optimal schedule is obtained by the SPT rule for the $1 | p_{j[r]} = p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), s_{psd} | \sum C_i$ problem.

Proof: The proof is omitted since it is similar to that of Property 1.

We will show in the next property that the WSPT rule provides the optimal solution for the total weighted completion time problem if the processing times and the weights are agreeable, i.e., $p_i \leq p_j$ implies $w_i \geq w_j$ for all jobs *i* and *j*.

Property 3. The optimal schedule is obtained by the weighted shortest processing time (WSPT) rule if the processing times and the weights are agreeable for the $1 | p_{j[r]} = p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), s_{psd} | \sum w_i C_i$ problem.

Proof: Suppose that $p_i \leq p_j$. It implies from Property 1 that $C_j(S) \leq C_i(S')$. Thus, to show that S dominates S', it suffices to show that $w_i C_i(S) + w_j C_j(S) \leq w_j C_j(S') + w_i C_i(S')$. From Equations (3) to (6), we have from $w_i \geq w_j$, $p_i \leq p_j$ and Lemma 3 that

$$[w_j C_j(S') + w_i C_i(S')] - [w_i C_i(S) + w_j C_j(S)]$$
$$= w_i [b\sum_{k=1}^{r-1} p_{[k]} + p_j b f(\sum_{k=1}^{r-1} p_{[k]}, r)$$
$$+ p_i f(\sum_{k=1}^{r-1} p_{[k]} + p_j f(\sum_{k=1}^{r-1} p_{[k]}, r), r+1)]$$
$$- w_j [b\sum_{k=1}^{r-1} p_{[k]} + b p_i f(\sum_{k=1}^{r-1} p_{[k]}, r)$$
$$+ p_j f(\sum_{k=1}^{r-1} p_{[k]} + p_i f(\sum_{k=1}^{r-1} p_{[k]}, r), r+1)]$$
$$+ f(\sum_{k=1}^{r-1} p_{[k]}, r)(w_i + w_j)(p_j - p_i) \cdot \quad (8)$$

Substituting $x_1 = \sum_{k=1}^{r-1} p_{[k]}$, $\theta = p_j / p_i$, $c = b$, $t = p_i$, $\lambda = f(\sum_{k=1}^{r-1} p_{[k]}, r)$, $\delta_1 = w_j / (w_i + w_j)$, $\delta_2 = w_i / (w_i + w_j)$ $y_1 = r$, and $y_2 = r+1$ into Equation (8), we have from Lemma 3 that $w_i C_i(S) + w_j C_j(S) \leq w_j C_j(S') + w_i C_i(S')$. This completes the proof.

III. Conclusions

In this paper, we provided a scheduling model where the learning effect and past-sequence-dependent setup times are taken into consideration. We showed that the SPT rule yields the optimal solution for the single-machine makespan and total completion time problems. We also showed that WSPT rule yields the optimal solution for the total weighted completion time if the weights and the processing times are agreeable.

References

[1] D. Biskup, "Single-machine scheduling with learning considerations." *European Journal of Operational Research*, 115, 1999, pp. 173-178.

[2] G. Mosheiov, "Scheduling problems with a learning effect." *European Journal of Operational Research*, 132, 2001, pp. 687-693.

[3] W.C. Lee, C.C. Wu, and H.J. Sung, "A bi-criterion single-machine scheduling problem with learning considerations." *Acta Informatica*, 40, 2004, pp. 303-315.

[4] P. Chen, C.C. Wu, and W.C. Lee, "A bi-criteria two-machine flowshop scheduling problem with a learning effect." *Journal of the Operational Research Society*, 57, 2006, pp. 1113-1125.

[5] C. Koulamas, and G.J. Kyparisis, "Single-machine and two-machine flowshop scheduling with general learning function." *European Journal of Operational Research*, 178, 2007, pp. 402-407.

[6] J.B. Wang, "Single-machine scheduling problems with the effects of learning and deterioration." *OMEGA-The International Journal of Management Science*, 35, 2007, pp. 397-402.

[7] A. Janiak and R. Rudek, "A new approach to the learning effect: Beyond the learning curve restrictions." *Computers & Operations Research*, 35, 2008, pp. 3727-3736.

[8] T. Eren, "Minimizing the total weighted completion time on a single machine scheduling with release dates and a learning effect." *Applied Mathematics and Computation*, 208, 2009, pp. 355-358.

[9] J.B. Wang, "Single machine scheduling with a time-dependent learning effect and deteriorating jobs." *Journal of the Operational Research Society*, 60, 2009, pp. 583-586.

[10] A. Janiak and R. Rudek, "A note on a makespan minimization problem with a multi-abilities learning effect." *OMEGA-The International Journal of Management Science*, 38, 2010, pp. 213-217.

[11] W.C. Lee, C.C. Wu, and P.H. Hsu, "A single-machine learning effect scheduling problem with release times." *Omega, The International Journal of Management Science*, 38, 2010, pp. 3-11.

[12] X. Huang, J.B. Wang, L.Y. Wang, W.J. Gao, and X.R. Wang, "Single machine scheduling with time-dependent deterioration and exponential learning effect." *Computers & Industrial Engineering*, 58, 2010, pp. 58-63.

[13] T.C.E. Cheng, W.C. Lee, and C.C. Wu, "Scheduling problems with deteriorating jobs and learning effects including proportional setup times." *Computers & Industrial Engineering*, 58, 2010, pp. 326-331.

[14] D. Biskup, "A state-of-the-art review on scheduling with learning effect." *European Journal of Operational Research*, 188, 2008, pp. 315-329.

[15] C. Koulamas, and G.J. Kyparisis, "Single-machine scheduling problems with past-sequence-dependent setup times." *European Journal of Operational Research*, 187, 2008, pp. 1045-1049.

[16] D. Biskup and J. Herrmann, "Single-machine scheduling against due dates with past-sequence-dependent setup times." *European Journal of Operational Research*, 191, 2008, pp. 587-592.

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[17] J.B. Wang, D. Wang, L.Y. Wang, L. Lin, N. Yin, and W.W. Wang, "Single machine scheduling with exponential time-dependent learning effect and past- sequence-dependent setup times." *Computers and Mathematics with Applications*, 57, 2009, pp. 9-16.

[18] X.R. Wang, J.B. Wang, W.J. Gao, and X. Huang, "Scheduling with past-sequence-dependent setup times and learning effects on a single machine." *International Journal of Advanced Manufacturing Technology*, 2009, DOI10.1007/ s00170-009-2308-0.

[19] Y.Q. Yin, D.H. Xu, and J.Y. Wang, "Some single-machine scheduling problems with past-sequence-dependent setup times and a general learning effect." *International Journal of Advanced Manufacturing Technology*, 2009, DOI10.1007 /s00170 -009-2360-9.

[20] J.B. Wang, and J.X. Li, "Single machine past-sequence -dependent setup times scheduling with general position-dependent and time-dependent learning effects." *Applied Mathematical Modelling*, 2010, doi: 10.1016/j.apm.2010.09.017.

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