A Queuing-inventory Model in Multiproduct Supply Chains

Ebrahim Teimouri, Ali Mazlomi, Raheleh Nadafioun, Iman G. Khondabi, and Mehdi Fathi

Abstract—we consider a supply chain that includes a manufacture which produces more than one product that are demanded by several retailers. After production, each all type of products is hold in separated warehouses. Each warehouse has different holding cost and each product has a different backorder cost. We formulate a linear cost function to aggregate all the costs of the holding, back ordering and ordering. The manufacture incurs a setup time whenever it switches from producing one product type to another and it has a finite production rate and stochastic production times. The manufacturer is modeled as a FCFS, GI/G/1 queue. In order to mitigate the effect of setups, products are produced in batches. Customers' arriving orders are sent to warehouses from retailers of each product and by sending production authorization (PA) to manufacturing plants. Moreover, we extend the proposed model in order to analyze the logistics process to three-echelon inventory model. At last, several numerical examples of manufacturing supply chain network are given in order to analysis performance evaluation.

Index Terms—Order batching, inventory control, queuing system, supply chain.

I. INTRODUCTION

Supply chain management includes materials/supply management from the supplier of raw materials to ultimate product and also, network of organizations that are involved, through upstream and downstream stages, in the different processes and activities that produce value in the form of products and services in the hands of the consumer. Therefore a supply chain consists of all parties involved in satisfying a customer request. And also, the supply chain includes not only the manufacturer and suppliers, but also transporters, retailers, and even customers themselves. The supply chain activities constitute a mega process and various decisions are involved in their successful design and operation. Decisions regarding stocking and control of inventory of stocks are a common problem to all enterprises. Asset managers of large enterprises have the responsibility of determining the approximate inventory level in the form of components and finished goods to hold at each level of supply chain in order to guarantee specified end customer service levels. Given the size and complexity of the supply chain, a common problem for this asset manager is to know how to quantify the trade-off between service level and investment in inventory required to supporting these service levels. The problem is made even more difficult because the supply chains are highly dynamic with uncertainty in demand, variability in processing times at each stage of the supply chain, multiple dimensions for customer satisfaction, finite resources, etc.

The goal of a supply chain should be to maximize overall supply chain profitability. Supply chain profitability is the difference between the revenue generated from the customer and the total cost incurred across all stages of the supply chain. One of the challenges in supply chain management is to control the capital in inventories. The objective of inventory control is therefore to balance conflicting goals like keeping stock levels down to have cash available for other purposes and having high stock levels for the continuity of the production and for providing a high service level to customers. A good inventory management system has always been important in the workings of an effective manufacturing supply chain.

Queuing systems are the natural models when dealing with problems where the main characteristics are congestion and jams. In this paper, we use GI/G/1 queue as tool for performance measures of the manufacturing supply chain and also, we use queue to analyze logistics processes.

We review the articles of inventory management in logistics chains including single-product multi-stage systems and multi-product systems (Table1). In the next section, we present our model in detail, and then in Section III, we analyze the proposed model with an example and also, we present the results of our computational analysis. Finally, we give some concluding remarks in section IV.
Table 2. Coding reviewed articles of inventory management in logistics chains

<table>
<thead>
<tr>
<th>Reference article(s)</th>
<th>Article’s code (problem definition/constraints/outputs/objective functions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3]</td>
<td>Queueing, Assemble, Single Production, Stochastic, M/M/1, Longest Path Analysis</td>
</tr>
<tr>
<td>[4]</td>
<td>Inventory Queueing, Single Production, Decomposition Method</td>
</tr>
<tr>
<td>[5]</td>
<td>Multi Production, M/G/1, Lead Time</td>
</tr>
<tr>
<td>[6]</td>
<td>Queueing, Multi Production, Decomposition Method</td>
</tr>
<tr>
<td>[7]</td>
<td>Inventory Queueing, Multi Production, Continuous, M^X/G/∞</td>
</tr>
<tr>
<td>[8]</td>
<td>Queueing, Single Production, Batch, GI^X/G/1</td>
</tr>
<tr>
<td>[9]</td>
<td>Production Inventory, Multi Production, Stochastic, Batch</td>
</tr>
<tr>
<td>[10]</td>
<td>Production Inventory, Stochastic, Batch, Decomposition Method, Monte Carlo simulation</td>
</tr>
</tbody>
</table>

II. PROBLEM DEFINITION

We consider a three echelon supply chain network including n retailers, L warehouses and a manufacturing plant as shown in Fig1. This network offers L types of product to the customers arrived to retailers’ node. Customers’ demands enter to the retailers and the whole demand accumulation for each product is forwarded to warehouses of that product. Then production authorization (PA) is sent to the manufacturing plant of that product with regards to the fact total number of orders should be Q_j for each products’ warehouse j = 1, 2, ..., L (orders of each product from warehouse to manufacturing plant are sent in batch size). We consider a multi-item production–inventory system where a manufacturing plant produces L type of products and separated inventory buffers are kept for each product. If it is available, an order is satisfied from buffer stock. If not, the demand is backordered. We assume that manufacturing plant serves producer of each product with GI/G/1 queue. After production process, products batch is delivered to the warehouse of that product and fulfill the retailers’ demand.

A. Assumptions

Assumptions of the developed model are as follow:

Customers’ demand includes all types of products. We assume the orders of retailer i is as an independent renewal process with a constant rate \( \lambda_i \geq 0 \) and Squared Coefficient of Variation (SCV) \( C^2 \). The probability vectors, \( q_i = (q_{i1}, q_{i2}, ..., q_{in}) \) define customers’ demand from each kind of product at retailer i, \( \sum_{j=1}^{L} q_{ij} = 1 \). Therefore, the orders of warehouse j are as the processes with a constant rate \( \lambda_{aj} = \sum_{i=1}^{n} \lambda_i q_{ij} \) and SCV, \( C^2_{aj} = \frac{1}{\lambda_{aj}} \sum_{i=1}^{n} \lambda_i q_{ij} C^2_i \). In other words, \( \lambda_{aj} \) and \( C^2_{aj} \) are mean arrival rates and SCV of the aggregated input streams of product j, respectively.

In our problem it is assumed that each warehouse holds one product type in batch size \( Q_j \) which maximum number of batches is \( K_j \). Therefore, maximum inventory level of warehouse of product j is \( Z_j = K_j \times Q_j \).

We apply production authorization (PA) system to produce each product type. The PA system is a generalized pull-based production control system. We assume that products in inventories are stored in batches for each product j, and there is a PA card attached to each batch. In this paper, we consider the case when the number of PA cards is the same as the number of batches. The PA system operates in the following way whenever \( Q_j \) units are depleted from a batch in the inventory, the corresponding PA card is transmitted to the manufacturing plant and also is served as new production orders that trigger the manufacturing plant to begin its production process. In general, the manufacturing plant uses a FCFS discipline to produce these orders. Once the manufacturing plant produces \( Q_j \) units, the finished units and the PA card are sent to the warehouse. In the event when a customer places an order and there is no production inventory available, we assume that this customer wait until the product becomes available.

Production policy is make to stock strategy which is based on forecast and operates under \((K_j - 1, K_j)\) inventory control rule for warehouse of product j.

The base of above assumptions, batch arrival streams in manufacturer is following a Poisson process with rate \( \lambda_a(B) = \sum_{j=1}^{L} \frac{\lambda_{aj}}{Q_j} \). Following the asymptotic approach suggested by [11], SCV of batch arrival streams in manufacturer is \( C^2_a(B) = \frac{\sum_{j=1}^{L} \frac{\lambda_{aj}}{Q_j} C^2_{aj}}{\lambda_a(B)} \).  

Fig1. Supply chain network
We define the probability that an arrival in manufacturer is for product \( j \) as: 
\[
p_j = \frac{\lambda_{a,j}}{\sum_{k=1}^{M} \lambda_{a,k}}(B) \]

The manufacturer incurs a setup time whenever it switches from producing one product type to another. We assume \( U \) and \( W \) be random variables that denote the setup time and process time experienced by a batch, respectively. The probability that a batch of product \( j \) experiences a setup (a random variable with mean \( \tau_j \) and variance \( \eta_j \)) is \( 1 - p_j \), we can obtain the mean and SCV of setup time experienced by an arbitrary batch as:

\[
E[U] = \sum_{j=1}^{L} p_j (1 - p_j) \tau_j \\
C_u^2 = \frac{Var[U]}{E^2[U]} = \frac{\sum_{j=1}^{L} p_j (1 - p_j) \eta_j}{\left( \sum_{j=1}^{L} p_j (1 - p_j) \tau_j \right)^2} 
\]

We assumed that unit production times at manufacturer for product \( j \) are i.i.d. generally distributed random variables, which is denoted by \( B_j \), with \( 1/\mu_j = E(B_j) \) and SCV, \( C_j^2 \). Thus, mean production time for batch product \( j \) is \( Q_j / \mu_j \) and coefficient of variation \( C_j^2 / Q_j \).

Similarly, we can obtain mean and SCV of processing time for an arbitrary batch as:

\[
E[W] = \sum_{j=1}^{L} p_j \frac{Q_j}{\mu_j} \\
C_w^2 = \frac{\sum_{j=1}^{L} Q_j C_j^2}{\sum_{j=1}^{L} Q_j} 
\]

We can obtain the mean and SCV of the effective batch service time \( S \), \( (S = U + W) \) of an arbitrary batch, from which we can then compute the corresponding mean and coefficient of variation as:

\[
C_s^2 = C_u^2 + C_w^2 
\]

The utilization of the manufacturing plant is given by

\[
\rho = \sum_{j=1}^{L} \frac{\lambda_{a,j}}{Q_j} E[S] = \lambda_a(B)E[S] 
\]

The system incurs a holding cost \( h_j \) per unit of inventory of product \( j \) per unit time, a backordering cost \( b_j \) per unit of product \( j \) per unit time and \( C_{s_j} \) order setup cost for product \( j \) ($ per set up).

The goal of modeling such a supply network is to minimize supply chain total cost in order to find optimal values of \( K_j, Q_j \).

Costs contain inventory holding cost \( (h_j) \), back ordering cost \( (b_j) \), and order set up cost \( (C_{s_j}) \).

B. Notations

The notations used in this paper are as follow:

- \( Q_j \): Number of units in one bucket of product \( j \);
- \( K_j \): Total number of buckets at warehouse of product \( j \);
- \( Z_j \): Maximum inventory at warehouse of product \( j \);
- \( \lambda_{a,j} \): Retailers’ demand arrival rate of product \( j \);
- \( C_{s,j} \): SCV of retailers’ demand arrival rate of product \( j \);
- \( A_j \): Number of orders of product \( j \) arrived at manufacturer;
- \( \mu_j \): Service rate of product \( j \) in manufacturing plant units/unit time;
- \( C_j^2 \): SCV of service rate of product \( j \) in manufacturing plant units/unit time;
- \( p_j \): The probability that an arrival is for product \( j \);
- \( \tau_j \): Mean setup time of product \( j \);
- \( \eta_j \): Mean lead time (including backordering delay) for an order;
- \( \eta_j \): Mean setup time of product \( j \);
- \( \mu_j \): Service rate of product \( j \) in manufacturing plant units/unit time;
- \( \rho \): Intensity of the manufacturing plant;
- \( I_{ji} \): Number of orders arrived at warehouse of product \( j \), in the queue \( M(\mu_j/C_{s,j}) \) in steady state;
- \( \Gamma_{ji} \): Expected number of orders from product \( j \) at retailer \( i \), in the queue \( M(\mu_j/C_{s,j}) \) in steady state;
- \( \rho_j^* \): Service rate of logistics process;
- \( \rho_j^* \): Intensity of the logistics hub \( \lambda_{a,j}/\xi < 1 \);
- Expected waiting time at warehouse of product \( j \) just due to backordering;
- \( L_{ji} \): Mean lead time (including backordering delay) for an order of items from retailer \( i \) to be filled from warehouse of product \( j \).
product $j$, $L_{j1} = L_{j2} = \ldots = L_{jn} = L_j$

\[
\theta_{ji} = \text{Expected demand for product } j \text{ during replenishment lead time for each item at retailer } l (\lambda_{a_i}, L_j)
\]

\[\text{C. Problem formulation}\]

In this paper, we would like to minimize the expected total cost at the warehouses. Mathematically, we can express:

\[
\begin{align*}
\text{Min} \sum_{j=1}^{n} TC(K_j, Q_j) &= \sum_{j=1}^{n} (h_j E[I_j] + b_j E[B_j] + C_j (\theta_{n,j})) \quad \text{s.t.} \\
K_j, Q_j &\in Z^+
\end{align*}
\]

\[(8)\]

For computing inventory and backorders, we use stochastic equations which capture the properties of the system as in [12]. Observing that,

\[
R_j = A_j - \frac{A_j}{Q_j} Q_j, \quad j = 1, 2, \ldots, L
\]

\[(9)\]

\[
B_j = \max [N_j Q_j + R_j - K_j Q_j, 0], \quad j = 1, 2, \ldots, L
\]

\[(10)\]

\[
I_j = \max [N_j Q_j - N_j Q_j R_j, 0], \quad j = 1, 2, \ldots, L
\]

\[(11)\]

The corresponding steady state probability distribution for $R_j$, $N_j$, $B_j$, $I_j$ are as follow:

\[
P(R_j = m) = \frac{1}{Q_j}, \quad m = 0, 1, \ldots, Q_j - 1
\]

\[(12)\]

We use a development described in [12] to approximate the probability distribution of batches in the queue $GI / G / 1$ using a geometric distribution of the following form:

\[
P(N = m) = \left\{ \begin{array}{ll}
1 - \rho & m = 0 \\
\rho (1 - \sigma) \sigma^{m-1} & m = 1, 2, \ldots
\end{array} \right.
\]

\[(13)\]

Where \( \sigma = (\tilde{N} - \rho) / \tilde{N} \), \( \tilde{N} = \lambda_{a_i} (B) w_0 + \rho \) and \( w_0 = \left( \rho^2 + C_0^2 \right) / \left( 1 + \rho^2 C_0^2 \right) \).

From [13], we also obtain the approximation of the distribution of the number of orders in the queue $GI / G / 1$ of product $j$:

\[
P_j(N_j = m_j) = \left\{ \begin{array}{ll}
1 - \frac{\rho}{\sigma} r_j & m_j = 0 \\
\left( \frac{\rho}{\sigma} \right) (1 - r_j) r_j^{m_j} & m_j = 1, 2, \ldots
\end{array} \right.
\]

\[(14)\]

Where \( r_j = \frac{p_j \sigma}{1 - \sigma (1 - p_j)} \), $p_j = \frac{\lambda_{a_{ij}}}{Q_j \lambda_{a_i}(B)}$ and steady state probability distributions $I_j$, $B_j$ are as follow:

\[
P(I_j = m) = \frac{1}{Q_j} P_j \left( \frac{Z_j - m}{Q_j} \right); m = 1, 2, \ldots, K_j Q_j
\]

\[(15)\]

\[
P(B_j = m) = \frac{1}{Q_j} P_j \left( \frac{Z_j + m}{Q_j} \right); m = 1, 2, \ldots
\]

\[(16)\]

We can obtain $E[I_j]$ and $E[B_j]$ as:

\[
E[I_j] = \frac{Z_j}{Q_j} P_j \left( \frac{Z_j}{Q_j} \right) + \sum_{i=1}^{n} \frac{i}{Q_j} P_j \left( \frac{Z_j + i}{Q_j} \right)
\]

\[(17)\]

\[
E[B_j] = \frac{Z_j}{Q_j} P_j \left( \frac{Z_j}{Q_j} \right) + \sum_{i=0}^{m} \frac{m}{Q_j} P_j \left( \frac{Z_j + i}{Q_j} \right)
\]

\[(18)\]

\[
E[B_j] = \frac{1}{\rho} \left( \frac{\rho}{\sigma} \right) r_j \left( \frac{Q_j - 1}{Q_j} + \frac{2 Q_j - r_j}{1 - r_j} \right)
\]

\[(19)\]

Now, we can calculate optimal inventory level of every product. (By (8))

\[\text{D. Performance measure of warehouses}\]

The stock-out probability at warehouse of product $j$ is the fraction of time that the on-hand inventory at warehouse of product $j$ is zero and is obtained as follows:

\[
P[I_j = 0] = P[Z_j \leq N_j \theta_j Q_j + R_j] = \left( \frac{Z_j - R_j}{Q_j} \right) \leq N_j \theta_j
\]

\[(20)\]

And also, the fill rate at warehouse of product $j$ is the fraction of time that the on-hand inventory at warehouse of product $j$ is greater than zero:

\[
P[I_j > 0] = P[Z_j > N_j \theta_j Q_j + R_j] = 1 - P[I_j = 0]
\]

\[(21)\]

Also the lead time of product $j$ at its manufacturing plant is given by

\[
W_j = \frac{(Q_j - 1)}{2} (1/\lambda_{a_{ij}}) + w_0 + (Q_j / \mu_j)
\]

\[(22)\]

Where \( \frac{(Q_j - 1)}{2} (1/\lambda_{a_{ij}}) \) is batch forming time of product $j$ and \( Q_j / \mu_j \) is mean production time for product $j$ batch.

\[\text{E. The squared coefficient of variation of the inter-departure times which is produced from the warehouses}\]

In this section, we show how to derive the characteristics of batching departure streams from the manufacturing plant with known $\lambda_j(B)$, $C_0^2(B)$ which are obtained as:
We use the approximation of SCV of the inter-departure times for the batches from the manufacturing plant with batch setups in the $GI/G/1$ queue which is given in [12] and shown in (23):

$$C_a^2(B) = (1 - \rho^2) \left( \frac{\lambda^2_{a,j} + \rho^2 C_j^2}{1 + \rho^2 C_j^2} \right) + \rho^2 C_j^2$$

(23)

Also, we can obtain the characteristics of batching departure stream of product $j$ from the manufacturing plant as following equation:

$$\lambda_{d,j} = \lambda_{a,j} = \sum_{i=1}^{L_j} q_{ij} \lambda_i$$

(24)

$$C_{d,j}^2(B) = p_j C_a^2(B) + 1 - p_j$$

(25)

And also, we use the following approximation of the SCV of the inter-departures of individuals from the warehouse of product $j$:

$$C_{d,j}^2(I) = Q_j C_a^2(B) + Q_j - 1$$

(26)

Where $Q_j$ denoting the size of fixed batches of product $j$. When a product departs from the warehouse, there is a probability $q_{ij}$ that the product will be routed to retailer $i$ therefore the mean inter-arrival time and SCV for arrivals to retailer $i$ are given by

$$\lambda_{a,j} = \lambda q_{ij}$$

(27)

$$C_{d,j}^2 = q_{ij} C_{d,j}^2(I) + 1 - q_{ij}$$

(28)

F. Logistics Process

In continue, we extend the model by adding logistics processes. We assume that there is some logistics time to supply products from warehouses to retailers. We model the logistics process of product $j$ by using $M/M^c/1/\infty$ queue in continuous time, where $c_j$ is vehicle capacity which is deterministic and logistics time is exponential. We assume that the logistics process depends on the demands of customers for its arrival process.

For the performance analysis of $M / M^c / 1 / \infty$ queue, we use the results of [15] and [16].

We obtain mean lead time of product $j$ from its warehouse at retailer $i$, $L_{ji} = I_{ji} + W_j$ by using Little’s law we have:

$$W_j = E[B_j]$$

$$L_{ji} = I_{ji} + W_j$$

(30)

$$\Gamma_{ij} = \frac{q_{ij} \lambda_{a,j}}{\xi} - C_j = \frac{1}{\xi}(q_{ij} \lambda_{a,j} C_j)$$

(31)

$$I_{ji} = \frac{\Gamma_{ij}}{q_{ij} \lambda_{a,j}} = C_j$$

(32)

We can compute expected demand of product $j$ at retailer $i$ during replenishment lead time as

$$\theta_{ji} = \lambda_{a,j} I_{ji}$$

(33)

III. NUMERICAL EXAMPLE

In this section, we analyze the model by an example. We consider a supply chain network which produces three product types. The supply chain includes a manufacturing plant, three warehouses and two retailers where the demands for products are characterized by

$$\begin{align*}
\lambda_1 &= 0.6 \\
\lambda_2 &= 0.8 \\
\lambda_3 &= 1 \\
\rho_1 &= 0.2 \\
\rho_2 &= 0.3 \\
\rho_3 &= 0.5
\end{align*}$$

We solve the problem with MATLAB 7 software coding. We obtain optimum value $K_j$ by variety values of batch sizing $Q_j$ for three products. In the condition that $\frac{b_j}{h_j} = 1$, we increase $Q_j$, and optimum maximum inventory level and total cost of three products are increased. The results imply that if backorder costs are greater than holding costs, system tends to hold more inventories (Table 4).
Table 4. Total cost variation by increasing $Q_j$ if $\frac{b_j}{h_j} = 1$

<table>
<thead>
<tr>
<th>Product type</th>
<th>$Q_j$</th>
<th>$K_j^*$</th>
<th>$\rho_j$</th>
<th>$E[U_j]$</th>
<th>$E[B_j]$</th>
<th>$TC^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.678</td>
<td>1</td>
<td>100.7158</td>
<td>1.0830e+03</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.1830</td>
<td>1</td>
<td>70.3479</td>
<td>1.0567e+03</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>0.0987</td>
<td>1</td>
<td>45.9475</td>
<td>1.6999e+03</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0.0394</td>
<td>1</td>
<td>26.9097</td>
<td>2.8394e+03</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>0.0193</td>
<td>1</td>
<td>16.7974</td>
<td>1.0294e+04</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>13</td>
<td>0.0098</td>
<td>1</td>
<td>11.0575</td>
<td>4.5463e+04</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1</td>
<td>0.0036</td>
<td>1</td>
<td>6.0881</td>
<td>3.1259e+04</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.9066</td>
<td>1</td>
<td>133.3429</td>
<td>1.4273e+04</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>0.0907</td>
<td>1</td>
<td>31.9469</td>
<td>1.4192e+03</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>0.0465</td>
<td>1</td>
<td>34.8418</td>
<td>2.6978e+03</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>0.0163</td>
<td>1</td>
<td>6.6877e+03</td>
<td>1.7450e+04</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.0081</td>
<td>1</td>
<td>11.3783</td>
<td>4.9734e+04</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.1258</td>
<td>1</td>
<td>1.0600e+03</td>
<td>1.3949e+03</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0.0304</td>
<td>1</td>
<td>23.6264</td>
<td>3.9171e+03</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>0.0212</td>
<td>1</td>
<td>15.3782</td>
<td>1.9484e+03</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>12</td>
<td>0.0103</td>
<td>1</td>
<td>9.7395</td>
<td>5.4166e+04</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.0081</td>
<td>1</td>
<td>8.1782</td>
<td>5.8032e+04</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1</td>
<td>0.0056</td>
<td>1</td>
<td>6.6426</td>
<td>4.3750e+04</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>0.0043</td>
<td>1</td>
<td>4.4199</td>
<td>3.5174e+04</td>
<td></td>
</tr>
</tbody>
</table>

In condition that $\frac{b_j}{h_j} = 10$, we increase $Q_j$, and optimum number of batches $(K_j^*)$ and optimum maximum inventory level is variable but there is not any trend. Furthermore, total cost of three products is increasing in $Q_j$ (Table 5).

Table 5. Total cost variation by increasing $Q_j$ if $\frac{b_j}{h_j} = 10$.

<table>
<thead>
<tr>
<th>Product type</th>
<th>$Q_j$</th>
<th>$K_j^*$</th>
<th>$\rho_j$</th>
<th>$E[U_j]$</th>
<th>$E[B_j]$</th>
<th>$TC^*$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.3066</td>
<td>1</td>
<td>183.3429</td>
<td>98.1773</td>
<td>1.3665e+04</td>
<td>105.2937</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.0907</td>
<td>1</td>
<td>853.3124</td>
<td>51.8066</td>
<td>9.2606e+03</td>
<td>50.8385</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0.0465</td>
<td>1</td>
<td>1.9792e+03</td>
<td>34.8418</td>
<td>9.5161e+03</td>
<td>56.3591</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>0.0212</td>
<td>1</td>
<td>4.9555e+03</td>
<td>21.5401</td>
<td>2.1594e+03</td>
<td>161.6691</td>
</tr>
<tr>
<td>15</td>
<td>24</td>
<td>0.0199</td>
<td>1</td>
<td>1.0238e+04</td>
<td>15.3412</td>
<td>7.6542e+03</td>
<td>223.0138</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>0.0081</td>
<td>1</td>
<td>1.3367e+04</td>
<td>11.3783</td>
<td>1.4711e+05</td>
<td>351.4037</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0.1238</td>
<td>1</td>
<td>1.0600e+04</td>
<td>50.3774</td>
<td>1.0615e+04</td>
<td>54.5614</td>
</tr>
<tr>
<td>15</td>
<td>27</td>
<td>0.0212</td>
<td>1</td>
<td>3.3723e+03</td>
<td>15.3827</td>
<td>2.4822e+04</td>
<td>99.8395</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>0.0081</td>
<td>1</td>
<td>3.1815e+04</td>
<td>8.1782</td>
<td>1.7157e+05</td>
<td>334.1556</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>0.0043</td>
<td>1</td>
<td>6.2078e+04</td>
<td>4.4199</td>
<td>3.4855e+05</td>
<td>888.8527</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

In this paper, we presented a model for the analysis of a three-layer supply chain which produces more than one product. We used $GI/I/G/1$ queue operating under $(K_j = 1, K_j)$ inventory control policy to analyze the performance of warehouses. We obtained performance measures such as stock-out probability, fill-rate and lead time of warehouses in proposed model. In the model, we used $M/M/\infty$ queue to analyze logistics process. In this paper, we survey the effect of order batching in multi-product multi-echelon supply chains. In future researches, we can consider a center warehouse that in the stock-out condition in each warehouse, customers’ demands are satisfied (adding transmittal cost). Also, the pricing concept can be added to our model as an attractive aspect of future research.

REFERENCES