Analyzing an Exponential Queuing Model Based on Stepwise Time Rates

Habib K. Chaleshtari, Ali Mazlomi, Iman G. Khondabi, and Mehdi Fathi

Abstract—**Encountering the uncertainties in our new world is a major problem which many managers are engaged with that. The stochastic nature of demand arrivals and service processes is an important problem which is studied by many researchers. This article is dedicated to analyzing some queuing systems with stepwise demand and service rates functions. Some analytical concepts are presented by using numerical examples. Also, the basic relations of queuing theory are used and finally the findings are applied in a case study. The article will help the managers to obtain the optimal servicing parameters for a stochastic system with stepwise demand and service rate functions to achieve the minimum total cost.**

Index Terms— **Base time rate, Mean rate, Non-stable queues, Stepwise function**

I. INTRODUCTION

With a renewed approach to exponential queuing systems in viewpoint of time base arrival and service rates, it is possible to improve the flexibility of these systems encountering with real applications. Using periodic stepwise function to estimate the variable states of rates allows us to achieve definite results for queue parameters because by calculating the mean rate from stepwise functions and replace them in classic relations of queuing theory, problem of encountering non-stable queue will be solved.

Studying these systems was first discussed by [1] and [2] in relation to obtain analytical models and continued by [3], [4], [5] and [6] in developing numerical and approximation methods.

This approach is applicable to survey transportation systems, networks traffic and scheduling stochastic systems. Therefore, it is possible to achieve exact key parameters of each queue by estimating time variances of rates in systems with variable rates to periodic stepwise functions.

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II. ANALYZING $M_1^{sw}/M_1^{sw}/1$ MODEL

A. Review Stage

All queuing systems which considered up to now and analyzed with math relations had a joint point which was their stationary. A stationary queuing system is achievable when arrival and service rates with any statistic distribution are stable in any planning period and utilization ratio is smaller than 1 [7]:

$$\rho = \frac{\lambda}{c\mu} < 1 \qquad (1)$$

Discussing queuing systems with several servers where each customer needs a stochastic number of same servers studied by [4] for the first time. This queuing system is similar to $M^x/M/1$ queuing system however there are some differences.

In many real world problems, probabilistic distributions rates related to interval between arrivals or interval between services are variable during the time. We specify this type of queuing models with a *t* index next to the statistical distribution. In exponential time based systems which are proceeded here, system manner can be studied by Chapman-Kolmogorov differential equations. Some examples of a $M_t/M_t/c_t/k$ system can be written in below form [8]:

$$\begin{aligned}
\frac{dp_0(t)}{dt} &= -\lambda(t)p_0(t) + \mu(t)p_1(t), \\
\frac{dp_n(t)}{dt} &= -(\lambda(t)+n\mu(t))p_n(t) + \lambda(t)p_{n-1}(t) \\
&\quad + (n+1)\mu(t)p_{n+1}(t) \qquad 0 < n < c(t) \\
\frac{dp_n(t)}{dt} &= -(\lambda(t)+c(t)\mu(t))p_n(t) + \lambda(t)p_{n-1}(t) \\
&\quad + c(t)\mu(t)p_{n+1}(t) \qquad n \ge c(t) \\
\frac{dp_k(t)}{dt} &= \lambda(t)p_{k-1}(t) - \mu(t)c(t)p_k(t)
\end{aligned} \qquad (2)$$

Beside λ, μ which are base time rates, above equation system has variable servers number c_t and it is possible to replace c_t with a deterministic number, if the under studied model has a stable number of servers. Generally, if related functions to arrival and service rates of these systems are irregular or so complex, it is only possible to achieve exact results by using numerical solution of differential equations system [5] and rather exact results for analyzing these queues in long term are achievable by using algorithmic

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approximation method [6]. Solving this system of equations with various numbers of servers during the time is more complex and it is necessary to define service policies while one or more engaged servers exit the system in a specific time. Preemptive discipline and exhaustive discipline policies before the customer exiting will affect the transient probabilities and needs to define a new system of differential equations based on Hyper-Geometric distribution [9]. Functions which are used for service and arrival time distribution rates are stepwise such as the function is shown in Fig.1. This special type of base time form of arrival and service rates is specified by using superscript *sw* for base time distributions in queuing notation.

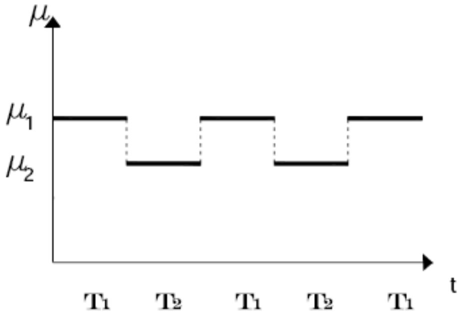

Fig.1 A stepwise function for service rate with period cycle $T_1 + T_2$

In order to analyze the behavior of this base time system, related differential equation system is solved by MATLAB7 software according to relation 2 and the results are shown in following graphs. In this numerical example, number of servers assumed deterministic and number of customers must be finite.

A. Numerical example $M_1^{sw} / M_1^{sw} / 5/18$

Assume a queuing system with exponential arrival and service times distribution, stepwise function base time rates, 5servers and a capacity of 18 customers. Arrival and service functions characteristics are as follow:

$$\text{Arrival Distribution} = \exp(\lambda_1, T_1, ..., \lambda_n, T_n) = \exp(15,8,12,4) \qquad (3)$$

$$\text{Service Distribution} = \exp(\mu_1, T_1, ..., \mu_n, T_n) = \exp(4,8,3,7) \qquad (4)$$

Relation 3 implies the base time exponential distribution which has rate 15 in first 8 units of time and in second 4 units of time, the arrival rate is equal with 12 in a unit of time. Above characteristics are true for time distribution between services too. System utilization ratio is equal to 1 in the most difficult status but according to system finite capacity we do not have the infinite queue in this situation.

We have to open the equations system perfectly and define that for MATLAB software in order to obtain the numerical solution for differential equations:

$$\frac{dp_{i,0}(t)}{dt} = -\lambda(t)p_{i,0}(t) + \mu(t)p_{i,1}(t)$$

$$\frac{dp_{i,n}(t)}{dt} = -(\lambda(t) + n\mu(t))p_{i,n}(t) + \lambda(t)p_{i,n-1}(t)$$
$$+ (n+1)\mu(t)p_{i,n+1}(t) \qquad for\, 0 < n < 5 \qquad (5)$$

$$\frac{dp_{i,n}(t)}{dt} = -(\lambda(t) + 5\mu(t))p_{i,n}(t) + \lambda(t)p_{i,n-1}(t)$$
$$+ 5\mu(t)p_{i,n+1}(t) \qquad for\, 5 \le n$$

$$\frac{dp_{i,k}(t)}{dt} = \lambda(t)p_{i,k-1}(t) - \mu(t)c(t)p_{i,k}(t) \quad for\, 0 \le i \le k = 18$$

By using command $ode45$ in software for equation systems, these results have been obtained for queue characteristics:

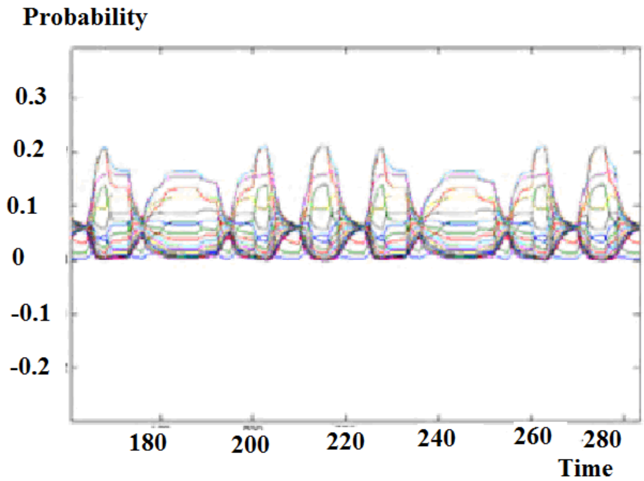

Fig.2 System transient probabilities for number of customers in the system $\pi_i(t)$ (each color refers 0 to 18)

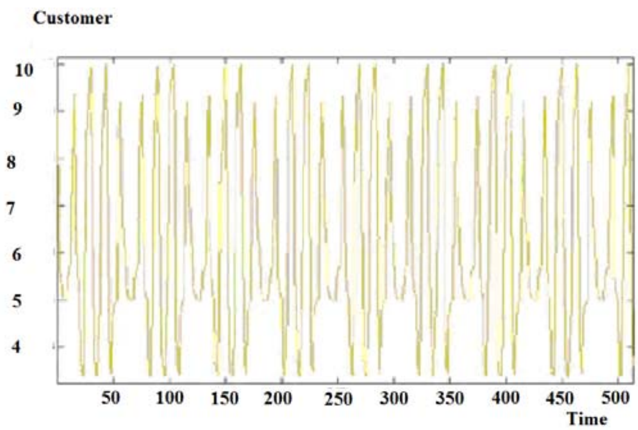

Fig.3 Number of customers in system during long term $L(t)$

It is obvious in Fig.2 and Fig.3 that related values to queue main parameters like transient probabilities and long term number of customers in system depends on time and do not converge to specific value but only have similar and repetitive cycles of a base cycloid. Also by using Simpson integral method we can calculate a mean value for a cycle which has some error.

III. General Analytic Model

To represent analytic queuing model related to exponential system with stepwise function base time rates it is important to consider that our analyzing base is random choosing of a time period between two consequent arrivals and consider the system manner during this period. Random variable period length and determining start and finish points of this period in stepwise function of service rate must be considered. Also, our main analyzing method to represent time continuous chain is Markovian because in every moment our queuing system is exponential.

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A. First step, analyzing $M/M_1^{sw}/1$ model

In this model it is assumed that arrival rate for exponential distribution is stable and only service rate is base time stepwise function. So, if we consider a period between two consequent arrivals and fit that on service rate stepwise function, number of states that can be studied is twofold number of steps in a cycle. Therefore, if service rate function has two steps, four states will be possible:

A. According to Fig.4 and using uniform distribution to calculate the probability of arrival interval time occurrence during a cycle of stepwise function, the first state service rate probability is given by:

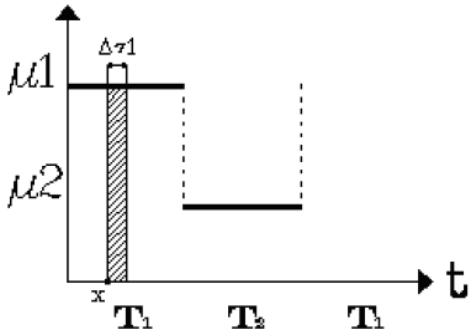

Fig.4 Fitting the interval arrival rate on first step of stepwise function

$$P_1 = (\frac{1}{T_1+T_2})\int_0^{T_1}\int_0^{T_1-x} \lambda e^{-\lambda t}\,dt\,dx = \frac{\lambda T_1 + e^{-\lambda T_1} - 1}{\lambda(T_1+T_2)} \qquad (6)$$

Probability P_1 is given by multiplying two probabilities:

$P1$=p(finish point of time interval be smaller than the end of first step)*p(match the start point of interval time on first step)

The probability of choosing a point on time axis is based on uniform distribution on a cycle and the probability of lasting the period to a specific value is calculated based on exponential distribution. And also it is assumed that service rate is stable during the service process of a customer and continues up to end of the service.

B. Probability $P2$ is calculated from the mentioned method but in this case, occurrence of interval arrival time period is in such way that start point of the period fits on the first step and finish point of the period surely will occur out of first step. This period is related to first step service rate and if the rate changes, first rate is applied.

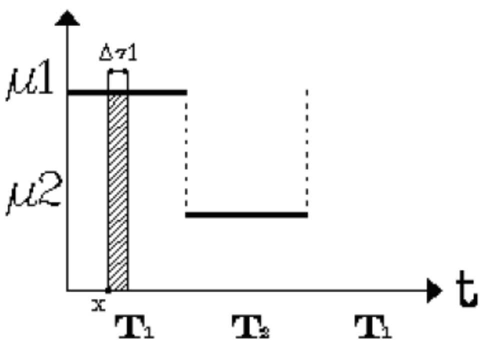

Fig.5 Fitting interval arrival time period on first and second steps of stepwise function

Probability of second state is relevant to first step rate and in this state like first state, service rate μ_1 is active.

$$P_2 = (\frac{1}{T_1+T_2})\int_0^{T_1}\int_{T_1-x}^{\infty} \lambda e^{-\lambda t}\,dt\,dx = \frac{1-e^{-\lambda T_1}}{\lambda(T_1+T_2)} \qquad (7)$$

C. Probability $P3$ is relevant to situation which interval arrival time periods completely stand in second service step and consequently service rate in this situation is equal to μ_2. Fig.6 shows the relevant situation:

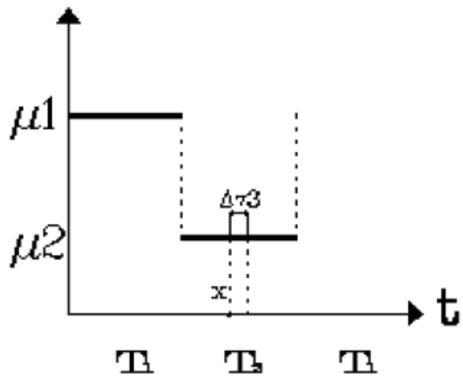

Fig.6 Fitting interval arrival time period on second step of stepwise function

Probability relation of this situation is given by:

$$P_3 = (\frac{1}{T_1+T_2})\int_{T_1}^{T_1+T_2}\int_0^{T_1+T_2-x} \lambda e^{-\lambda t}\,dt\,dx = \frac{\lambda T_1 + e^{-\lambda T_1} - 1}{\lambda(T_1+T_2)} \qquad (8)$$

D. Similarly to state2, state four is relevant to a situation which service rate variation from second step to first step will occur during a time period between arrivals. Fig.7 illustrates situation four schematically:

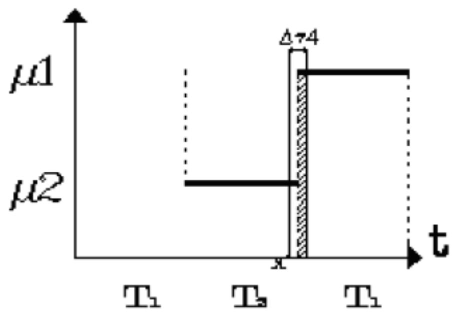

Fig.7 Fitting interval arrival time period on first and second steps of stepwise function

Probability of this state is calculated by below relation:

$$P_4 = \left(\frac{1}{T_1+T_2}\right)\int_{T1}^{T1+T2}\int_{T1+T2-x}^{\infty} \lambda e^{-\lambda t}\,dtdx = \frac{1-e^{-\lambda T1}}{\lambda(T_1+T_2)} \qquad (9)$$

Probabilities of various service states in a time period between stochastic arrivals was calculated according to relations (6) to (9). Summation of these probabilities is equal to one. Therefore, system expected service rate is the statistical mean of these four states for rates and is given by:

$$\overline{\mu} = (P_1+P_2)\mu_1 + (P_3+P_4)\mu = \left(\frac{T_1}{T_1+T_2}\right)\mu_1 + \left(\frac{T_2}{T_1+T_2}\right)\mu_2 \qquad (10)$$

$$\sum_{i=1}^{4} P_i = 1 \qquad (11)$$

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Now, according to exponential behavior of queuing system in every state (possible various rates), there exist a continuous Markovian chain to analyze this queuing system which is different from the classic exponential queuing model just in calculating mean service rate.

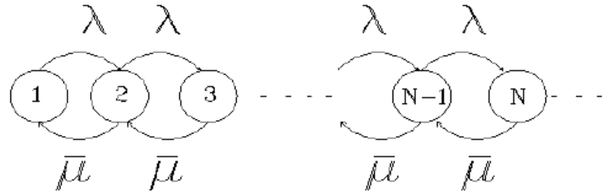

Fig.8 Continuous time Markovian chain for $M/M_t^{sw}/1$ queuing system

Therefore, all queuing relations in $M/M/1$ systems are true here [7] but we replace a mean service rate.

B. Analyzing $M_t^{sw}/M_t^{sw}/1$ ***model***

This model is more complete than the model presented in section III.A just by one stage where arrival rate variation is a stepwise function in its relevant time. Computational differences between this model and the last one are in service rate calculating which is needed in computing probability in each state of four possible states. So, if we have a time stepwise function for arrival rate, we can study following possible states:

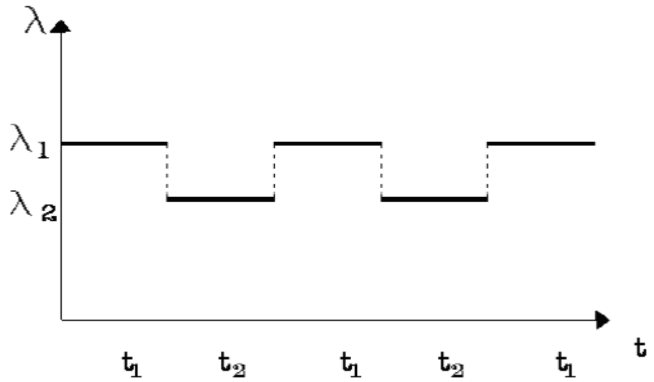

Fig.9 Arrival rate stepwise function with two steps in each cycle

In this section there exist relations like (6) to (9) which result that time interval between two consequent arrivals in this system can be shown based on first rate or second rate each of which have a probability to be chosen.

$$P(\Delta t \in T_1) = \left(\frac{t_1}{t_1 + t_2}\right) \tag{12}$$

$$P(\Delta t \in T_2) = \left(\frac{t_2}{t_1 + t_2}\right) \tag{13}$$

Therefore we can rewrite relation (6) in below form:

$$P_{11} = \left(\frac{t_1}{t_1+t_2}\right)\left(\frac{1}{T_1+T_2}\right)\int_0^{T1}\int_0^{T1-x} \lambda_1 e^{-\lambda 1t}.dtdx = \left(\frac{t_1}{t_1+t_2}\right)\frac{\lambda_1 T_1 + e^{-\lambda 1T1} - 1}{\lambda_1(T_1+T_2)} \tag{14}$$

$$P_{21} = \left(\frac{t_2}{t_1+t_2}\right)\left(\frac{1}{T_1+T_2}\right)\int_0^{T1}\int_0^{T1-x} \lambda_2 e^{-\lambda 2t}.dtdx = \left(\frac{t_2}{t_1+t_2}\right)\frac{\lambda_2 T_1 + e^{-\lambda 2T1} - 1}{\lambda_2(T_1+T_2)} \tag{15}$$

Other relations can be calculated similarly and based on equation (16) and (17) it is possible to study general formulation of any possible state of every stepwise function with any number of steps:

$$P_1(m) = \sum_{j=1}^{Ns}\left(\left(\frac{t_j}{\sum_{i=1}^{Ns} t_i}\right)\left(\frac{1}{\sum_{k=1}^{Ms} T_k}\right)\left(\frac{\lambda_j T_m + e^{-\lambda jTm} - 1}{\lambda_j}\right)\right) \tag{16}$$

$m = 1\ To\ Ms$

$$P_2(m) = \sum_{j=1}^{Ns}\left(\left(\frac{t_j}{\sum_{i=1}^{Ns} t_i}\right)\left(\frac{1}{\sum_{k=1}^{Ms} T_k}\right)\left(\frac{1 - e^{-\lambda jTm}}{\lambda_j}\right)\right) \tag{17}$$

$m = 1\ To\ Ms$

Probability 1and 2 in relations (16) and (17) are relevant to states where interval between arrivals were completely placed in one step of service function or was in a form which included service rate variations. Finally, mean arrival and service rate in this system can be calculated based on following relations (18) and (19) which result in calculating the classic relations of exponential queues.

$$\bar{\mu} = \sum_{i=1}^{Ms}\big(P_1(i) + P_2(i)\big)\mu_i = \sum_{j=1}^{Ms}\left(\frac{T_j}{\sum_{i=1}^{Ms} T_i}\right)\mu_j \tag{18}$$

$$\bar{\lambda} = \sum_{j=1}^{Ns}\left(\frac{t_j}{\sum_{i=1}^{Ns} t_i}\right)\lambda_j \tag{19}$$

C.I. Analyzing $M_t^{sw}/G/1$ ***model***

In relation to $M/G/1$ model, we can generate a Markovian chain model as follows:

$$X_{n+1} = \begin{cases} X_n + A - 1 & X_n \geq 1 \\ A & X_n = 0 \end{cases}$$

$$P\{A = a\} = \int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^a}{a!} b(t)d(t) \tag{20}$$

$P\{A=a\}$ shows the probability of a arrivals to system in two consequent service intervals. As it is seen in relation (20), to calculate this probability, Poisson process is used to calculate number of arrivals in the interval. According to base time state and especially stepwise function of exponential distribution rate in this section we can replace a λ_i rate instead of each step of function:

$$P\{A = a\} = \sum_{j}^{Ns}\left(\frac{T_j}{\sum_{i=1}^{Ns} T_i}\right)\left(\int_0^{\infty}\frac{e^{-\lambda j}(\lambda_j t)^a}{a!} b(t)dt\right) \tag{21}$$

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In this section it is assumed that service rate stepwise function has Ns steps with various rates in each of its periodic cycles which result in calculating probability with summation of Ns consequent relations. Using this method it is simply possible to analyze $M_t^{sw}/G/1$ queuing system.

C.II. Analyzing $G/M_t^{sw}/1$ model

For a $G/M_t^{sw}/1$ queuing model, Markovian chain is defined as follows:

$$X_{n+1} = X_n + 1 - B \;:(X_{n+1} \geq B, X_n \geq 0) \qquad (22)$$

Therefore

$$P\{B=b\} = \int_0^{\infty} \frac{e^{-\mu t}(\mu t)^b}{b!} a(t)\,dt \qquad (23)$$

$P\{B=b\}$ shows the probability of b exits in interval time between two consequent arrivals to system. Now, according to consideration of base time queuing system stepwise exiting rate we have remember that service won't be uniform during all moments of a cycle. So, we have to use the method of defining arrival intervals on service function and calculate possible probabilities on service rate stepwise function and finally we obtain mean service rate to replace in relation (23).

$$P_1(m) = \left(\frac{1}{\sum_{i=1}^{Ms} T_i}\right)\left(\int_{Tc}^{Tc+Tm}\int_0^{Tc+Tm-x} a(t)\,dt\right) \qquad (24)$$

$$m = 1 \; To \; Ms$$

$$Tc = \sum_{j=1}^{m-1} T_j$$

$$P_2(m) = \left(\frac{1}{\sum_{i=1}^{Ms} T_i}\right)\left(\int_{Tc}^{Tc+Tm}\int_{Tc+Tm-x}^{\infty} a(t)\,dt\right) \qquad (25)$$

$$m = 1 \; To \; Ms$$

$$Tc = \sum_{j=1}^{m-1} T_j$$

Relation (23) is relevant to probability of time period occurrence in one step of service rate stepwise function and completely places in that step. Relation (24) is relevant to states where interval time starts from first step and finishes out of first step. This equation was used in relations (6) to (9) but in that case, time distribution $a(t)$ was exponential and integral was computable, so in relations (23) and (24) after recognizing $a(t)$ distribution, relevant integral and needed probabilities will be calculated. Ms steps are considered for service rate stepwise function which result in $2 \times Ms$ probability calculations. Consequently, mean service rate will be calculated by relation (26):

$$\overline{\mu} = \sum_{m=1}^{Ms} (P_1(m) + P_2(m))\mu_m \qquad (26)$$

Therefore, relevant probabilities to $G/M_t^{sw}/1$ will be obtained by replacing $\overline{\mu}$ instead of μ:

$$P\{B=b\} = \int_0^{\infty} \frac{e^{-\overline{\mu} t}(\overline{\mu} t)^b}{b!} a(t)\,dt \qquad (27)$$

Other computations relevant to $G/M_t^{sw}/1$ model are completely similar to $G/M/1$ model just with mean service rate.

IV. CASE STUDY

In this section, it is tried to show using of these models by a practical example. This example is about modeling a simple crossroad by using base time queuing model.

In a simple cross road with a traffic light, as it is seen in Fig.10 every arrival roads has separate arrival rate each of which have variation in specific time periods in more real cases. Each path has a specific capacity for car queues. Queue units can be expressed as one car or several cars which can move through the road width because in real cases, road width gets full earlier. Each group size is based on cars mean width and road width and can be any real number. Finally, these group sizes will result in final solution of queues length and present customers in system. Here we consider an example from a crossroad which has four stepwise functions for arrival rates and one stepwise service rate function. Service rate values constantly varies between zero and crossroad service rate. The reason is that when the light varies between red and green positions, this stepwise rate are generated from the viewpoint of arrival path and cycle order substitutes in each vertical path. Also, arrival rate function can vary between two different numbers but arrival stepwise functions can obtain more than two variation values (step in each cycle).

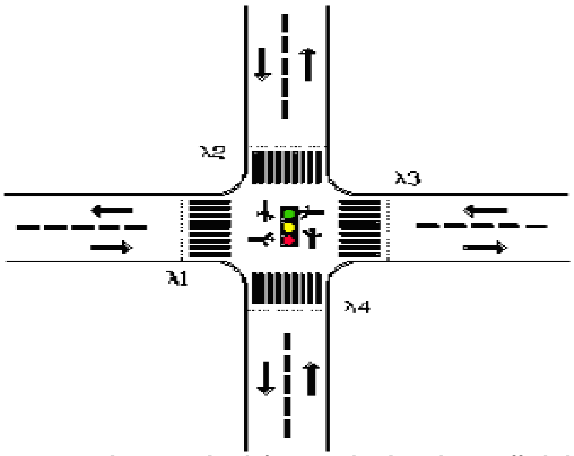

Fig.10 A simple crossroad with four arrival paths and one traffic light

According to Table.1 which contains characteristics of the supposed crossroad with stepwise exponential rates, we can study the results:

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Table.1 Relevant information to a simple crossroad

No	***1***	***2***	***3***	***4***	
λ_1	9	10	15	12	***Total***
λ_2	5	8	12	7	
T_1	10	6	8	3	
T_2	5	10	4	2	
Cyc.	15	12	12	5	
μ_1	20	0	20	0	
μ_2	0	20	0	20	
T_3	8	8	8	8	
T_4	15	15	15	15	
Cyc.	10	10	15	4	
C	1	1	1	1	
K	20	15	18	12	
L	13.33	2.01	17.01	2.86	35.21
L_q	12.35	1.34	16.01	2.1	31.8

L, L_q results are calculated by $M_t^{sw}/M_t^{sw}/1/K$ model. Each above table row is relevant to one branch of crossroad. Stoplight times are 8 and 15 seconds vice versa for crossroads. Yellow light time is considered in green light time. Maximum capacity for number of cars in each road is given in column *k*. Finally, queues information are aggregated in last column which shows that with this couple time considered for stoplight it's expected that in long term in a random moment, about 32 cars are stopped in crossroad. Therefore it's possible to minimize long term expected queue length in crossroad and find stoplights optimal time by varying green and red phases times. By using queuing relation and replace that in a computer software which is prepared by MATLAB7, time period (5,180) seconds for green and red lights studied and following results are obtained.

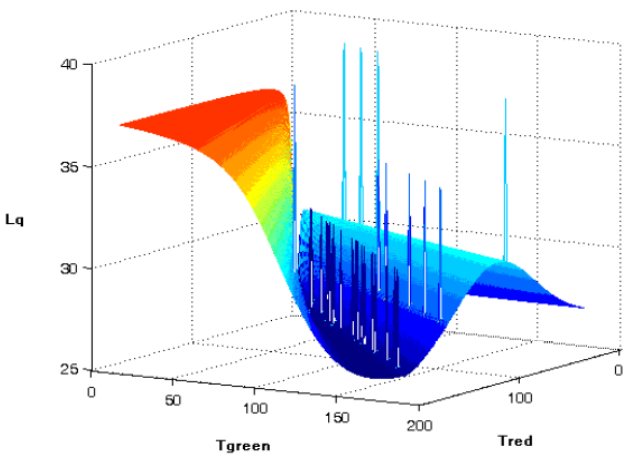

Fig.11 Queue expected values for assumed crossroad in long term

Local maximum points in Fig.11 are banned points which have been generated in reason of nominator and denominator same values in queue utilization relation (relation number (1)). Now, it's obvious that values beside main diagonal are best times which can be used in stoplight assignment. Best relevant couple time is (31, 35) which reduces expected crossroad queue number to 25.71which is a salient improvement.

V. Conclusion

As it is seen in paper's sections, by base time mean calculation which are arrival and service rates in exponential queuing systems or general and exponential queuing systems, a set of deterministic results are calculated for these types of queue parameters. Stepwise function has a good ability to fit the rates complex variations in time in simplest situation. So, we can hopefully approximate sinusoidal and cosine functions or more complex forms to stepwise function and obtain a quick solution with high accuracy by this method. Another application of base time queues with stepwise rates is the ability to scheduling work shifts for servers which work in systems with stochastic specifications. Calculating an expected cost function based on generated queues and optimizing that by various times for steps can help us to obtain the least cost based on a right planning in stochastic systems. One field to future research is to analyze base time rates in group arrival or group service systems or to analyze queuing networks with base time nodes. Entering base time rates to other queuing discussions would be a solution object for many practical problems.

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