Computerized Surface Wave Method for Offshore Geotechnical Investigations

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***Abstract*—Surface wave method is an in-situ nondestructive testing procedure used for geotechnical explorations and determining the stiffness profile of the ground layers on land and under water. The method is based on the generation of surface waves and recording them on the surface of the ground on land or offshore, analysis of the phase differences between the signals recorded at the receivers and backcalculation of the results through an inversion process by use of numerical inverse techniques. In order to develop and apply the surface wave method under water, a computer program named *SWundWAT* has been developed during this research work and used for constructing the theoretical dispersion curve of a layered system. This theoretical dispersion curve is then used to make the required comparisons with the experimental dispersion curve obtained in the field for the purpose of determining the real site profile. To evaluate the applicability and reliability of the developed program, the application of the program to a published set of experimental data obtained in the field is presented and discussed. It is shown that the experimental data and the program results are in good agreement, indicating that the developed program can be used to construct the theoretical dispersion curve of the layered structures under water.**

***Index Terms*— Surface wave, Offshore, Dispersion curve, Stiffness**

I. Introduction

Seismic wave techniques include a group of in situ tests for measuring the soil dynamic properties in small strain level. These methods are based on the propagation of elastic seismic waves in soil and measurement of wave velocity.

Stress wave measurements have been used on land and offshore to investigate the material systems and their properties for decades. The most commonly used measurements have involved compression or acoustic waves. In the past 20 years, significant efforts have been directed towards developing a technique involving surface waves. The technique employs Rayleigh type surface waves on land and Scholte type surface waves offshore. The technique has been used to profile many geotechnical sites on land, including natural soil deposits, compacted earth materials, and solid waste landfills, as well as concrete layered structures, highway pavements and airport runways. Recently, the surface wave technique has been extended to underwater environments to measure shear stiffness profiles of the seafloor. One of the attractive attributes of the testing technique is its non-intrusive nature, especially for application at deep water sites.

Surface wave method is based on the application of an impact on the surface and recording the signals by transducers placed on the ground surface at known distances away from the source. The phase differences of the recorded signals are analyzed in frequency domain and then inversion analysis is applied to obtain the site stiffness profile. In order to develop and apply the method under water, theoretical relationships of seismic waves propagated in soil and surface waves propagated at soil-water interface have been reviewed and presented.

The surface wave method consists of exciting Scholte wave energy on the seafloor and measuring the propagation of this energy past two or more receivers arranged in a linear array. Scholte waves are generated by applying a dynamic vertical load to the seafloor surface. Vertically sensitive receivers, located on the seafloor surface, are used to sense the passage of wave energy. The time signals recorded at each receiver are transformed into the frequency domain through a FFT algorithm, and the phase difference between the receivers, as a function of frequency, is determined. The phase velocity of the Scholte wave, is computed from:

$$V_{Sc} = f(\frac{360}{\phi})d \qquad (1)$$

where f is frequency, ϕ is the phase difference at that frequency, and d is the spacing between receivers. The resulting phase velocities are plotted against wavelength (or frequency) to create a dispersion curve. A typical results of field measurements in the form of a dispersion curve is shown in Fig. 1. One of the key characteristics of Scholte waves is that different wavelengths stress different depths into the seafloor. Shorter wavelengths (higher frequencies) stress shallower depths while longer wavelengths (lower frequencies) stress deeper depths. Therefore, the trend exhibited in the measured dispersion curve with increasing wavelength reflects the stiffness properties of the material with increasing depth.

To determine the shear stiffness profile of the material, an iterative forward modeling or inversion scheme must be employed in which a theoretical dispersion curve for an assumed profile is compared to the measured dispersion curve. The profile is changed until a match between the theoretical and experimental dispersion curves is achieved.

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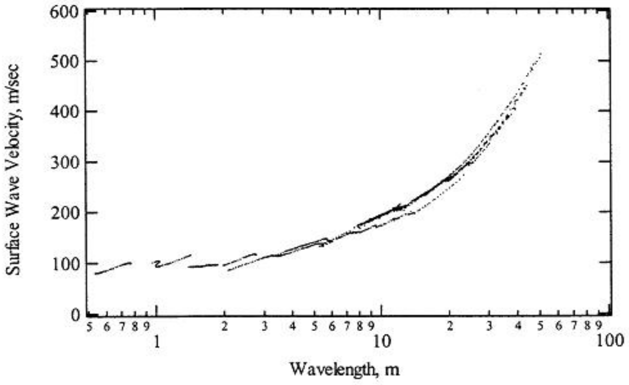

Figure 1: Typical results of surface wave field measurements

The theoretical dispersion curve for the under water measurements will be different from the dispersion curve generated for the same profile on land due to the presence of the overlying water layer. On land, the air/soil interface closely approximates the theory of a surface wave propagation at a solid/vacuum boundary. The propagation velocity of this wave, called a Rayleigh wave, depends on the shear wave velocity and Poisson's ratio of the soil. Under water, the air/water interface gives rise to different interface wave termed a Scholte wave. The scholte wave velocity depends on additional factors including the depth of the water, the stiffness of the seafloor relative to the bulk modulus of the water, and the mass density of the seafloor relative to the mass density of the water.

II. Principles Of General Formulation

The transfer matrix approach is often used to solve the dynamic response to an external load of an elastic system overlaid by water. The transfer matrix is formulated considering the continuity of displacements and force equilibrium at the top and bottom surfaces of each elastic layer and the adjacent layers. The dynamic response of a soil deposit can then be related to the exciting motion or load, either at the surface or at the base, by matrix multiplication of the transfer matrices for each layer.

An alternative approach, and the one used in this study, is to relate the forces at the interfaces between layers directly to the displacements at the same locations by a dynamic stiffness matrix. Using this stiffness matrix approach, a global stiffness matrix of the complete layered system can be assembled. The global load vectors correspond to external forces at the interfaces of the layered system. The characteristic equation can be formulated from the stiffness matrix approach to determine normal modes of vibration of the layered system. Theoretical dispersion curves for seismic waves propagating across a layered system can then be determined from the normal modes (roots). Gravity effects due to contrasting densities across boundaries at the air-water, soil-water, and soil-soil interfaces, sometimes termed "counter buoyancy" effects, are also considered in the formulation. A complete explanation of the formulation can be found in Lee, et al. (1997).

III. Governing Equations For Water And Soil Layers

The dispersion curve for a layered system overlaid by water is computed from the global stiffness matrix for the coupled layered soil-water system. The water is assumed to be inviscid and compressible, and the water motion is assumed to be irrotational. The waves traveling along the air-water interface are assumed to be of small amplitude following linear wave theory. The soil in the layered system is assumed to be elastic and its properties are characterized by shear and compression wave velocities, a mass density, a Poisson's ratio, and a layer thickness.

The governing equation for water is derived from the equation of motion and Hook's law. Displacements are also assumed to be small following linear wave theory. In general, the equations of motion for plane wave propagation can be derived from dynamic equilibrium in the two given directions. The stiffness matrix for water layer can be expressed as (Haskell, 1953):

$$K = \frac{\omega^2 \rho_w}{\alpha} \begin{bmatrix} \frac{e^{\alpha d} + e^{-\alpha d}}{e^{\alpha d} - e^{-\alpha d}} - \frac{kg}{\omega^2} & 0 & -\frac{2}{e^{\alpha d} - e^{-\alpha d}} \\ 0 & 0 & 0 \\ -\frac{2}{e^{\alpha d} - e^{-\alpha d}} & 0 & \frac{e^{\alpha d} + e^{-\alpha d}}{e^{\alpha d} - e^{-\alpha d}} \end{bmatrix} \quad (2)$$

which is a 3 × 3 symmetric matrix satisfying $K\ U = T$ relationship, where U and T are vectors representing the displacements and stresses, respectively, at the top and bottom of the water layer. In the stiffness matrix above, g is the acceleration due to gravity, ρ_w is the mass density of the water, d is the water depth, ω is the angular frequency, e is the base of the natural logarithm, and α is given by:

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{V_w^2}} \quad (3)$$

where V_w is the compression wave velocity of water and k is the wave number.

The stiffness matrix for a soil layer is obtained by relating the horizontal and vertical tractions, to the horizontal and vertical displacements at the top and bottom of the layer. Kausel and Roesset (1981) showed that in plane strain conditions, the dynamic stiffness matrix for a soil layer is a symmetric 4 × 4 matrix consisting of four 2 × 2 submatrices satisfying the following relationship:

$$K = 2kG \begin{bmatrix} [k_{11}]_{2*2} & [k_{12}]_{2*2} \\ [k_{21}]_{2*2} & [k_{22}]_{2*2} \end{bmatrix}_{4*4} \quad (4)$$

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The submatrices k_{11}, k_{12}, k_{21} and k_{22} are expressed as:

$$k_{11} = \frac{1-s^2}{2D}\left\{\begin{matrix} \frac{1}{s}(C^r S^s - rsC^s S^r) & -(1-C^r C^s + rsS^r S^s) \\ -(1-C^r C^s + rsS^r S^s) & \frac{1}{r}(C^s S^r - rsC^r S^s) \end{matrix}\right\} - \frac{1+S^2}{2}\left\{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\right\} \quad (5)$$

$$k_{12} = \frac{1-s^2}{2D}\left\{\begin{matrix} \frac{1}{s}(rsS^r - S^s) & -(C^r - C^s) \\ C^r - C^s & \frac{1}{r}(rsS^s - S^r) \end{matrix}\right\} \quad (6)$$

$$k_{21} = \frac{1-s^2}{2D}\left\{\begin{matrix} \frac{1}{s}(rsS^r - S^s) & (C^r - C^s) \\ -(C^r - C^s) & \frac{1}{r}(rsS^s - S^r) \end{matrix}\right\} \quad (7)$$

$$k_{22} = \frac{1-s^2}{2D}\left\{\begin{matrix} \frac{1}{s}(C^r S^s - rsC^s S^r) & (1-C^r C^s + rsS^r S^s) \\ (1-C^r C^s + rsS^r S^s) & \frac{1}{r}(C^s S^r - rsC^r S^s) \end{matrix}\right\} + \frac{1+S^2}{2}\left\{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\right\} \quad (8)$$

and
$$D = 2(1 - C^r C^s) + (\frac{1}{rs} + rs) \quad (9)$$

where, $r = \sqrt{1-(\omega / kC_p)^2}$, $s = \sqrt{1-(\omega / kC_s)^2}$, C^r = *Cosh krh* , S^r = *Sinh krh* , C^s = *Cosh ksh* , S^s = *Sinh ksh* , *h* is layer thickness and *G* is the shear modulus of the layer.

The soil layer stiffness matrix is a function of the frequency of the excitation (ω), the wave number (k), the soil properties and the thickness of the layer (h).

Vertical displacements of the soil at the seafloor can result in counterbalancing forces due to density contrast between soil and water. This effect is referred to as the "counter buoyancy", as mentioned before, and can be taken into account in the theoretical solution by introducing a restoring force proportional to the buoyant unit weight of the soil and the vertical displacement of the midline.

IV. Stiffness Matrix For A Soil-Water System

Stiffness matrices presented above for the water and a soil layer are used to construct the global stiffness matrix for a soil-water system. The global stiffness matrix is constructed by assembling the stiffness matrices of different layers. The scheme is shown in Fig. 2. The matrix is formulated considering the continuity of displacements and force equilibrium at the top and bottom surfaces of each layer and adjacent layers.

The global stiffness matrix for a system consisting of *n* soil layers, with the deepest layer extending to infinity (half space), and overlaid by water is a (2*n*+3) × (2*n*+3) symmetric matrix satisfying the following relationship:

$$K\,U = T \quad (10)$$

where *T* and *U* represent vectors of external stresses (tractions) and displacements, respectively, acting at the interfaces. In solving for phase velocities corresponding to plane wave propagation in a system with no external stresses (tractions), the right hand side of above equation becomes zero.

$$K\,U = 0$$

The nontrivial solution to this equation is obtained by setting the determinant of the global stiffness matrix equal to zero.

$$\det [K] = 0$$

This equation is the system characteristic equation relating the phase velocities to the wavelengths through sets of elastic parameters for the soil layers and water. The roots of this equation represent the velocity of possible propagating waves in the system.

$S =$

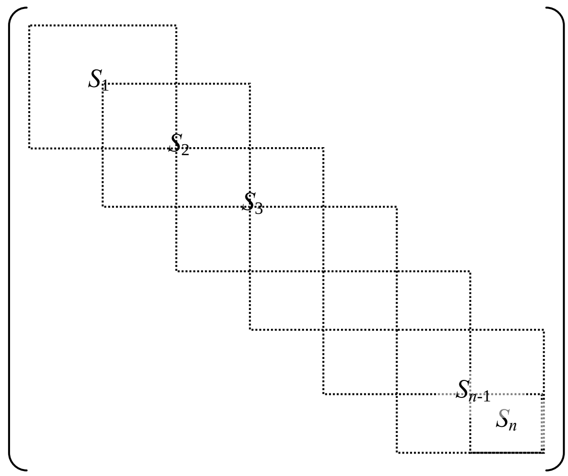

Figure 2: Global stiffness matrix for a layered system

An iterative procedure is employed to obtain the solution. This procedure involves assuming a trial velocity and computing the corresponding determinant of the global stiffness matrix for the system. The value for the first trial velocity is calculated using linear wave theory for a rigid base. New trial velocities are obtained by adding a velocity increment sequentially to the previous trial velocity. The velocity increment is computed by dividing the difference between the maximum shear wave velocity of the materials present in the system and the first trial velocity into a large number of increments. The existence of a root is identified when the determinants of two successive trial velocities display opposite sign. The convergence of the solution for phase velocity is based on the ratio of the change in phase velocity to the previous value of phase velocity. Convergence is considered to have been reached when this ratio is smaller than a given tolerance. A tolerance of a 10^{-6} was used in this analyses.

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V. Computer Program

A computer program named *SWundWat* was written using *MATLAB* software to implement the solutions for a layered system. The program employs the iteration procedure described earlier to obtain a solution consisting of the phase velocities and displacements of the system interfaces.

The flowchart shown in Fig. 3 illustrates the trend and steps required for implementing the program and plotting the dispersion curve of the layered system.

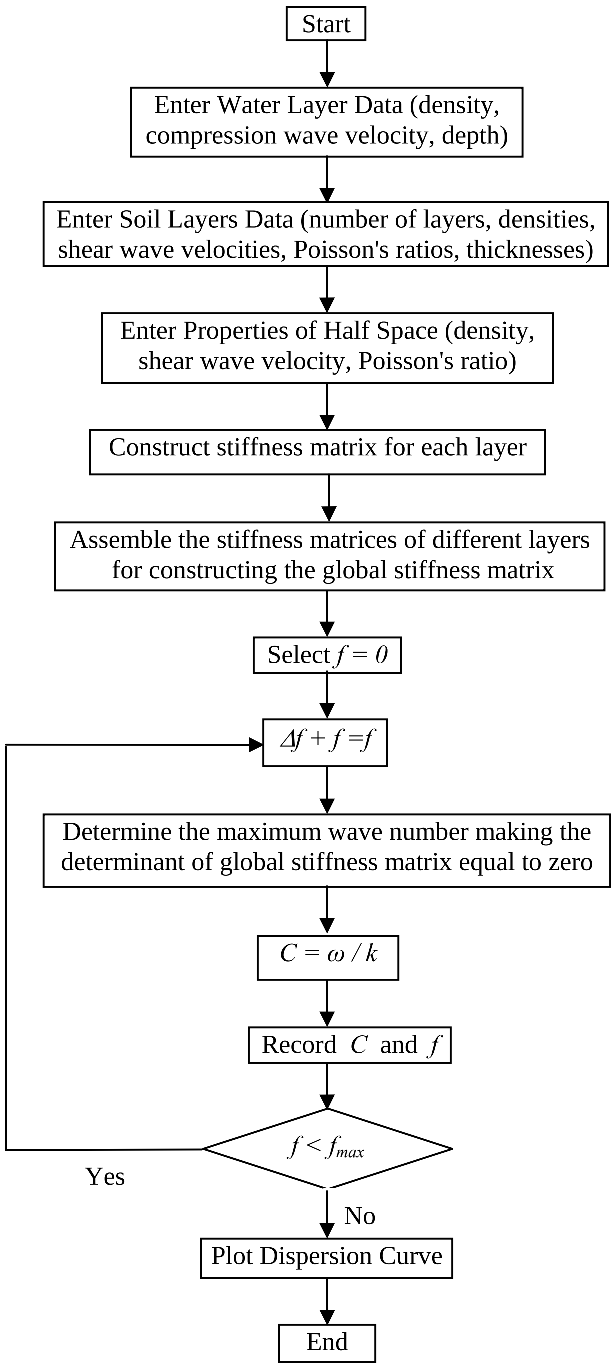

Figure 3: Flowchart of computer program operations

VI. Application To Experimental Results

In order to apply and investigate the applicability of the developed computer program for constructing the theoretical dispersion curve of a layered water-soil structure, an artificial four-layer half space soil system overlaid by water has been modeled and investigated. Figure 4 shows the assumed layered profile. Also shown in Tab. I are the different values required for each layer used in the construction of the dispersion curve of assumed layered model. Figure 5 shows the theoretical dispersion curve of the system, obtained using the developed program. The plotted theoretical dispersion curves were generated in deep water conditions (water depth of 1000 m). In deep water, the effect of water depth on Scholte wave velocity is negligible, i.e. Scholte wave velocity can be considered to be independent of the wavelength [Sedighi-Manesh, et al., 1992].

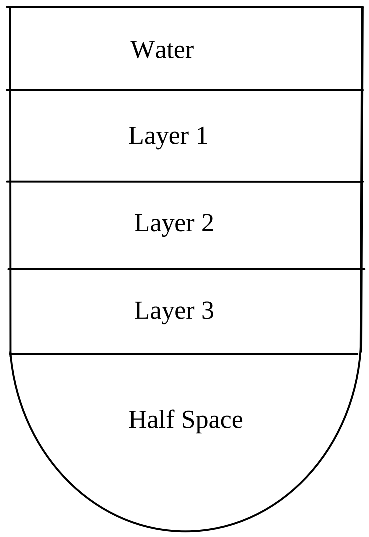

Figure 4: Water-soil layered profile

Table I: Parameters of water-soil layers

Layer	*Thickness [m]*	*Shear Wave Velocity [m/sec]*	*Poisson's Ratio*	*Density [gr/cm^3]*
Water	100	*	-	1
1	10	250	0.25	1.7
2	20	300	0.25	1.8
3	15	320	0.25	1.95
Half space	-	350	0.25	1.95

* Water compression wave velocity = 1500 *m/sec*

Besides, as an application two more cases selected from published set of data is presented and studied in this research work. The aim is to verify and also demonstrate the applicability and effectiveness of the developed program and the procedure employed in the development of the program.

The under water soil layers data shown in Tab. II are based on the experimental results given by Rosenblad [2000]. The results were obtained from experiments performed in the Strait of Georgia off the coast of Vancouver B. C. (Canada), at two different test locations termed Site A and Site B (having water depth of 40 ft and 175 ft, respectively). The results from site A were obtained using SASW and SCPT tests and the results from site B were measured using only SASW tests. The Poisson's ratio and unit weight for materials of both sites layers were assumed to be 0.48 and 100 *pcf*, respectively.

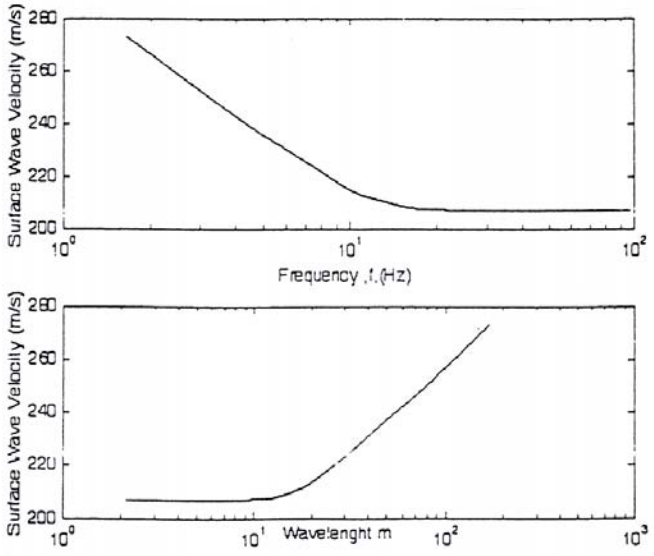

Figure 5: Theoretical dispersion curves for assumed model

Table II: Under water soil layers data from field measurements

Soil Layer	*Site A*		*Site B*	
	Thickness [ft]	*Shear Wave Velocity [ft/sec]*	*Thickness [ft]*	*Shear Wave Velocity [ft/sec]*
1	3.5	145	3.3	180
2	9	320	10	290
3	20	550	15	410
4	Half space	550	Half space	410

Tables III and IV compare the results between the measured phase velocities and calculated phase velocities (using the developed program) for both sites A and B. The results have been shown for the same values of wavelengths and comparisons are being made for six different values of wavelengths. As can be seen from the tables, the calculated results and the measured data are in excellent agreement. These results indicate that the program is working well and can be used effectively in the performance of surface wave method under water and offshore geotechnical explorations.

Table III: Comparison of measured and calculated results (Site A)

Wavelength [ft]	*Field Test Phase Velocity [ft/sec]*	*Calculated Phase Velocity [ft/sec]*
1	130	129.87
8	160	160.82
20	240	240.64
30	300	300.67
40	340	340.54
70	420	420.94

Table IV: Comparison of measured and calculated results (Site B)

Wavelength [ft]	*Field Test Phase Velocity [ft/sec]*	*Calculated Phase Velocity [ft/sec]*
5	165	165.34
7	180	180.64
10	200	200.48
20	240	240.35
30	280	280.82
40	300	300.23

VII. Summary and Conclusions

The surface wave method is a nondestructive and nonintrusive technique that can be used to determine shear stiffness profiles of the seafloor. Development of the method for offshore applications is in its early stages. The depth to which profiling can be performed is controlled by the range in wavelengths generated by the source and the spacings between the source and the receivers. Deeper profiles require longer array spacings and larger sources generating lower frequency ranges.

In order to develop and apply the surface wave method under water, a computer program named *SWundWat* has been developed during this research work and used for constructing the theoretical dispersion curve of the layered model. This theoretical dispersion curve is then used to make the required comparisons with the experimental dispersion curve obtained in the field for the purpose of determining the real site profile. By employing the developed program, an artificial layered model as well as two examples of real sites have been studied and the results obtained have been compared with the available actual data. The results show very good match indicating that the developed program is working well and can be used for profiling the sites under investigation.

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